4.84-, 4.90-, and 4.93-Mev levels[14] in Si^{29}. Below 1.2 Mev the curve rises because the 875-kev gamma ray from O^{16} is much stronger than any of the gamma rays from Si^{28}. When the region in the neighborhood of the change of slope marked 1.25 Mev in the upper curve was investigated under more favorable conditions of counting rate and window width, a peak was found at a gamma-ray energy of 1.25 Mev. This probably comes from the $Si^{28}(d,p)Si^{29}$ reaction and represents the decay to the ground state of the 1.28-Mev level[14] in Si^{29}.

[14] Endt, Van Patten, Buechner, and Sperduto, Phys. Rev. 83, 491 (1951).

In the energy region between 1.25 and 3.91 Mev, other gamma rays are indicated, but it is difficult to be conclusive for at least two reasons: (1) a little carbon contamination would give a big effect in comparison to the effect from Si^{28}, and the 3.08-Mev gamma ray from $C^{12}(d,p)C^{13}$ has peaks at 2.06, 2.57, and 3.08 Mev; (2) Si^{29} has five excited states in this region by $Si^{28}(d,p)Si^{29}$,[14] and it would be difficult to resolve them since one gamma ray may have as many as three peaks.

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Nuclear Forces and β Decay Matrix Elements

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The matrix elements of β decay are reduced to nonrelativistic form under the assumption that the nuclear Hamiltonian contains interactions involving odd operators. Particular attention is given to an interaction which gives rise to a spin orbit effect.

THE effect of nuclear forces on the β-decay matrix elements which involve odd operators has been discussed by a number of authors.[1-7] The matrix element of prime interest in these papers has been the pseudoscalar matrix element.

Peaslee[3] and Rose and Osborn[6] have shown that if the nuclear forces are due to two-body interactions and if these interactions are limited to even-even or odd-odd operators, then the nuclear forces have little effect upon the nonrelativistic form of the matrix elements.

Since there exists no satisfactory treatment of the relativistic two-nucleon problem, this note is limited to a consideration of the Hamiltonian of a nucleon interacting with various types of fields. The Foldy-Wouthysen transformation[8,9] has been employed to reduce the problem to a Schroedinger-Pauli form. This method is essentially the same as that used by Rose and Osborn.[6]

The nuclear Hamiltonian may be written as

$$H=-\beta M-\boldsymbol{\alpha}\cdot\mathbf{p}+\mathcal{E}+\mathcal{O};\tag{1}$$

$\mathcal{E}$ indicates interactions containing only even operators and $\mathcal{O}$, odd operators.[10] Interactions involving only even operators affect the nonrelativistic form to the matrix elements by adding only terms of order $1/M^2$, and these should be negligible.[6] However, Konopinski[7] has suggested that if the nuclear "Thomas" term,[11] a second-order term, is for some reason large enough to account for the nuclear inverted doublets, then one would also expect the effect, also second order, on the pseudoscalar matrix element of the scalar nuclear potential to be enhanced. Rather than consider such anomalous effects, we will consider only contributions of terms in $\mathcal{O}$ which are of first order in $1/M$.

In general $\mathcal{O}$ has the form

$$\mathcal{O}=\boldsymbol{\alpha}\cdot\mathbf{V}+i\beta\boldsymbol{\alpha}\cdot\mathbf{F}+\gamma_5\theta+i\beta\gamma_5\varphi.\tag{2}$$

The $\mathbf{V}$, $\mathbf{F}$, θ, and φ are field quantities with the proper transformation properties.

In the low energy limit, the β decay matrix elements become[2]

$$\int\boldsymbol{\alpha}\approx-(1/M)\int[\mathbf{p}-\mathbf{V}-\boldsymbol{\sigma}\times\mathbf{F}-\theta\boldsymbol{\sigma}],\tag{3a}$$

$$i\int\beta\boldsymbol{\alpha}\approx(1/M)\int[\boldsymbol{\sigma}\times\mathbf{p}+\boldsymbol{\sigma}\times\mathbf{V}-\mathbf{F}-\varphi\boldsymbol{\sigma}],\tag{3b}$$

$$\int\gamma_5\approx(1/M)\int[\boldsymbol{\sigma}\cdot\mathbf{p}-\boldsymbol{\sigma}\cdot\mathbf{V}-\theta],\tag{3c}$$

[1] M. Ruderman, Phys. Rev. 89, 1227 (1953).
[2] R. Herbst and A. Bushkovitch, Phys. Rev. 91, 442 (1953); also thesis, St. Louis University, 1953 (unpublished).
[3] D. C. Peaslee, Phys. Rev. 91, 1447 (1953).
[4] H. Takebe, Progr. Theoret. Phys. (Japan) 10, 673 (1953).
[5] Alaga, Kofoed-Hansen, and Winther, Kgl. Danske Videnskab. Selskab, Mat-fys. Medd. 28, No. 3 (1953).
[6] M. E. Rose and H. K. Osborn, Phys. Rev. 93, 1315 (1954).
[7] E. J. Konopinski, Phys. Rev. 94, 492 (1954).
[8] L. L. Foldy and S. A. Wouthysen, Phys. Rev. 78, 29 (1950).
[9] W. Barker and Z. Chraplyvy, Phys. Rev. 89, 446 (1953).

