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An Indirect Measurement of the n-n Interaction*†

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The gamma-ray spectrum from the process $\pi^- + d \rightarrow 2n + \gamma$ has been remeasured using a high resolution 110-channel pair spectrometer. The negative pions produced in an internal target by 335-Mev protons of the Berkeley 184-inch cyclotron, are captured at rest in a high-pressure deuterium vessel. Comparison of the spectrum with the gamma-ray spectrum from the process $\pi^- + p \rightarrow n + \gamma$ measured with the same apparatus shows the effect of the n-n interaction as a broadening of approximately three times the resolving width. In spite of the low counting rates and resulting statistical inaccuracy, it is possible to establish limits for the scattering length "a" for S-state neutron-neutron scattering. The experimental data are analyzed by evaluating a weighted first moment of the spectrum. Comparison of the weighted first moments of the theoretical spectra with that of the data leads to a value of $a = -15.9 \times 10^{-13}$ cm with limits, based on the probable error, of $a = -8.5 \times 10^{-13}$ cm and $a = -\infty$ cm. The value $a = -15.9 \times 10^{-13}$ cm corresponds to an unbound state (~160 kev positive) for the hypothetical dineutron. The bound dineutron of more than 50 kev is less than 0.1 percent probable. This result is consistent with the n-p and p-p scattering data and is in agreement with the theory of charge independence of nuclear forces.

I. INTRODUCTION

THE p-p interaction for low-energy scattering has been successfully treated by the evaluation of the scattering length and the effective range.[1] The scattering results predict[2] that the lowest singlet state for two protons is unstable and to date no He^2 or diproton has been found. Furthermore, if charge symmetry[3] is assumed, the parameters derived from the low-energy p-p scattering experiments when applied to the n-n system predict that the ^1S state of the dineutron would not be stable either. Since the low-energy scattering parameters for the p-p and n-p systems are also in agreement within experimental error, the more general theory of charge independence[3] would predict an unstable ^1S state for the deuteron as is known to be the case.[4] The low-neutron fluxes available at present, however, have precluded the use of scattering experiments to study the n-n forces. The n-d scattering would in principle give information about the n-n interaction but the theoretical interpretation of the experimental work "is still in a very preliminary state"[5] because of the complications inherent in the three body problem.

Evidence for charge symmetry from other fields of investigation is good, so results contrary to those we have found would have been surprising. However, it is worthwhile to supply direct verification and derive whatever quantitative information that is possible with the present experimental techniques.

Two further approaches have been used to obtain experimental evidence concerning the n-n force: (1) a search for the dineutron; (2) analysis of the energy spectrum of "C" resulting from the reaction $A + B \rightarrow C + 2n$. If "C" is a particle, its nuclear interaction with the two neutrons makes a quantitative analysis very difficult. This is again a three-body problem and if "C" is a complex nucleus the results will depend on the model for "C". If, however, "C" is a gamma ray an accurate analysis is possible.[6] It should also be noted that, in both cases, even if there is no bound state the system will be peaked if the neutrons are emitted singly under the action of an attractive potential. The present experiment permits the indirect evaluation, within rather broad limits, of the scattering length for the n-n interaction.

Using the first approach Kundu and Pool[7] bombarded Co^{59} and Rh^{103} with tritium. The observed half-lives of the products and the energies of the decay betas established the fact that the reaction had proceeded by capture of two neutrons. From the behavior of the cross section for various triton energies they concluded that the mechanism involved was probably the Oppenheimer-Phillips[8] process with t rather than a compound nucleus mechanism. This suggests that the two neutrons may be captured as a group, which would imply that a dineutron could have at least transient existence in the instant between the polarization of the tritium nucleus and the capture. As yet, the experiment has a rather remote bearing on the actual occurrence of an observable dineutron since the detailed theoretical analysis of this process and its implications concerning the n-n interaction have not been made.

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[1] J. M. Blatt and J. D. Jackson, Phys. Rev. **76**, 18 (1949).
[2] H. A. Bethe and R. F. Bacher, Revs. Modern Phys. **8**, 133 (1936).
[3] G. Breit and E. Feenberg, Phys. Rev. **50**, 850 (1936); K. M. Watson, Phys. Rev. **85**, 852 (1952).
[4] H. A. Bethe, *Elementary Nuclear Theory* (John Wiley and Sons, Inc., New York, 1947), first edition, p. 52 ff; J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952), p. 93.
[5] See Blatt and Weisskopf, reference 4, p. 206.
[6] K. Watson and R. Stuart, Phys. Rev. **82**, 738 (1951).
[7] D. N. Kundu and M. L. Pool, Phys. Rev. **73**, 22 (1948); Phys. Rev. **82**, 305 (1951).
[8] J. R. Oppenheimer and M. Phillips, Phys. Rev. **48**, 500 (1935).

Feather[9] has established an upper limit on the binding energy of the dineutron. By consideration of a group of known reactions for which the Q values have been experimentally established he is able to deduce

$$He^6 \rightarrow He^4 + 2n - 0.7 \text{ Mev}.$$

Since He^6 is known to decay instead by β emission, one concludes that the binding energy of the dineutron is less than 0.7 Mev.

