Pairing Effects in Coulomb Energies and the Radii of Mirror Nuclei*

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(Received June 16, 1954)

The Coulomb energy difference between the nuclei of a mirror pair exhibits an odd-even alternation with Z that is presumed to reflect the well-known pairing property of the short-range nuclear forces. By taking second differences of Coulomb energy (differences between successive mirror pairs), the alternation is seen to continue to at least $Z=15$, and additional irregularities appear that may be shell-structure effects. The analysis of Feenberg and Goertzel is discussed from the point of view of the shell model, and the pairing of spins is extended to the spherically symmetric pairing characteristic of the state of lowest seniority. A harmonic oscillator model with jj coupling is used to calculate the Coulomb energy, including exchange effects, in the state of lowest proton seniority. The single parameter of the model is determined by comparison with experimental data and remains constant to ± 1.5 percent through the $p_{1/2}$ and $d_{5/2}$ shells. The rms radius of the nuclear charge distribution is calculated by the same model. Between C^{13} and Al^{27}, the equivalent r_0 decreases fairly smoothly from 1.34 to 1.20. For $A \leq 11$ the model is not satisfactory, and for $A \geq 31$ there are some serious inconsistencies in the data. The most recent data indicate that r_0 may decrease to the range 1.1 to 1.15 for $A \cong 39$.

I. INTRODUCTION

THE Coulomb energy of a nucleus of atomic number Z has been written by Feenberg and Goertzel[1] in the form,

$$E_c(Z)=\tfrac{1}{2}Z(Z-1)a+[\tfrac{1}{2}Z]b, \quad (1)$$

where $[\frac{1}{2}Z]$ denotes the largest integer not exceeding $\frac{1}{2}Z$. The first term represents the Coulomb energy of Z equivalent protons without consideration of symmetry effects. The second term arises ultimately from the Pauli exclusion principle and represents the correlation or pairing energy resulting from the fact that two protons have a larger probability of being found close together if their spins are oppositely directed. Unlike the force between electrons in atomic spectroscopy, the nuclear forces are attractive and favor the pairing of spins. The Coulomb forces between protons are far too weak to prevent pairing; however, since their energy depends on the spatial correlations, its odd-even alternation is a symptom and a rough measure of the pairing.

The parameters a and b are not expected to be exactly constant, but if only the ground states of the mirror nuclei are considered, one may hope that a and b will vary only slowly with Z. The difference in Coulomb energy between the two nuclei of a mirror pair will then be

$$\Delta_1(Z)=E_c(Z)-E_c(Z-1)$$
$$=(Z-1)a+\tfrac{1}{2}[1+(-1)^Z]b. \quad (2)$$

Different mirror pairs can be conveniently compared[1] by dividing Δ_1 by $Z-1$, but this procedure has the disadvantage of making the pairing effects inconspicuous in the range of higher Z. In order to exhibit the pairing effects clearly, it is better to take second differences, in which the first term is no longer weighted by $Z-1$:

$$\Delta_2(Z)=\Delta_1(Z)-\Delta_1(Z-1)=a+(-1)^Z b. \quad (3)$$

Equation (3) will serve, in Secs. II and III, as a guide in examining the experimental data, which have improved considerably since the paper of Feenberg and Goertzel. In Sec. IV we shall criticize the definitions of a and b from the point of view of the shell model and introduce modified definitions that seem to be more appropriate to this model. In Sec. V the crude but very convenient assumption of an isotropic harmonic oscillator potential will be used to calculate the modified a and b. The calculations will be compared with the experimental pairing effect; in addition, they will provide a prescription for obtaining nuclear radii from observed Coulomb differences.

II. EXPERIMENTAL DATA

The mass difference between the nuclei of a mirror pair can be obtained from experimental Q values of nuclear reactions, especially the (p,n) reaction, or from the β-decay end-point energy. After correction for the n-H^1 difference, the mass difference is then interpreted as the difference in Coulomb energy by the assumption of charge symmetry of the nuclear forces. In most cases, accurate Q values are available for low A ($A \leq 27$). For higher A, one must rely primarily on end-point determinations of the short-lived positron activities. If T_0 is the end-point kinetic energy of a positron spectrum, the first Coulomb energy difference Δ_1 is $T_0+(1.804\pm 0.001)$ Mev, since the n-H^1 mass difference is 0.782 ± 0.001 Mev and two electron masses account for 1.022 Mev. Alternatively, Δ_1 is just the negative of the Q value for a (p,n) reaction.

For $A \leq 21$, we have used the adjusted Q values

* This work was supported in part by the U. S. Atomic Energy Commission and the Higgins Scientific Trust Fund.
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[1] E. Feenberg and G. Goertzel, Phys. Rev. **70**, 597 (1946).

given by Li *et al.*[2] and Li.[3] For $A \geq 23$, a list of experimental values of Δ_1 is given in Table I. Where more than one measurement is available, the adopted value listed in the second column of Table II is an average weighted by the reciprocal squares of the quoted errors.[4] Thus, if the individual measurements are denoted by $a_i \pm \sigma_i$, where σ_i is the error quoted by the author, the adopted value is $\bar{a} \pm \bar{\sigma} = \sum_i w_i a_i \pm (\sum_i \sigma_i^{-2})^{-\frac{1}{2}}$, where $w_i = \bar{\sigma}^2/\sigma_i^2$. Measurements for which no limits of error are quoted are included in Table I for comparison, but have not been used in obtaining the adopted value. The averaging procedure is not entirely satisfactory when systematic errors are present, and $\bar{\sigma}$ tends to underestimate the actual uncertainty. In the cases of S^{31} and Ca^{39}, the values given by references Hu 54 and Br 53 are so much higher than the earlier measurements that it would be meaningless to take a weighted average. These higher-energy data have therefore been treated separately throughout, and are indicated by parentheses in Table II.

The second differences defined in Eq. (3) are listed in the third column of Table II and are plotted against Z in Fig. 1; it should be remembered that Z denotes the largest of the three atomic numbers involved in a second difference. The vertical line through a plotted point indicates the error of the second difference, computed from the errors of the two first differences by taking the square root of the sum of their squares. A vertical line is omitted when the computed error is less than 10 kev. Points for successive even (or odd) values of Z have been connected by straight lines.

III. DISCUSSION

The principal feature of Fig. 1 is that the line connecting points of even Z lies distinctly above the line for odd Z up to $Z=15$ (or 18, according to the recent measurement of S^{31} by Hunt *et al.*). This separation shows clearly the odd-even pairing effect up to this point; the less accurate data available to Feenberg and Goertzel[1] would have maintained the separation only up to $Z=8$.