[10] We have set $\hbar=c=m=1$, where m is the mass of the electron.
[11] D. R. Inglis, Phys. Rev. 50, 783 (1936); H. W. Furry, Phys. Rev. 50, 784 (1936).

and

$$i\int \beta\gamma_5 \approx -(1/M)\int [\boldsymbol{\sigma}\cdot\mathbf{F}-\varphi]. \tag{3d}$$

The first terms in (3a), (3b), and (3d) are independent of the nuclear interactions and are in agreement with Rose and Osborn[6] and Ahrens and Feenberg.[12] No such term appears in (3d) because all terms containing derivatives of the lepton covariants have been discarded.[13]

The obvious extension to higher-order forbidden matrix elements is valid. For example,

$$\int \boldsymbol{\alpha}\times\mathbf{r} \approx \frac{1}{M}\int [\mathbf{r}\times\mathbf{p}-\mathbf{r}\times\mathbf{V}-\mathbf{r}\times(\boldsymbol{\sigma}\times\mathbf{F})-\theta\mathbf{r}\times\boldsymbol{\sigma}]. \tag{3c}$$

One must now give some interpretation to the quantities $\mathbf{V}$, $\mathbf{F}$, θ, and φ. These could be considered as the field variables of quantized fields. Ruderman[1] discussed the last term in (3d) from this point of view. In this case φ is the wave function of the pseudoscalar meson field. Any attempt to give such an interpretation is, however, subject to all of the uncertainties of meson field theories.

As a first approximation one may attempt to treat $\mathbf{V}$, $\mathbf{F}$, θ, and φ as classical fields. However, it is difficult to understand how θ or φ can be central and still conserve parity. Vector interactions ($\mathbf{V}$) give rise to velocity-dependent forces, and little is known about the existence of such forces in the nucleus. The tensor interaction ($\mathbf{F}$) lends itself to the most immediate interpretation.

Gaus[14] has shown that if the nuclear Hamiltonian contains a term $i\beta\boldsymbol{\alpha}\cdot\mathbf{F}$, then the nonrelativistic Hamiltonian will contain a spin-orbit term. If $\mathbf{F}$ is set equal to $f(r)\mathbf{r}$, then in the nonrelativistic limit the Hamiltonian will contain the terms

$$-[f(r)/M]\boldsymbol{\sigma}\cdot\mathbf{r}\times\mathbf{p} \\ -(1/2M)[f^2(r)r^2+f'(r)r+3f(r)]. \tag{4}$$

The last terms contribute only a small amount to the central nuclear potential, and so in a phenomenological theory the magnitude of $f(r)$ may be adjusted to fit known spin orbit energies.

[12] T. Ahrens and E. Feenberg, Phys. Rev. **86**, 64 (1952).
[13] Ahrens, Feenberg, and Primakoff, Phys. Rev. **87**, 663 (1952).
[14] H. Gaus, Z. Naturforsch. **7a**, 44 (1952).

If such a term is considered, (3d) becomes

$$i\int \beta\gamma_5 \approx -(1/M)\int f(r)\boldsymbol{\sigma}\cdot\mathbf{r}. \tag{3d$'$}$$

This is of the same form as the matrix element derived by Konopinski[7] in connection with the "Thomas" term. If we use the estimate for the spin-orbit energies that the average of $f(r)/M$ lies between 2 and 4, then we see that the pseudoscalar matrix element and also the terms in (3a) and (3b) involving $\mathbf{F}$ are not negligible.

It might be mentioned here that a term $i\beta\boldsymbol{\alpha}\cdot\mathbf{r}f(r)$ also affects the electromagnetic properties of a proton. In an electromagnetic field a new term appears in the low energy limit*

$$(e/M)f(r)\boldsymbol{\sigma}\cdot\mathbf{r}\times\mathbf{A}. \tag{5}$$

If $\mathbf{A}$ were, for example, the vector potential of a constant magnetic field, the magnetic moment of the proton may be calculated, and the g factor becomes

$$g=1\pm\frac{2\mu_p-1}{2l+1}\mp\langle f(r)r^2\rangle\frac{j+\frac{1}{2}}{j(j+1)};\ j=l\pm\tfrac{1}{2}. \tag{6}$$

The first two terms are the usual g factor while the last is a correction due to the tensor interaction. Since $\langle f(r)r^2\rangle\approx 1$, this effect tends to lower the $j=l+\frac{1}{2}$ Schmidt line and raise the $j=l-\frac{1}{2}$ line by about $\frac{1}{2}$ a nuclear magneton. This "suppression" of proton moments does not seem large enough to be discerned among the many uncertainties about nuclear moments.

These considerations indicate that the nonrelativistic form of the β-decay matrix elements is quite dependent upon the assumptions about nuclear forces and that even an order of magnitude calculation to the pseudoscalar matrix element must wait until much more is known of nuclear forces.

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* *Note added in proof.*—Jensen and Goeppert-Mayer, Phys. Rev. **85**, 1040 (1952), have shown that this electromagnetic effect should exist for any spin-orbit interaction.