Acting on a suggestion by Feather[9] that the dineutron might be emitted in the process of fission, and if so could be detected by subsequent capture in Bi^{209}, Fenning and Holt[10] irradiated Bi^{209} in the Harwell pile. The thermal flux of neutrons was about 10^{12} neutrons per sq cm per sec. Denoting the flux of dineutrons by ϕ and the capture cross section by σ, they reported that no activity ascribable to AcC'' or Tl^{207} was observed and set $\phi\sigma < 1.5 \times 10^{-21}$ per sec.

Feather gave a preliminary report[11] on an experiment by Fenning in which he had irradiated He^4 in the Harwell pile and apparently observed the approximately one-second activity from He^6 in the reaction $He^4 + n^2 \rightarrow He^6 \rightarrow \beta^-$. It was later concluded[12] that the observed activity was a result of contamination of the He^4. A limit of $\phi\sigma < 10^{-18}$ per sec was set, where σ is now the capture cross section of helium for dineutrons.

The experiments so far described only yield evidence on the existence of the dineutron. Beyond the establishment of limits on the "binding energy" of this hypothetical particle, they have given no detailed information on the n-n interaction.

Los Alamos Scientific Laboratory[13] using the second approach mentioned above has reported evidence for the production of dineutrons in the reaction $t^3 + t^3 \rightarrow He^4 + n^2$. A. Hemmendinger[14] gives more detail on this experiment. The reaction may proceed in three alternative ways:

$$(1) \quad t^3 + t^3 \rightarrow He^4 + 2n,$$

$$(2) \quad t^3 + t^3 \rightarrow He^4 + n^2,$$

$$(3) \quad t^3 + t^3 \rightarrow He^5 + n$$
$$\hookrightarrow He^4 + n.$$

The first process will give a continuous distribution of alpha-particle energies. The second process would give a peak of alpha-particle energies depending on the angle of observation. A detailed kinematic analysis of the third process shows that two or more peaks of alpha-particle energies, associated with a given angle, would be expected. But if the He^5 nucleus is in its ground state as determined by n He^4 scattering measurements[15] the number of peaks for the He^5 process would be limited to two and one of these peaks would be of too low energy to be observed by their apparatus. At all angles the peak resulting from process (2) would appear at a higher energy than any of the peaks from process (3). Since two peaks are observed, the high-energy peak is attributed to the emission of interacting neutrons. Hemmendinger states that "the dineutron appears to be unbound by a few hundred kev," although no detailed calculations, based on the absolute energy calibration from the $d(t,n)He^4$ reaction, are given.

The fact that two of the $d(t,n)He^4$ alpha-energy peaks almost coincide on the pulse-height scale with the two attributed to the emission of interacting neutrons suggests the possibility that either beam or target contamination by deuterium in the t-t reaction might account for the observed results. Hemmendinger[16] reports that a careful mass spectrograph analysis was carried out to eliminate this possibility and the results showed negligible deuterium contamination.

Another group at Chalk River[17] have also investigated the t-t reactions by measurement of the disintegration products. They observe no alpha-particle group with an energy equal or greater than 3.8 Mev. They therefore conclude that a dineutron of zero or greater binding energy can exist in less than one percent of the disintegrations. They also place an upper limit of one percent on the disintegrations as resulting from the existence of the dineutron in a virtual state of energy, $|E_B| \leq 0.6$ Mev, and of lifetime $\geq 3 \times 10^{-21}$ sec. Deuterium contamination of the target prevents them from making more detailed observations in the energy region corresponding to the unstable dineutron.

Experimental work with mesons has brought about a theoretical simplification of this general method for investigating the n-n interaction. Panofsky, Aamodt, and Hadley[18] have shown that the capture of negative pions in deuterium may lead to either of the following processes:

$$\pi^- + d \rightarrow 2n,$$

$$\pi^- + d \rightarrow 2n + \gamma.$$

Tamor and Marshak[19,20] and Brueckner, Serber, and Watson[21] have pointed out that the gamma spectrum from the radiative capture is influenced by the n-n interaction.

Watson and Stuart,[6] using a singlet S state for the

[9] N. Feather, Nature **162**, 213 (1948).
[10] F. W. Fenning and F. R. Holt, Nature **165**, 722 (1950).
[11] N. Feather, Report of an International Conference on Elementary Particles, Tata Institute of Fundamental Research, December, 1950 (Commercial Printing Press, Bombay, India).
[12] F. W. Fenning (private communication to W. K. H. Panofsky). We wish to thank Dr. Fenning for his kindness in keeping us informed of his results.
[13] Los Alamos Scientific Laboratory, Phys. Rev. **79**, 238 (1950).
[14] A. Hemmendinger, Oak Ridge National Laboratory Report TID-372, 1950 (unpublished).
[15] Bashkin, Petree, Mooring, and Peterson, Phys. Rev. **77**, 748 (1950).
[16] We are grateful to Dr. Hemmendinger for giving us this information on the work at Los Alamos.
[17] Allen, Almquist, Dewan, Pepper, and Sanders, Phys. Rev. **82**, 262 (1951).
[18] Panofsky, Aamodt, and Hadley, Phys. Rev. **81**, 565 (1951).
[19] S. Tamor and R. Marshak, Phys. Rev. **80**, 766 (1950).
[20] S. Tamor, thesis, University of Rochester, 1950 (unpublished).
[21] Brueckner, Serber, and Watson, Phys. Rev. **81**, 575 (1951).