The second point of interest is that between certain even values of Z, the fluctuations are conspicuously larger than between others. There are sharp drops between $Z=6$ and 8 and between 14 and 16, which might be associated with the closing of $p_{3/2}$ and $d_{5/2}$ shells at 6 and 14, respectively. In each case the next subshell to be filled ($p_{1/2}$ or $s_{1/2}$) contains two protons. The fluctuations in the line for odd Z are less marked, although there seems to be a distinct drop between $Z=7$ and 9, and another drop somewhere between 17 and 21. These drops suggest that the wave function of the ninth, and possibly the twenty-first, proton has a relatively small overlap with the core, as might be expected if a new shell begins after $Z=8$ or 20. These interpretations do not explain why changes occur after 6 and 14 in the line for even Z but not across 6 and 14 in the line for odd Z, and *vice versa* for 8. Moreover, the pattern is very different if the higher result for the

[2] Li, Whaling, Fowler, and Lauritsen, Phys. Rev. **83**, 512 (1951).
[3] C. W. Li, Phys. Rev. **88**, 1038 (1952).
[4] E. R. Cohen, Revs. Modern Phys. **25**, 709 (1953).

TABLE I. Coulomb energy differences Δ_1 for mirror pairs with $A \geq 23$. The end-point kinetic energy of the positron spectrum is $T_0 = \Delta_1 - (1.804 \pm 0.001)$ Mev.

Positron emitter	T_0 (Mev)	Δ_1 (Mev)	Method[a]	Reference[b]
Mg^{23}	2.82	4.62	cc	Wh 39
	2.78	4.58 ±0.3	pn	Wh 39
	3.17	4.97	pn	Bl 51
	2.99	4.79 ±0.09	scin	Bo 51
	3.073	4.877±0.010	pn	Wi 52
	2.95	4.75 ±0.07	scin	Hu 54
Al^{25}	3.23	5.03 ±0.06	d	Li 52, Go 53
Si^{27}	3.74	5.54	cc	Mc 40
	4.0	5.8 ±0.1	pn	Mc 40
	3.54	5.34 ±0.1	cc	Ba 40
	3.92	5.72	pn	Bl 51
	3.48	5.28 ±0.10	scin	Bo 51
	3.805	5.609±0.010	pn	Ki 53
	3.76	5.56 ±0.08	scin	Hu 54
P^{29}	3.63	5.43 ±0.07	cc	Wh 41
	5.25	7.05 ±0.1	d	Pe 48, Li 52
	4.16	5.96 ±0.04	d	Ma 52, Li 52
	3.945	5.749±0.010	s	Ro 53
	3.9	5.7 ±0.2	scin	Na 53
S^{31}	3.85	5.65 ±0.07	cc	Wh 41
	3.87	5.67 ±0.15	cc	El 41
	4.06	5.86 ±0.12	scin	Bo 51
	4.50	6.30 ±0.10	scin	Hu 54
Cl^{33}[c]	4.13	5.93 ±0.07	cc	Wh 41
	4.2	6.0 ±0.2	scin	Na 53
A^{35}	4.38	6.18 ±0.07	cc	Wh 41
	4.41	6.21 ±0.09	cc	El 41
K^{37}[c]	...	...		
Ca^{39}	5.13	6.93 ±0.15	scin	Bo 51
	6.7	8.5 ±0.5	a	Br 53
	6.10	7.90 ±0.15	scin	Hu 54
Sc^{41}	4.94	6.74 ±0.07	cc	El 41

[a] Methods: cc, cloud chamber; s, magnetic spectrometer; scin, scintillation spectrometer; a, absorption; pn, $\Delta_1 = -Q_{pn}$; d, $\Delta_1 = Q_{dp} - Q_{dn}$.
[b] We are indebted for several of the following references to a privately circulated list compiled by Professor H. T. Richards.

Ba 40 Barkas, Creutz, Delsasso, Sutton, and White, Phys. Rev. **58**, 383 (1940).
Bl 51 Blaser, Boehm, Marmier, and Scherrer, Helv. Phys. Acta **24**, 465 (1951).
Bo 51 F. I. Boley and D. J. Zaffarano, Phys. Rev. **84**, 1059 (1951).
Br 53 R. Braams and C. L. Smith, Phys. Rev. **90**, 995 (1953).
El 41 D. R. Elliott and L. D. P. King, Phys. Rev. **60**, 489 (1941).
Go 53 E. Goldberg, Phys. Rev. **89**, 760 (1953).
Hu 54 Hunt, Kline, and Zaffarano, Phys. Rev. **95**, 611 (1954) and W. A. Hunt, Ph.D. thesis, Iowa State College, 1954 (unpublished).
Ki 53 Kington, Bair, Carlson, and Willard, Phys. Rev. **89**, 530 (1953).
Li 52 C. W. Li, Phys. Rev. **88**, 1038 (1952).
Ma 52 Mandeville, Swann, Chatterjee, and Van Patter, Phys. Rev. **85**, 193 (1952).
Mc 40 McCreary, Kuerti, and Van Voorhis, Phys. Rev. **57**, 351 (1940).
Na 53 M. Nahmias and T. Yuasa, Compt. rend. **236**, 2399 (1953).
Pe 48 R. A. Peck, Jr., Phys. Rev. **73**, 947 (1948).
Ro 53 H. Roderick and C. Wong, Phys. Rev. **92**, 204 (1953).
St 53 P. Stähelin and P. Preiswerk, Nuovo cimento **10**, 1219 (1953).
St 53a P. Stähelin, Helv. Phys. Acta **26**, 691 (1953).
Wh 39 White, Delsasso, Fox, and Creutz, Phys. Rev. **56**, 512 (1939).
Wh 41 White, Creutz, Delsasso, and Wilson, Phys. Rev. **59**, 63 (1941).
Wi 52 Willard, Kington, and Bair, Phys. Rev. **86**, 259 (1952).

[c] Activities originally attributed to Cl^{33} and K^{37} in reference Bo 51 have been assigned to the ground states of Cl^{34} and K^{38} by references St 53, St 53a, and Hu 54.

TABLE II. Adopted values of Coulomb energy differences, Δ_1, and second differences, $\Delta_2(Z)=\Delta_1(Z)-\Delta_1(Z-1)$. Values of the energy parameter of the harmonic oscillator model, $e^2(\nu/\pi)^{\frac{1}{2}}=\Delta_1/\xi(A)$. Radii of the mirror nuclei of smaller Z in terms of $r_0=\sigma(A)/\Delta_1$. Values in parentheses refer to the higher-energy data for S^{31} (reference Hu 54 of Table I) and Ca^{39} (references Br 53 and Hu 54).

Nucleus of larger Z	Δ_1 (Mev)	Δ_2 (Mev)	$\xi(A)$	$e^2(\nu/\pi)^{\frac{1}{2}}$ (kev)	Nucleus of smaller Z	$\sigma(A)$	r_0
$_2He^3$	0.764±0.001	0.764±0.001	2.000	381.9±0.5	H^3	1.260	1.649±0.002
$_3Li^5$	0.8 ±0.3	0.04 ±0.3	3.000	270 ±100	He^5	1.593	2.0 ±1.0
$_4Be^7$	1.645±0.001	0.85 ±0.3	4.667	352.5±0.2	Li^7	2.450	1.489±0.001
$_5B^9$	1.852±0.002	0.207±0.002	5.667	326.8±0.4	Be^9	2.857	1.543±0.002
$_6C^{11}$	2.762±0.003	0.910±0.004	7.333	376.6±0.4	B^{11}	3.543	1.283±0.002
$_7N^{13}$	3.003±0.002	0.241±0.004	8.667	346.5±0.2	C^{13}	4.023	1.340±0.001
$_8O^{15}$	3.487±0.005	0.484±0.005	10.167	343.0±0.5	N^{15}	4.549	1.305±0.002
$_9F^{17}$	3.549±0.006	0.062±0.008	10.350	342.9±0.6	O^{17}	4.477	1.262±0.002
$_{10}Ne^{19}$	4.038±0.005	0.489±0.008	11.832	341.3±0.4	F^{19}	5.082	1.259±0.002
$_{11}Na^{21}$	4.30 ±0.03	0.27 ±0.03	12.643	340 ±3	Ne^{21}	5.373	1.248±0.009
$_{12}Mg^{23}$	4.873±0.010	0.57 ±0.03	14.125	345.0±0.7	Na^{23}	5.928	1.217±0.003
$_{13}Al^{25}$	5.03 ±0.06	0.16 ±0.06	14.937	337 ±4	Mg^{25}	6.186	1.230±0.015
$_{14}Si^{27}$	5.604±0.010	0.57 ±0.06	16.418	341.3±0.6	Al^{27}	6.706	1.197±0.002
$_{15}P^{29}$	5.767±0.010	0.163±0.010	18.113	318.4±0.6	Si^{29}	7.297	1.265±0.002
$_{16}S^{31}$	5.70 ±0.06	−0.07 ±0.06	19.477	293 ±3	P^{31}	7.727	1.356±0.015
	(6.30 ±0.10)	(0.53 ±0.10)		(323 ±5)			(1.23 ±0.02)
$_{17}Cl^{33}$	5.94 ±0.07	0.24 ±0.09	20.098	296 ±4	S^{33}	7.878	1.33 ±0.02
		(−0.36 ±0.12)					
$_{18}A^{35}$	6.19 ±0.06	0.25 ±0.09	21.508	288 ±3	Cl^{35}	8.320	1.344±0.013
$_{19}K^{37}$	...	...	22.311	...	A^{37}	8.520	...
$_{20}Ca^{39}$	6.93 ±0.15	...	23.721	292 ±6	K^{39}	8.944	1.29 ±0.03
	(7.95 ±0.14)			(335 ±6)			(1.13 ±0.02)
$_{21}Sc^{41}$	6.74 ±0.07	−0.19 ±0.17	22.985	293 ±3	Ca^{41}	8.562	1.270±0.013
		(−1.21 ±0.16)					
$_{22}Ti^{43}$	...	...	24.341	...	Sc^{43}	9.029	...

S^{31} end point is correct. The drop after 14 then disappears, and a large drop occurs between 15 and 17.

Since a large Coulomb energy indicates a close association between the protons, it suggests a strong nuclear interaction as well; in fact, the investigation of odd-even Coulomb effects was stimulated by the known pairing property of the nuclear energy. It is therefore of interest to compare the variations in Coulomb energy with variations in the nuclear binding energy of the last nucleon. Redlich[5] has plotted the binding energy of the last proton as a function of Z (for several fixed values of $N-Z$), and similarly that of the last neutron as a function of N. If allowance is made for the average trend of the curves, a neutron curve of given $N-Z$ shows a pattern of variations similar to that of the proton curve with the same value for $Z-N$. The average trend is of course different for protons and neutrons because of the Coulomb energy, but on the scale of energies of interest here, the Coulomb energy is a very smoothly varying quantity and has a negligible effect on irregularities in the binding energy. For the mirror nuclei, in particular, the binding energy of the last neutron for $N-Z=1$ is just the binding energy of the last proton for $N-Z=-1$ plus the first Coulomb energy difference $\Delta_1(Z)$.

The pattern of variations in the binding energy contains several features exhibited by the curves of Fig. 1. In Redlich's curves for even Z or N, there are sharp drops between 6 and 8 and between 14 and 16 (as well as between 28 and 30). In the curves for odd Z or N, there are dips at 9 and some indication of dips at 21.

[5] M. G. Redlich, Phys. Rev. **88**, 38 (1952).

IV. THEORY

The two-parameter expression [Eq. (1)] obtained by Feenberg and Goertzel for the total Coulomb energy represents in a very satisfactory way the two main features of the experimental data, namely, the average trend of the first differences and the even-odd alternation of the second differences. In order to describe finer details of the data, one could require a and b to be slowly varying functions of Z and proceed to determine them from the experimental values of Δ_1. However, this procedure will not give a satisfactory representation of the second differences, since these do not change slowly. In fact, the parameters of Eqs. (1), (2), and (3) can be identified with one another only insofar as they do vary slowly with Z. Rapid variations, whether large or small, will be enhanced by the process of taking differences. Perhaps the opposite point of view is more appropriate. The parameters of Eq. (3) may have a more direct interpretation and a greater interest than those of Eqs. (2) or (1), in which the variations have been smoothed out by the process of taking sums. Such is the case in the shell model, which we shall consider after a brief review of Feenberg and Goertzel's derivation of Eq. (1).

Their model is that of the supermultiplet theory. The wave function of the ground state has maximum orbital symmetry but is not specified in detail, and the total proton spin is a good quantum number with the value $\frac{1}{2}$ for odd Z and 0 for even Z. This means that

as many protons as possible are paired off with oppositely directed spins; there are $[\frac{1}{2}Z]$ such pairs, and one extra proton if Z is odd. A descriptive interpretation of the analysis can be given as follows. The total number of proton-proton bonds, or couplings, is $N_t=\frac{1}{2}Z(Z-1)$. Of these, $N_p=[\frac{1}{2}Z]$ are bonds between paired protons; they are completely antisymmetric in spin and symmetric in orbital coordinates. The remaining N_t-N_p are "statistical bonds," in the sense that each one is three-quarters spin-symmetric and one-quarter spin-antisymmetric, in accordance with the statistical weights of the triplet and singlet spin states. If L_c denotes the average Coulomb energy of a statistical bond, and L_s that of an orbitally symmetric bond, then the total Coulomb energy is

$$E_c=(N_t-N_p)L_c+N_pL_s=N_tL_c+N_p(L_s-L_c)$$
$$=N_tL_c+N_p\cdot\tfrac{3}{4}L_c', \quad (4)$$

where the last equality defines L_c'. This result is Eq. (1); the second term is expected to be different from zero because protons have a larger probability of being found close together if their spins are oppositely directed. Thus, we expect that L_s will be larger than $L_c=\frac{3}{4}L_a+\frac{1}{4}L_s$, where L_a is the average Coulomb energy of an orbitally antisymmetric bond (spins parallel).