final state of the two neutrons, have calculated the theoretical spectra based on the phase shift which is directly related to the low-energy n-n scattering cross section through the equation $\sigma_{n-n}=4\pi/k^2 \sin^2\delta$. Schwinger[22] has shown that δ may be related, through the equation $k\cot\delta\approx -1/a+\frac{1}{2}r_0k^2$, to the two parameters used to describe the low-energy scattering: a, the scattering length and r_0, the effective range. Since the spectra are not very sensitive to the exact value of r_0, they assign to it the value obtained from p-p scattering: $r_0=2.65\times10^{-13}$ cm.[23] $\alpha=1/a$ becomes the convenient parameter in this description of the n-n interaction. If α is positive the possibility of forming the bound state exists and the spectrum will be composed of a continuous part and a monochromatic line separated from the continuum by approximately the binding energy.

Previous experimental results[24] using the basic method described in this paper lead to a nominal value of +1.2 Mev for the lowest energy of the n-n system at which the phase shift is $\pi/2$, equivalent to a value of $\hbar c\alpha\simeq 34$ Mev. The energy resolution was such that the effect of the interaction was evident only as a change in the energy scale. The resulting uncertainty in the scattering length, however, was too poor to exclude either the bound or no-interaction case. The present work is an attempt to extend these results by improving the resolving power of the spectrometer.

II. EXPERIMENTAL METHOD

A. Basic Method

The capture of negative pions[18] at rest in hydrogen yields a monoenergetic gamma ray since it is a two-body process: $\pi^-+p\rightarrow n+\gamma_p$. The capture of negative pions at rest in deuterium[18] yields a spectrum of gamma rays influenced by the n-n interaction: $\pi^-+d\rightarrow 2n+\gamma_d$. The case where the two neutrons come off together, but without interaction determines the energetic upper limit for an unbound state. Conservation of energy and momentum gives, for the two processes:

$$E_{\max}(\gamma_d)=M_\pi c^2+M_dc^2 -2M_nc^2[1+\tfrac{1}{4}(E_{\max}(\gamma_d)/M_nc^2)^2]^{\frac{1}{2}} =\frac{(M_\pi c^2+M_dc^2)^2-(2M_nc^2)^2}{2(M_\pi c^2+M_dc^2)} \quad (1)$$

$$E(\gamma_p)=M_\pi c^2+M_pc^2-M_nc^2[1+\tfrac{1}{2}(E(\gamma_p)/M_nc^2)^2]^{\frac{1}{2}} =\frac{(M_\pi c^2+M_pc^2)^2-(M_nc^2)^2}{2(M_\pi c^2+M_pc^2)}. \quad (2)$$

$E_{\max}(\gamma_d)-E(\gamma_p)$ determines the position of the upper limit of the continuous part of the theoretical spectra on the energy scale relative to $E(\gamma_p)$. The dependence of this position on an error in the meson mass can be shown to be

$$d[E_{\max}(\gamma_d)-E(\gamma_p)] \simeq(1/M_nc^2)[E(\gamma_p)-\tfrac{1}{2}E_{\max}(\gamma_d)]dE(\gamma_p) \quad (3)$$
$$\simeq 0.06dE(\gamma_p) \text{ (Mev)}.$$

Using the value[25] $E(\gamma_p)=129.2\pm0.12$ leads to

$$\Delta[E_{\max}(\gamma_d)-E(\gamma_p)]\simeq\pm0.007 \text{ Mev}.$$

This will be seen to be a negligible factor in the present measurement. The theoretical spectra, corrected for the resolution of the instrumentation, may thus be compared with the experimental data for $\pi^-+d\rightarrow 2n+\gamma_d$ without exact evaluation of the absolute energy scale provided both capture processes are observed with identical instrumentation. A mere change of the gas used as the capturing substance made this possible in the present instance.

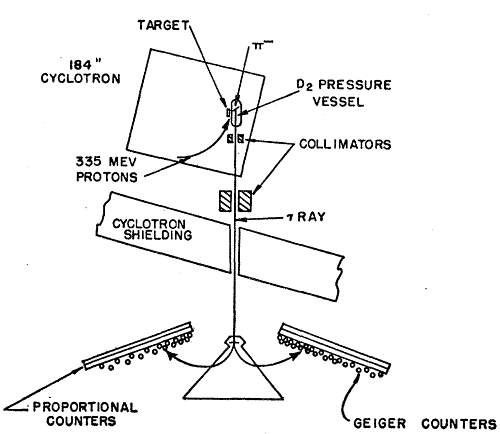

FIG. 1. Schematic diagram of geometrical arrangement of apparatus.