In order to express the L's in terms of expectation values, we observe that if P_{ij} exchanges the spatial coordinates of protons i and j, then the total Coulomb energy of all the orbitally symmetric bonds is

$$N_sL_s=\sum_{i<j}\int\psi^*\frac{e^2}{r_{ij}}\frac{1}{2}(1+P_{ij})\psi d\tau. \quad (5)$$

The average energy L_s is just this sum divided by the number of symmetric bonds, $N_s=N_p+\frac{1}{4}(N_t-N_p)$. Similarly, L_a is obtained by changing the sign of P_{ij} and dividing by the number of antisymmetric bonds, $N_a=\frac{3}{4}(N_t-N_p)$.

It is clear that L_s and L_a, and therefore the a and b of Eq. (1), depend by definition on the state considered. The state is not uniquely specified by requiring that the total proton spin be 0 or $\frac{1}{2}$, and before making calculations with a detailed model, it will be necessary to make a further assumption about the wave function. We shall use the shell model and define Z' to be the number of protons in an unfilled shell, the remaining $Z-Z'$ protons being in lower closed shells. Then the total Coulomb energy can be written as a sum of three terms: the interaction of the protons in the closed shells, Z' times the interaction of one of the outer protons with the protons in the closed shells, and the interaction of the Z' outer protons among themselves. Anticipating that the last term will have a Z' dependence analogous to that of Eq. (1), as it certainly will formally if we apply Feenberg and Goertzel's procedure to the Z' outer protons, we can write

$$E_c=\alpha+\beta Z'+\tfrac{1}{2}Z'(Z'-1)a+[\tfrac{1}{2}Z']b, \quad (6)$$

where α and β are independent of Z'. Provided that a and b can be shown to vary slowly with Z' in the ground states of the mirror nuclei, the first Coulomb energy difference will be

$$\Delta_1=\beta+(Z'-1)a+\tfrac{1}{2}[1+(-1)^Z]b, \quad (7)$$

since $Z-Z'$ is even. The expression for the second differences obtained from Eq. (7) is then identical with Eq. (3). In order to discuss the second differences, it is therefore sufficient to consider the interaction of protons outside closed shells (except at a transition between shells). If the a and b of Eq. (1) were rigorously constant, then they would be identical with the a and b considered here. On the other hand, if the a and b considered here change from one shell to another, then the a and b of Eq. (1) will not be constant and will have a less direct interpretation in terms of the shell model.

Since the presence of neutrons outside closed shells introduces certain complications, let us first consider a hypothetical simplified problem in which all the nucleons outside closed shells are protons. If all these protons have the same orbital angular momentum, l, then the ground state determined by short range nuclear forces will be the state of lowest seniority, i.e., the state in which there are as many ^1S pairs as possible. In LS coupling, this uniquely determined state is ^1S for Z' even and ^2L, where $L=l$, for Z' odd.

The Coulomb energy in this state is given by a general formula of Racah[6] for the expectation value of any Wigner force, and is

$$E_c(l^{Z'})=\tfrac{1}{2}Z'(Z'-1)\epsilon+[\tfrac{1}{2}Z']\delta, \quad (8)$$

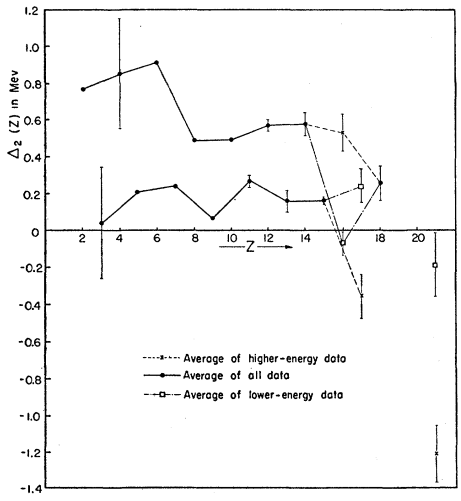

FIG. 1. Second Coulomb differences $\Delta_2(Z)=\Delta_1(Z)-\Delta_1(Z-1)$. Note that Z stands for the highest of the three atomic numbers involved; for instance, $\Delta_2(8)$ is the $O^{15}-N^{15}$ Coulomb energy difference minus the $N^{13}-C^{13}$ difference. Points for successive even (odd) values of Z are connected by straight lines. The points shown as "higher energy data" are those listed in parentheses in the third column of Table II.

[6] G. Racah, *L. Farkas Memorial Volume* (Research Council of Israel, Jerusalem, 1952), p. 294. See Eqs. (33) and (34).

where

$$\epsilon=(2l)^{-1}[(2l+1)F^0-E(l^2,{}^1S)], \tag{9a}$$

$$\delta=(2l+1)(2l)^{-1}[E(l^2,{}^1S)-F^0]. \tag{9b}$$

F^0 is the zero-order Slater integral and $E(l^2,{}^1S)$ is the interaction energy in the state $(l^2){}^1S$. The parameter ϵ vanishes in the limit of short-range forces, and δ vanishes in the long-range limit. Thus, in the state of lowest seniority, there is no pairing energy in the long-range limit, whereas all the energy is pairing energy in the short-range limit. For Coulomb forces, E_c has the expected form with values of a and b that are independent of Z' within a shell. Except at a transition between shells, Eq. (8) will give us a means of estimating the a and b of Eq. (3) after the effects of the neutrons outside closed shells have been discussed.

In jj coupling, also, E_c has the same form with constant a and b for the state of lowest seniority in the $j^{Z'}$ configuration. This state has $J=0$ for Z' even and $J=j$ for Z' odd. The expectation value of the Coulomb energy[7] is

$$E_c(j^{Z'})=\tfrac{1}{2}Z'(Z'-1)\frac{(2j+1)F^0-2E_0}{2j-1}+[\tfrac{1}{2}Z']\frac{2j+1}{2j-1}(E_0-F^0), \tag{10}$$

where E_0 now denotes the Coulomb energy in the state j^2, $J=0$.

However, the constants of Eq. (8) are not the L_c and L_c' obtained by applying Feenberg and Goertzel's definitions to the Z' protons outside closed shells. In order to obtain N_sL_s and N_aL_a separately, we can use a formula of Racah for the expectation value of any Majorana force in the state of lowest seniority.[6] In conjunction with Eq. (8), the expectation value of $\sum_{i<j}P_{ij}e^2/r_{ij}$ leads to the constant value $L_a=\epsilon$. But the expression for L_s depends in a complicated manner on Z':

$$L_s-L_a=\frac{4[\tfrac{1}{2}Z']\delta}{\tfrac{1}{2}Z'(Z'-1)+3[\tfrac{1}{2}Z']}=L_c'.$$

From these equations it follows that $L_c=\frac{1}{4}(L_s+3L_a)=L_a+\frac{1}{4}(L_s-L_a)$ is also a function of Z' and contains part of the pairing energy. In the state of lowest seniority, therefore, the quantities that are convenient for theoretical treatment are not L_c and L_c' but rather the constants ϵ and δ in Eq. (8).

The failure of L_c and L_c' to be independent of Z' may be interpreted by saying that the average Coulomb energy of a symmetrical bond changes with Z' when the state of lowest seniority is considered for each value of Z'. However, it is interesting that constant values of L_c and L_c' can be defined in terms of an average over all states in the $l^{Z'}$ configuration that have the same spin.

The possibility of taking an average with this property is shown by a procedure, due to Racah, for expressing the average energy over certain classes of states in the l^n configuration in terms of averages over similar classes in the l^2 configuration. The classes in question are characterized by the value of the Majorana operator $\sum_{i<j}P_{ij}$ (which is a good quantum number in LS coupling of protons only), or equivalently by the spin. In the l^2 configuration, the average Coulomb energy over all the space-symmetrical states (${}^1S$, ${}^1D$, $\cdots$, with $P_{12}=1$) is

$$\bar{L}_s=\epsilon+(l+1)^{-1}\delta. \tag{11}$$

The average Coulomb energy over all space-antisymmetrical states (${}^3P$, ${}^3F$, $\cdots$, with $P_{12}=-1$) is

$$\bar{L}_a=\epsilon; \tag{12}$$

this is the same L_a that we obtained before for the state of lowest seniority.[8]

The average energy over either class of states can be expressed in the unified form,

$$\bar{E}_c(l^2)=\bar{L}_s\tfrac{1}{2}(1+P_{12})+\bar{L}_a\tfrac{1}{2}(1-P_{12})=\bar{L}_c+\tfrac{1}{4}(1+2P_{12})\bar{L}_c'. \tag{13}$$

This equation cannot be regarded as an operator equation expressing the energy operator as a linear combination of 1 and P_{12}, since the energy operator has many more different eigenvalues in the l^2 configuration than does the right-hand side. Extra terms would be required to reproduce the expectation value of the energy in each individual state, but Eq. (13) shows that all these additional operators average to zero over all states with the same orbital symmetry.

In the l^n configuration the total interaction energy is simply a sum over all two-particle interactions:

$$[E_c(l^{Z'})]_{op}=\bar{L}_c\sum_{i<j}1+\tfrac{1}{4}\bar{L}_c'\sum_{i<j}(1+2P_{ij})+\cdots.$$

The additional terms that would be required to separate the energies of the individual states are indicated by dots; they are similarly sums of two-particle interactions over all pairs of particles. But just as in the l^2 configuration, they again average to zero over each class consisting of all states of a given orbital symmetry, as can be shown by group theoretical arguments.[9] The Coulomb energy, averaged in this sense, becomes

$$\bar{E}_c(l^{Z'})=\tfrac{1}{2}Z'(Z'-1)\bar{L}_c+\tfrac{1}{4}\sum_{i<j}(1+2P_{ij})\bar{L}_c', \tag{14}$$

[7] Reference 6, Eq. (44a).

[8] Racah's formula for the energy in the state of lowest seniority is obtained by a procedure that is similar to the one used here, but requires a more detailed classification of the states. However, no further subdivision of the space-antisymmetrical states of l^2 is introduced, and L_a is consequently the same in both cases.

[9] Reference 6, Sec. 3.

where

$$\bar{L}_c=\tfrac{1}{4}(\bar{L}_s+3\bar{L}_a)=\epsilon+\tfrac{1}{4}(l+1)^{-1}\delta, \quad (15)$$

$$\bar{L}_c'=\bar{L}_s-\bar{L}_a=(l+1)^{-1}\delta. \quad (16)$$

Equation (14) can be expressed in terms of the spin of the class of states over which the averaging process is extended. By use of the relation $P_{ij}=-\frac{1}{2}-2(\mathbf{s}_i\cdot\mathbf{s}_j)$, which is valid for protons, one obtains

$$\bar{E}_c(l^{Z'},S)=\tfrac{1}{2}Z'(Z'-1)\bar{L}_c+\tfrac{1}{2}[\tfrac{3}{4}Z'-S(S+1)]\bar{L}_c'. \quad (17)$$

In particular, the class with $S=\frac{1}{2}$ for Z odd or with $S=0$ for Z even contains the ground state according to the supermultiplet theory. For this class, the coefficient of $\bar{L}_c'$ is $\frac{3}{4}[\frac{1}{2}Z']$, and Eq. (17) takes the form of the last two terms of Eq. (6) with constant coefficients a and b.

When both protons and neutrons are present outside closed shells, the presence of proton-neutron forces will prevent the proton seniority from being a good quantum number, and Eq. (8) is no longer strictly applicable. The total proton spin is still a good quantum number (0 or $\frac{1}{2}$) in the lowest supermultiplet, but states of different proton seniority and orbital angular momentum will be mixed together, and the expectation value of the Coulomb energy will be a weighted average of contributions from these various states. Among them is the state of lowest proton seniority, with proton angular momentum $L=0$ for Z even and $L=l$ for Z odd. It seems implausible that states with other L values should have amplitudes so large as to justify taking the average over terms with a weighting factor $2L+1$, as was done in obtaining $\bar{L}_c$ and $\bar{L}_c'$. In fact, the state of lowest proton seniority is expected to have the predominant amplitude; consequently, it seems a reasonable approximation to use Eq. (8) for a theoretical estimate of the pairing effect even when neutrons are present outside closed shells. Likewise, in jj coupling, the state of lowest proton seniority is expected to predominate in the ground state of a mirror nucleus, and in this case the estimate will therefore be based on Eq. (10). Since lowest seniority implies maximum pairing, this approximation will give a larger odd-even effect than a calculation that takes account of the neutrons.

V. CALCULATIONS WITH A HARMONIC OSCILLATOR MODEL

We shall now inquire whether a nuclear shell model gives a satisfactory detailed account of the observed odd-even variations in Coulomb energy and of the additional irregularities that appear to be associated with shell closure. The model will also provide a means of calculating nuclear radii from experimental Coulomb energies, with inclusion of exchange effects.