B. General Arrangement

Negative pions are produced by bombarding a heavy-element target (in this case thorium) with the internal proton beam of the 184-inch Berkeley synchrocyclotron. Some of the pions are then absorbed in deuterium gas at 2700 psi. The gas is confined in a vessel contiguous to the primary (proton beam) target. Gamma rays emitted in the reaction $\pi^-+d\rightarrow 2n+\gamma_d$ are permitted to enter a pair spectrometer through a collimating system which shields the spectrometer from the primary target (Fig. 1). A detailed description of this experimental arrangement is given in the paper by Panofsky *et al.*[18]

The events comprising the data are Geiger-tube co-

[22] J. Schwinger, Phys. Rev. **72**, 742 (1947); hectographed notes on nuclear physics, Harvard, 1947 (unpublished).
[23] J. D. Jackson and J. M. Blatt, Revs. Modern Phys. **22**, 77 (1950).
[24] Aamodt, Panofsky, and Phillips, Phys. Rev. **83**, 1057 (1952).
[25] K. M. Crowe, paper to be submitted to *Review of Scientific Instruments;* K. M. Crowe, thesis, University of California (unpublished); K. M. Crowe and R. H. Phillips, preceding paper [Phys. Rev. **96**, 470 (1954)].

incidences resulting from detection of the pair fragments (see Fig. 1). The Geiger tubes are gated by a quadruple coincidence between both sides of both proportional counters in coincidence with a beam-pulse gate. A measurement of the energy of the pair fragments determines the energy of the incident gamma ray.

The present pair spectrometer has been redesigned from that used by Panofsky to improve the energy resolution. This was accomplished by changing the location of the Geiger tubes to reduce the defocusing effects of multiple scattering in the converter. Figure 2 is a schematic drawing showing how first-order horizontal angular focusing results from the geometry used. In addition an increased number of detector channels (110) were used with the energy defining aperture reduced to 0.75 Mev (Fig. 2).

The same equipment has been used in a precision measurement of the mass of the π^- meson as described in the companion paper.[25] The magneto-optical features have been described in that paper and the detailed theory of the spectrometer is being submitted for publication in the *Review of Scientific Instruments*.[25] Since the experimental results have been shown to be essentially independent of an absolute energy scale, a more detailed analysis of the pair spectrometer is unnecessary except insofar as the resolution or detection efficiency is energy dependent (see Sec. F).

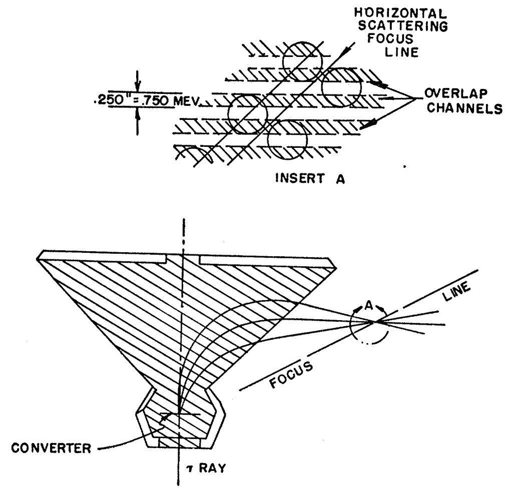

FIG. 2. Schematic diagram to show how first-order horizontal angular focusing of the pairs is achieved. The Geiger-tube array is grouped about the focus line as shown in the insert. The region of constant magnetic field is shaded; region of no magnetic field is unshaded. (Fringing field is neglected in this approximation.)

C. Operation of the Runs

The entire running time extended continuously over a period of ten days. The net data collection time was 37.8 hours for deuterium, 26.0 hours for hydrogen, and 22.1 hours for background. The background runs were made by evacuating the high-pressure vessel. Total counts were 421 hydrogen gammas, 432 deuterium gammas, and 824 background gammas.

Plateaus for the proportional counters and associated electronics were established using the gamma-ray flux from neutral pion decay[26] by moving the primary target into the collimated line of observation. The 140 odd Geiger tubes and their associated electronics were individually checked at the beginning of the runs and at the end of each day by means of a radioactive source. No replacements were necessary. The magnetic field was maintained constant to ± 0.02 percent throughout the experiment by monitoring with a nuclear fluxmeter.

D. Background Effects

The observed background is attributed to decay gammas from neutral pions produced by scattered protons which strike the high-pressure vessel. The shape of the background spectrum substantiates this. Consideration must be given to the fact that processes other than the radiative capture of the negative pion may have taken place when the deuterium was introduced. Besides negative pions, positive and neutral pions, protons, neutrons, and gamma rays are produced by the 335-Mev protons incident on the heavy-element primary target. Some of these particles can give rise to high-energy gamma rays when they interact with the deuterium. These interactions are now listed in order of their importance.

Charge exchange scattering of either positive or negative pions can produce neutral pions with their consequent high-energy decay gammas. A calculation based on the charge exchange cross section for negative pions reported by workers at Rochester[27] gives an effect which is less than one percent of the measured radiative capture. A search for charge exchange scattering of negative pions stopped in deuterium[28] gave a negligible upper limit for this process. Evidence that charge exchange scattering is negligible in the present experiment is also given by the result of previous experimental work on the capture of negative pions in deuterium.[18,29] Attempts to observe gamma emission from neutral pions resulted in a null effect (-2.5 ± 7.2 percent of the radiative capture intensity).

Any decay gammas from neutral pions produced by stray high-energy particles incident on the deuterium would also have shown up in the previous deuterium capture experiments. The null effect mentioned above rules this out.

The data showed no evidence of being influenced by accidentals. Coincidences between three or more Geiger channels and other multiple events, not caused by a single gamma-ray pair, comprised less than ten percent

[26] Bjorklund, Crandall, Moyer, and York, Phys. Rev. **77**, 213 (1950).

[27] R. Wilson and J. P. Perry, Phys. Rev. **84**, 163 (1951); Roberts, Spry, and Tinlot, Phys. Rev. **90**, 343 (1954).

[28] J. Steinberger and W. Chinowsky, Phys. Rev. **95**, 623(A) (1954).

[29] R. L. Aamodt, thesis, University of California, 1951 (unpublished).

of the observed events. The single count rate (gated Geiger counts of only one pair fragment) was approximately equal to the rate for real pairs. This is attributed mainly to end effects. Deadtime and the individual efficiency of the Geiger tubes also contributed to these losses.

E. The Folding Procedure

The theoretical spectra, as mentioned in Sec. II, A, must be corrected for the resolution of the instrument. The pair spectrometer because of its finite resolving power detects a distribution of energy values about $E(\gamma_p)$ instead of a monochromatic line from the process $\pi^-+p\rightarrow n+\gamma_p$. The gamma-ray spectrum from the capture in hydrogen $I_\gamma(E)$ shown in Fig. 3, thus defines the resolution of the instrumentation at the energy value $E(\gamma_p)$. Provided this resolution curve is not energy dependent it may be folded into the various theoretical spectra for comparison with the experimental results.

The folding operation is

$$\phi(E)=\int_{-\infty}^{+\infty} I(t)R(t-E)dt,$$

where $I(t)$ is the theoretical spectrum, and $R(t)$ is the resolution curve (the kernel) which describes the distribution of energy values around an origin defined by the setting of the instrument. In general, this origin must be determined by a theoretical analysis of the characteristics of the instrumentation which produce the spread in energy values. As has been shown [Eq. (3)], no exact evaluation of the energy scale is necessary for evaluation of the final result of this experiment. The method of correcting the data for the resolution of the instrumentation is also independent of any error in determining the origin of the kernel. Physically this independence follows from the fact that the value $E(\gamma_p)$ chosen for the origin is also the value which determines the position of $I(t)$ on the energy scale. This may be shown as follows. The origin of the kernel is the measured value $E(\gamma_p)$ and is thus the origin in a plot of $R(t)=I_\gamma\{-[E-E(\gamma_p)]\}$. An error $(-\Delta E)$ in the measured value of $E(\gamma_p)$ thus results in a change $(+\Delta t)$ in the origin of the kernel. The kernel now becomes $R(t+\Delta t)$. But this change in $E(\gamma_p)$ also changes the position of the spectrum $I(t)$ by an amount Δt. The resulting fold is

$$\phi(E)=\int_{-\infty}^{+\infty} I(t+\Delta t)R[(t+\Delta t)-E]dt.$$

Thus $\phi(E)$ is independent of Δt.

Equation (3), gives the error in the position of the theoretical spectra caused by an error in the origin, $E(\gamma_p)$, of the kernel. Watson and Stuart[6] assumed a value $E_{\max}(\gamma_d)=132.00$ Mev which corresponds to an

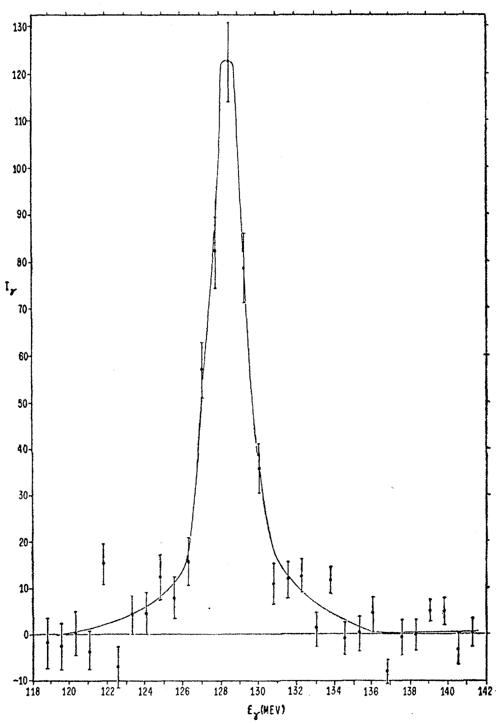

FIG. 3. $I_\gamma(E)$, the energy spectrum of the gamma ray from the process: $\pi^-+p\rightarrow n+\gamma_p$ with background subtracted. The data have been corrected for energy dependent factors (Sec. II, F).