The single-nucleon wave functions will be assumed for simplicity to be the stationary states of an isotropic harmonic oscillator well, $V(r)=\frac{1}{2}m\omega^2r^2=\hbar\omega\nu r^2$. In jj coupling the various shells will be filled in the order $0s_{1/2}$, $0p_{3/2}$, $0p_{1/2}$, $0d_{5/2}$, $1s_{1/2}$, $0d_{3/2}$, $0f_{7/2}$, and the Coulomb interaction between protons will be treated as a perturbation to first order. Since the model contains only one parameter, essentially the spring constant of the oscillator, the calculated Coulomb energy of any nucleus will be a numerical multiple of a single energy unit, $e^2\nu^{\frac{1}{2}}$.

As a preliminary test to see if the model gives the right order of magnitude for the pairing effect, we can calculate second Coulomb differences for values of Z from 10 to 14 by assuming ν to be constant through the $d_{5/2}$ shell in jj coupling (or the d shell in LS coupling). Within this shell the second differences should alternate between $a+b$ and $a-b$; the experimental data of Fig. 1 show roughly, though not exactly, this behavior, with a ratio $(a-b)/(a+b)$ of the order of 0.4 or 0.5. For the state of lowest proton seniority in jj coupling, we need only calculate the constants in Eq. (10). The symbol E_0 in that equation is a linear combination[10] of Slater integrals of the Coulomb potential, to be evaluated with the radial wave functions[11] of the isotropic harmonic oscillator. The integrals can be done analytically, and $(a-b)/(a+b)$ is found to be 0.55. With LS coupling in the state of lowest proton seniority, the value of this ratio is found to be 0.44 from Eqs. (8) and (9), in which $E(d^2,{}^1S)$ is likewise a linear combination[12] of Slater integrals. On the other hand, the averages $\bar{L}_c$ and $\bar{L}_c'$ over all states of the same proton spin in LS coupling, as calculated from Eqs. (15), (16), and (17), lead to a ratio $(a-b)/(a+b)=0.83$. This last result implies an odd-even effect considerably smaller than the observed one, and supports the assertion of Sec. IV that it is a better approximation to assume the state of lowest proton seniority. The choice between LS and jj coupling is less clear-cut; the larger pairing effect of LS coupling is perhaps in slightly better agreement with present experimental data, but the order of filling the d and s shells is ambiguous in LS coupling.

In further calculations it will therefore be assumed that jj coupling is in force and that the nucleons in an unfilled shell combine to form the state of lowest proton seniority. (The calculations can then be carried out as though neutrons were not present.) The parameter ν will be required to be the same for all protons in a given nucleus, and also the same in both nuclei of a mirror pair. Instead of attempting to reproduce the experimental second differences by suitable choice of ν, we shall calculate the first differences as multiples of $e^2\nu^{\frac{1}{2}}$; comparison of calculated and experimental first differences will then determine $e^2\nu^{\frac{1}{2}}$ as a function of A. The appropriateness of the model can be judged according

[10] G. Racah, Phys. Rev. **62**, 438 (1942), Eq. (54).

[11] I. Talmi, Helv. Phys. Acta **25**, 185 (1952). In Sec. I of this reference, the symbol ν is two times the ν of the present paper.

[12] E. U. Condon and G. H. Shortley, *The Theory of Atomic Spectra* (Cambridge University Press, Cambridge, 1935), p. 202.

to whether this parameter has a simple dependence on A or whether it still exhibits the odd-even effects and other irregularities that one would like to attribute to the amount of overlap of the wave functions rather than to variations of the nuclear potential well.

In order to calculate first differences, it is not enough to consider the protons outside closed shells. The addition of one proton to a nucleus not only increases the Coulomb energy of the unfilled shell, but also gives rise to energy of Coulomb interaction between the extra proton and each of the closed shells, as indicated by the symbol β in Eq. (7). Let $\beta(nl,n'l')$ denote the Coulomb interaction energy between one proton with quantum numbers nl and one $n'l'$ proton, averaged over all protons in the closed $n'l'$ shell. This quantity is independent of m_l and m_s. For example, any proton from the ninth to the fourteenth has a Coulomb interaction with the closed s and p shells amounting to $2\beta(0s,0d)+6\beta(0p,0d)$. The β's are linear combinations[13] of Slater integrals of both the direct and exchange varieties.

Besides the interaction of an extra proton with a shell that is closed in LS coupling, one needs in some cases the interaction with a subshell that is closed only in jj coupling. An easy method of calculating the latter is to observe that the energy of a closed (LS) shell minus one particle is independent of the coupling scheme. The energy of the $(p_{3/2})^4p_{1/2}$ configuration is therefore equal to that of $(p_{3/2})^3(p_{1/2})^2$, and by use of Eq. (10) one finds $\beta(0p_{3/2},0p_{1/2})=F^0-F^2/10=(17/12)\times e^2(\nu/\pi)^{\frac{1}{2}}$. Similarly, in terms of Slater integrals for the d shell, it is simple to obtain

$$\beta(0d_{5/2},0d_{3/2})=F^0-F^2/70-F^4/21=(489/400)e^2(\nu/\pi)^{\frac{1}{2}}.$$

By adding the increase of Coulomb energy of the unfilled shell [when Z' in Eq. (10) is increased by unity] to the interaction of the extra proton with the closed shells, we obtain the calculated first Coulomb difference as a numerical multiple of the energy parameter:

$$\Delta_1(Z)=\xi(A)e^2(\nu/\pi)^{\frac{1}{2}}. \tag{18}$$

Since the Slater integrals for harmonic oscillator wave functions can be evaluated analytically, $\xi(A)$ is obtainable as a rational number for each value of $A=2Z-1$ and is listed to three decimal places in the fourth column of Table II. By substituting the experimental Coulomb energy difference in the left side of Eq. (18), the energy parameter $e^2(\nu/\pi)^{\frac{1}{2}}$ is obtained as a function of A. Its values are listed in the fifth column of Table II and are plotted in Fig. 2.

The model is clearly unable to account for the large experimental odd-even effect in the $p_{3/2}$ shell; however, the eight points corresponding to the $p_{1/2}$ and $d_{5/2}$ shells have the same energy parameter to within ±1.5 percent. According to the older data, the parameter decreases 14 percent in two equal steps as the $s_{1/2}$ shell is filled, and then remains constant to within ±2 percent in the $d_{3/2}$ shell. The new data of Hunt and Zaffarano[14] for S^{31} and Ca^{39} make a radical change in this picture of the $s_{1/2}$ and $d_{3/2}$ shells, and throw some doubt also on the reliability of the early cloud-chamber determinations of the Cl^{33}, A^{35}, and Sc^{41} end points. Additional measurements of these end points by other methods would be of very great interest; it should also be emphasized that the K^{37} end point has never been measured.