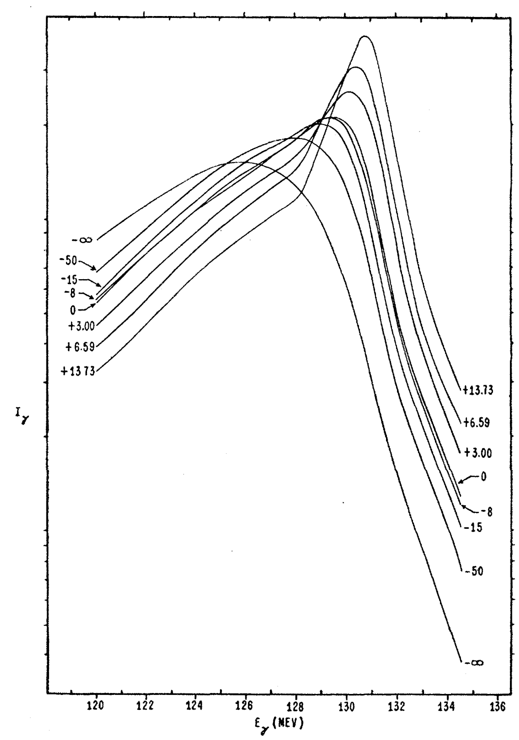

FIG. 4. Plot of the folded theoretical spectra; i.e., the theoretical spectra as deduced by Watson and Stuart (see reference 6) folded with the resolution function, $R(t-E)$.

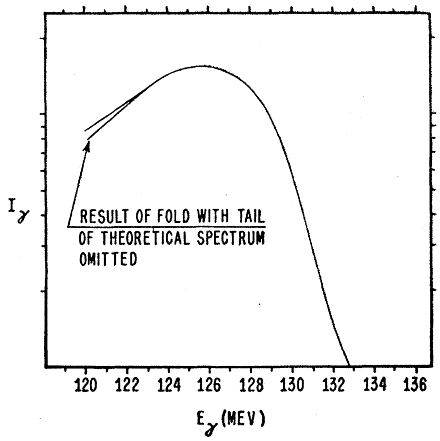

FIG. 5. Fold of the resolution function with the theoretical spectrum for $\hbar c\alpha = -\infty$ showing result of omitting the tail below 120 Mev of the theoretical spectrum.

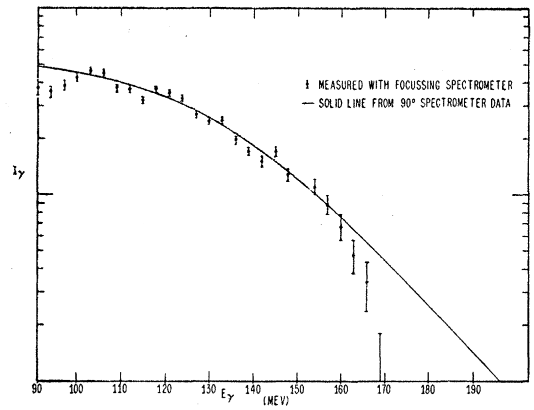

FIG. 6. Neutral meson gamma-ray spectrum obtained with the pair spectrometer by bombarding wolfram with 340-Mev protons. The observation angle is 180°. The solid curve is the same spectrum as obtained by Crandall (see reference 30) with the conventional nonfocusing 90° wedge-spectrometer geometry.

origin at $E(\gamma_p) = 129.9$ Mev. For the purposes of this analysis the error introduced by using the calculated curves is negligible since the measured value[25] of $E(\gamma_p)$ is 129.2 ± 0.12. The folded theoretical spectra are shown in Fig. 4.

The theoretical curves[6] are uncertain in the energy region below 120 Mev, but this should result in an insignificant error in the final folded theoretical curves because both the theoretical curves and the kernel have low values of $I_\gamma(E)$ in this region. The effect was checked by a fold performed omitting the entire tail (below 120 Mev) of one of the theoretical spectrum. The spectrum for the case $\hbar c\alpha = -\infty$ was used for this check. It would show the maximum effect since it has the largest values of $I_\gamma(E)$ in this region of any of the theoretical spectra. The result is shown in Fig. 5. A calculation based on the analysis of the data as described in Part G below shows that this procedure results in less than a three percent error in evaluating the spectrum for $\hbar c\alpha = -\infty$ for comparison with the experimental results. Thus even a large error in the theoretical data below 120 Mev would have a very small effect on the fold.

F. Energy Dependence of Efficiency and Resolving Power

It is important that no energy-dependent errors distort the observed gamma-ray spectra. A major source of energy dependence of the detection efficiency of the pair spectrometer results from its finite size, since it detects a variable fraction of the possible pair combinations for a given E_γ. A correction for this effect is easily calculated from the known geometry of the Geiger counter[29,30] array.

A correction was made for the variation of the pair production cross section as a function of energy. The maximum effect was less than $1\frac{1}{2}$ percent for I_γ from capture in deuterium. No correction is necessary for I_γ from capture in hydrogen since $E(\gamma_p)$ is monochromatic.

Multiple scattering in the converter is a source of energy-dependent losses in the observed intensity of gamma rays. The defocusing effect of the fringing field is not independent of the energy of the vertically scattered electrons. As a result they are scattered into a solid angle which is energy dependent whereas the vertical defining aperture of the Geiger tubes is constant. The rather large (three square inches) converter used will magnify this effect. A measurement of the relative efficiency was made with π_0 decay gammas of known intensity to determine the magnitude of this effect (Fig. 6). The details are given in the paper dealing with the associated experiment.[25] It is shown that no significant losses occur except at the ends of the spectrum which are far from the region of interest for both capture processes as will become apparent later in this section.

The resolution, as has been pointed out, must not be energy dependent if it is to be used in the folding operation over the range of energy values of the theoretical spectra. The final analysis of the data (see Sec. II, G), is based on evaluation of the first moment of the folded theoretical curves. The fold with $R(t-E)$ will only result in an error in the evaluation of the first moment of the folded theoretical curves if the first moment of $R(t)$ is energy dependent. A theoretical analysis of the resolution curve of the pair spectrometer[25] involves consideration of (1) energy channel width, (2) multiple scattering of the pair in the converter, (3) radiation straggling of the pair in the converter, (4) energy loss resulting from ionization by the pair, (5) lateral width of the converter, and (6) higher-order defocusing effects resulting from (a) multiple scattering of pair fragments in the horizontal plane, (b) motion in

[30] W. E. Crandall, thesis, University of California, 1952 (unpublished).

the vertical plane and finite converter height. These components are folded together to yield the final resolution curve. The first moment of the resolution curve can be obtained by adding algebraically the first moments of the components. All the above components except (2) have first moments, which must therefore be examined for energy dependence and the shift of the moment in the range of the fold must be evaluated.

The first moment of the energy-channel width results from the fact that the measured energy values of the channels do not agree with the design values for the detection of an ~129-Mev gamma ray.[25] The moment of this component may be calculated for other gamma-ray energies by averaging over the corresponding channels. The resulting shift of the moment is ~ -30 kev for the extreme range of the fold.

The radiation straggling component can be derived from Heitler's expression[31] for the probability, $W(y)dy$, that an electron will retain e^{-y} times its initial energy in traversing a thickness l of converter. By a change of variable the probability $W(E/E_0)dE$ that the electron retains a fraction E/E_0 of its initial energy E_0 is

$$W(E/E_0)dE \simeq \frac{[\Gamma(bl)]^{-1}}{E_0(\ln E_0/E)^{1-bl}}dE \quad bl \leq 0.1.$$

The first moment can be shown to be

$$M(W) \simeq E_0 - \frac{bl}{1+bl}(E_0 - E_c),$$

where E_c is the cutoff value assumed for this component of the resolution. Comparison of the values of $M(W)$ for extreme values of E_0 in the range of the fold shows the shift in $M(W)$ is negligible.

The probability of energy loss from ionization is approximately constant up to a value of ~1.4 Mev for the converter and air path used. By using this value as a cutoff, a calculation based on the dE/dx curves given by Halpern and Hall[32] shows that the shift of the moment is negligible.

The effect of the finite width of the converter varies with the incident gamma-ray energy, but is always such as to increase the observed energy, the more so the greater the energy. Results of orbit calculations for different energy pair fragments permit evaluation of the shift in the moment of this component which turns out to be ~ -80 kev.

By the same method used for the finite converter width calculation the shift of the moments in the second-order multiple scattering component and vertical motion and finite converter height component are each ~ -30 kev.

The resultant shift thus adds to ~ -170 kev. However, two important factors tend to minimize any effects of this shift. First, the intensity of the theoretical curves is in general low outside a region within approximately 3 Mev of the energy value (~129 Mev) at which the resolution curve is determined, whereas the above shifts are computed on the basis of the extreme range of the fold (~12 Mev). Second, the method of treating the data (II, G) weights the folded curves inversely as the distance from this energy value (~129 Mev). We therefore conclude that the influence of these shifts on the final data is negligible as compared with the other sources of error (see the next section).

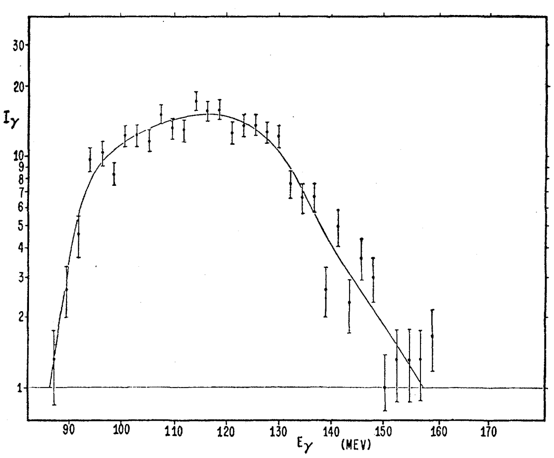

FIG. 7. Smoothed background spectrum uncorrected for energy dependent factors (Sec. II, F). The statistical probable error of the combined points is shown.