Since the experimental Coulomb difference, for given A, determines the spring constant of the oscillator well, it also determines the single-nucleon wave functions and the shape of the charge distribution. In particular, the mean square radius of the nucleus can be obtained directly from the virial theorem. Since the energy of a single-particle state with quantum numbers n and l is $\hbar\omega(2n+l+\frac{3}{2})$, the expectation value of r^2 in this state is

$$\overline{r^2}=(4n+2l+3)/4\nu. \tag{19}$$

The mean square radius of the charge distribution for the stable member of a mirror pair is obtained by averaging Eq. (19) over the quantum numbers of the $Z_<=\frac{1}{2}(A-1)$ protons:

$$\langle r^2\rangle_{\text{Av}}=\eta(Z_<)/4\nu, \tag{20}$$

where $\eta(Z_<)$ is an easily calculated rational number. We now define r_0 in the usual way in terms of the radius of an equivalent uniform charge distribution,

$$R_{\text{eq}}=[(5/3)\langle r^2\rangle_{\text{Av}}]^{\frac{1}{2}}=r_0A^{\frac{1}{3}}\times10^{-13}\text{ cm}. \tag{21}$$

It follows from Eqs. (18), (20), and (21) that

$$r_0=\sigma(A)/(\Delta_1\text{ in Mev}), \tag{22}$$

where

$$\sigma(A)=0.5244A^{-\frac{1}{3}}\xi(A)[\eta(Z_<)]^{\frac{1}{2}}. \tag{23}$$

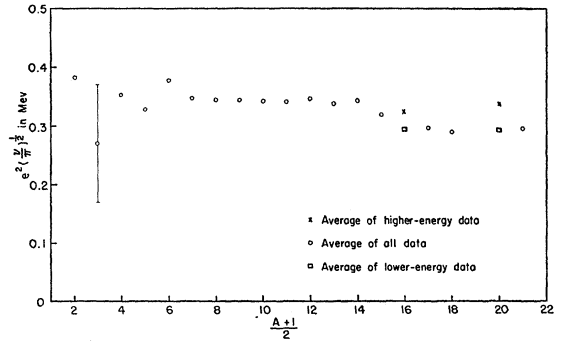

FIG. 2. The single parameter $e^2(\nu/\pi)^{\frac{1}{2}}$ of the harmonic oscillator model as a function of $\frac{1}{2}(A+1)=Z_>$. The radial wave functions of the oscillator model are completely determined by ν, which is related to the oscillator spring constant by the definition $V(r)=\frac{1}{2}m\omega^2r^2=\hbar\omega\nu r^2$. The plotted points are determined from Eq. (18) by using experimental Coulomb differences Δ_1 and values of ξ calculated with jj coupling. "Higher-energy" points are those listed in parentheses in the fifth column of Table II.

[13] Reference 12, p. 182.

[14] Reference Hu 54 following Table I.

The numbers σ are listed in the seventh column of Table II; the values of r_0 corresponding to the experimental Coulomb differences are given in the eighth column and are also plotted in Fig. 3. The radius of the charge distribution of the unstable member of a mirror pair can be obtained from Eq. (22) if desired by replacing $\eta(Z_<)$ by $\eta(Z_>)$ where $Z_>=\frac{1}{2}(A+1)$.

Figure 3 shows that in the region $13\leq A\leq 27$, where the constancy of the energy parameter encourages some confidence in the model, r_0 decreases fairly smoothly from 1.34 for C^{13} to 1.20 for Al^{27}.[15,16] According to the old data, the nucleus expands abruptly while the $s_{1/2}$ shell is being filled, and r_0 then decreases through the $d_{3/2}$ shell. The new data of Hunt and Zaffarano[14] again suggest a different picture, in which the abrupt increase of r_0 is much less pronounced and may be an artificial consequence of the model. Their results are consistent with a continued decrease of r_0 to values between 1.1 and 1.15 for the heaviest mirror nuclei, but until further measurements are made, the radii of the mirror nuclei with A between 31 and 41 remain uncertain by 10 or 15 percent and do not permit extrapolation to higher A.

The harmonic oscillator model can also be applied to isotopic-spin triplets if the assumption of charge symmetry is extended to charge independence. For mass numbers 10 and 14, all three members of the triad are known experimentally; in such a case, without appealing to data from nuclei of different mass number, we can compare the observed position of the $T_z=0$ state with its position as calculated from the difference in Coulomb energy between the $T_z=\pm 1$ states. When the $T_z=0$ state has an odd number of protons, it might be expected to lie lower than would be predicted by a model that does not take adequate account of the odd-even effect.

For example, the $C^{10}-B^{10}$ difference in Coulomb energy is 4.85 ± 0.10 Mev if the end point of the C^{10} positron spectrum is taken to be 2.10 ± 0.10 Mev.[17] The γ-ray energies and the Be^{10} end point are known to a few kilovolts.[18] According to the model of a uniform charge distribution, the position of the 0+ state of B^{10} should correspond to the ratio of Coulomb differences $(B^{10*}-Be^{10})/(C^{10}-B^{10*})=(5\times 4-4\times 3)/(6\times 5-5\times 4)$. The calculated energy of the 0+ state is then 1.93 ± 0.05 Mev; as expected, the experimental value of 1.739 ± 0.003 Mev is lower than this. The harmonic oscillator model also underestimates the pairing effect in the $p_{3/2}$ shell. Since the calculation of ξ depends only on the number of protons, the ratio of the Coulomb differences in jj coupling can be found from Table II to be 5.667/7.333, and the corresponding energy of the 0+ level is 1.89 ± 0.05 Mev. In LS coupling the ratio of Coulomb differences is lowered to 5.667/7.500 and the energy of the 0+ level is 1.86 ± 0.05 Mev.

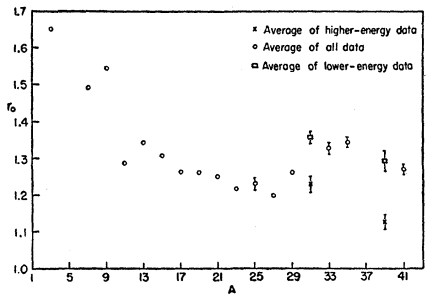

FIG. 3. Radii of the mirror nuclei of smaller Z. The point at $A=13$, for instance, is related to the rms radius of the nuclear charge distribution of C^{13} (not N^{13}) by the definitions $(5/3)^{\frac{1}{2}}\langle r^2\rangle_{Av}^{\frac{1}{2}}=R_{eq}=r_0A^{\frac{1}{3}}\times 10^{-13}$ cm. The plotted points are determined from Eq. (22) by using the experimental Coulomb differences Δ_1 and the values of σ calculated with jj coupling. "Higher-energy" points are those listed in parentheses in the last column of Table II.

The case of N^{14} benefits from more accurate experimental data as well as from slightly greater confidence in the oscillator model. The end point of the O^{14} spectrum has recently been measured as 1.835 ± 0.008 Mev,[19] and the γ-ray energy and the C^{14} end point are also known accurately[18]; the $O^{14}-C^{14}$ Coulomb difference is found to be 6.576 ± 0.015 Mev. The calculated ratio of Coulomb differences, $(N^{14*}-C^{14})/(O^{14}-N^{14*})$, is 6/7, 8.667/10.167, and 8.333/10.167 in the uniform density model, the oscillator model with jj coupling, and the oscillator model with LS coupling, respectively. The corresponding energies of the 0+ state in N^{14} are 2.408, 2.399, and 2.335 Mev, each with a computed error of 7 kev, as compared with the experimental value of 2.310 ± 0.012 Mev. Thus the jj model is scarcely better than the uniform density model in this instance, whereas the LS coupling result is almost in agreement with experiment. It may be worth mentioning that the energy parameter $e^2\nu^{\frac{1}{3}}$, calculated from the $O^{14}-C^{14}$ difference on the assumption of charge symmetry alone, lies between 1 percent and 1.5 percent higher, in both jj and LS coupling, than the average of its values for mass numbers 13 and 15.

VI. CONCLUSIONS

The odd-even effect in the Coulomb energies of mirror nuclei can be accounted for, in a manner that appears to be satisfactory except for the lightest nuclei ($Z\leq 6$), by a shell model with harmonic oscillator wave functions. It is not necessary to attribute the odd-even alternation of Coulomb energy to a similar alternation in the radius of the charge distribution; on the contrary,

[15] This value for Al is in very good agreement with preliminary data from electron scattering (private communication from Professor R. Hofstadter).

[16] The values of r_0 obtained from the uniform density model also show a steady decrease (superposed on a pronounced odd-even alternation) from 1.515 for O^{17} to 1.34 for Al^{27}. Professor R. K. Sheline of Florida State University informs us that he and Dr. N. R. Johnson have likewise observed a decrease of this kind and have attributed the subsequent discontinuity to shell structure.

[17] R. Sherr and J. B. Gerhart, Phys. Rev. **91**, 909 (1953).

[18] F. Ajzenberg and T. Lauritsen, Revs. Modern Phys. **24**, 321 (1952).

[19] J. B. Gerhart, Phys. Rev. **95**, 288 (1954).

a sufficient cause is to be found in the symmetry properties of the ground-state wave function. The permutation symmetry corresponding to a pairing of spins is not enough by itself, however, and has to be extended to the more thorough-going type of pairing described by Racah's notion of lowest seniority.

The results presented in Table II are based on jj coupling, but LS coupling appears to give an equally good account of the present experimental data. In fact, the larger odd-even effect predicted by LS coupling has the advantage of producing better accord between the assumption of charge independence and the observed energy of the 0+ state in N^{14}. Further measurements are needed to establish whether a discontinuity in Coulomb energy and radius occurs after $Z=14$.

The oscillator model gives appreciably smaller values for the rms radius of the nuclear charge distribution than does the uniform density model in which $E_c=(3/5)Z(Z-1)e^2/R_{\text{eq}}$ and $\Delta_1(Z)=(6/5)(Z-1)e^2/R_{\text{eq}}$. Part of the difference is due to the absence of exchange terms[20] in the latter case, but it is not clear that exchange effects are sufficient to account for the whole difference. For instance, if the Slater exchange integrals are completely omitted in calculating Δ_1 for $A=17$, the result obtained is $10.967e^2(\nu/\pi)^{\frac{1}{3}}$, only 6 percent larger than the value with exchange. The corresponding r_0 for O^{17} would be 1.34 without exchange, compared with 1.52 from the uniform density model. This remaining discrepancy is caused by the change in shape of the shell-model charge distribution when an outer proton turns into a neutron.

The values of r_0 listed in Table II show a steady decrease from C^{13} to Al^{27}. It is to be hoped that further measurements of Coulomb energies will soon determine whether this decrease continues without interruption in the heavier mirror nuclei, and that electron scattering experiments will provide an independent set of results for comparison.

It is a pleasure to express our gratitude to Professor E. P. Wigner for valuable discussions.

[20] The separation between direct and exchange terms is sometimes defined in a different way. When the wave function is taken to be a single determinant of one-particle functions, it is often formally convenient to add a sum of "self-energy" integrals, $\frac{1}{2}\Sigma_i\int\int|\psi_i(\mathbf{r}_1)\psi_i(\mathbf{r}_2)|^2(e^2/r_{12})d\tau_1 d\tau_2$, to the direct terms and subtract the same sum in the form of exchange contributions. We do not include such integrals in the definitions of "direct" and "exchange"; it seems superfluous to attribute direct and exchange Coulomb energies of equal magnitude and opposite sign to a system containing only one charged particle, and undesirable to attribute exchange energy to an unsymmetrized many-particle wave function with no positional correlations.

PHYSICAL REVIEW VOLUME 96, NUMBER 2 OCTOBER 15, 1954

Radiations from Ir^{192m}†

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(Received July 12, 1954)

The photon and electron spectra of Ir^{192m} (1.42 min) have been thoroughly investigated. Study of the photon spectrum with scintillation spectrometers and an aluminum-lined proportional counter spectrometer revealed the presence of a quantum continuum, iridium L x-rays, and a 58-kev gamma ray. The quantum continuum was shown to be bremsstrahlung engendered by conversion electrons from the isomeric transition. Previous to this study, the quantum continuum had been interpreted as possibly caused by "two-quantum" emission. A high-resolution, 180° permanent magnet spectrograph was employed for the examination of the conversion electron spectrum. The energy of the isomeric transition was found to be 58.0±0.4 kev from the conversion electron studies. Lifetime-energy considerations and the observed L-subshell conversion ratios show that the 58.0-kev gamma ray is $E3$. The L-shell conversion coefficient was determined from scintillation spectrometer data to be $\geqslant 870$.

I. INTRODUCTION

PREVIOUS investigations[1,2] have shown that the decay of Ir^{192m} (1.42 min) is accompanied by iridium L x-rays, L-shell conversion electrons from an ~60-kev gamma ray, and a quantum continuum with peak intensity at ~30 kev and upper energy end point at ~60 kev. Conversion electron analyses have been made by Hole,[3] Caldwell,[4] and Weber[5] yielding energy values for the isomeric transition of 55.5, 57.4, and 56.0 kev, respectively. The reported quantum continuum has been interpreted as possibly caused by "two-quantum" transitions.[1,2] Sachs[6] has suggested that nuclear decay through transitions of the type 0→0 (yes) can take place by simultaneous emission of two quanta, one quantum and one electron or two elec-

† Work performed under the auspices of the U. S. Atomic Energy Commission.

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[5] C. Weber and A. Flammersfeld, Z. Naturforsch. **A8**, 580 (1953); C. Weber, Z. Naturforsch. **A9**, 115 (1954).

[6] R. G. Sachs, Phys. Rev. **57**, 194 (1940).