G. Treatment of the Data

The reduction of the raw data from the pion capture reactions requires the normalization of the background counts to those of the γ_p and γ_d signal data. Any normalization procedure depends on some method of evaluating the relative beam intensity and detection efficiency from one type of run to another. The beam monitoring methods attempted were not found to be internally consistent to better than ±10 percent over the ten day run.

An alternative procedure based on the data itself was used to obtain the normalization for the background subtraction. The background data were averaged over three energy channels using a simple arithmetical mean and a smooth curve was drawn through the result. The curve was adjusted to the data by balancing probable errors. This process was repeated independently for the two other possible ways of combining the results of three energy channels. The values of all three smoothed curves were averaged to give the values for the final smoothed background curve. This curve was then plotted on semilog paper (Fig. 7) to permit normalization by visual matching of slopes. This same procedure was carried out for the γ_p and γ_d signal data.

The theoretical resolution curve[25] mentioned above and the actual data show that the full width at half-

[31] W. Heitler, *The Quantum Theory of Radiation* (Oxford University Press, London, 1944), second edition, p. 225.

[32] O. Halpern and H. Hall, Phys. Rev. **73**, 484 (1948).

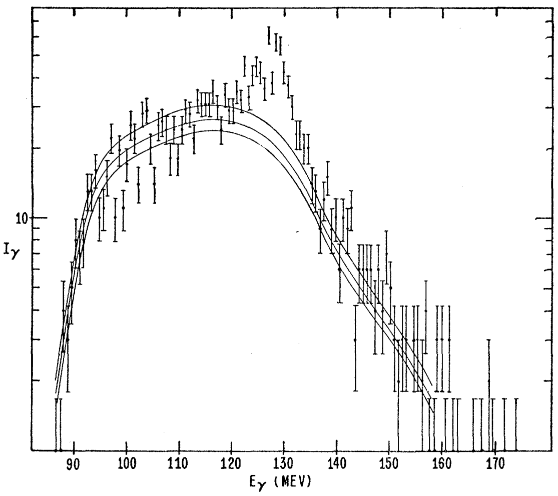

FIG. 8. Unadjusted data showing gamma-ray spectra from $\pi^-+d\rightarrow 2n+\gamma_d$. The data are uncorrected for energy-dependent factors (Sec. II, F). All three normalizations of the smoothed background spectrum are shown. The "best" fit is the middle curve.

maximum of the γ_p signal is approximately two Mev and the signal is below one percent of its peak value beyond ten Mev on either side of the peak. It is, therefore, possible to normalize the background to the γ_p signal by matching slopes of the two curves excluding the region within ten Mev of the peak of the γ_p signal. Because of the strong peaking of the signal as a result of the improved resolution, the resulting folded theoretical spectra are insensitive, within the statistical error, to a five percent change in the normalization factor.

The observed resolution curve (I_γ, Fig. 3), which was used in the fold is in good agreement with the theoretical resolution curve[25] and does not have any significant uncertainty resulting from background normalization.

In the normalization of the γ_d signal no assumption may be made regarding the limitation of signal intensity in the energy region below the peak since the shape of the spectrum is unknown. The background was therefore normalized using three different factors: (1) "best" fit, (2) fit to high-energy signal, and (3) fit to low-energy signal (Fig. 8). The final result was evaluated using all three factors to check the sensitivity of the data to the background normalization. The high and low normalization gives a final result in disagreement by 0.18 Mev at most, with the result calculated on the basis of "best" fit normalization. This effect is included in the probable error. It will be noted that the region of energy dependent losses as shown in Fig. 6 lies beyond any significant γ_d signal for any of the three normalization factors.

The final γ_d signal with background subtracted and the correction made for energy dependent factors (Sec. II, F) is shown in Fig. 9.

The analysis of the data was made by evaluating a weighted first moment of this net experimental data for comparison with the weighted first moment of the folded theoretical curves. The deuterium spectrum has a long tail toward low energy which is relatively insensitive to the n-n interaction. The method of weighting has been chosen to minimize this tail with a resulting reduction in the accuracy of the moment. Although elaborate statistical methods can be applied, we have chosen the weighting factor proportional to the reciprocal of the square of relative probable error at each point.

The weighted first moment, M, was determined as follows:

$$M=\sum \frac{N_iF_i}{W_i^2}E_i \Big/ \sum \frac{N_iF_i}{W_i^2},$$

where

$$W_i^2=\frac{\{[\frac{1}{3}(0.675)(m_{i-1}+m_i+m_{i+1})^{\frac{1}{2}}\mathfrak{N}]^2+(0.675)^2n_i\}}{N_i^2},$$

n_i = number of counts in channel i of the γ_d spectrum, m_i = number of counts in channel i of the background effects, $N_i=n_i-\mathfrak{N}m_i$, F_i = energy dependent correction factor (Sec. II, F), $\mathfrak{N}$ = background normalization factor. The same weights, W_i, were used in the determination of the weighted moment of the experimental data and the theoretical curves. The probable error of the calculated value of M was determined by the method of propagation of errors to be $\delta M=\pm 0.39$ Mev. M was considered as a function of one variable only, N_i. The contribution to the error in M from the error in the combined probable error of the signal and background is of the second order and was neglected.

M was determined for all three background normalization factors for the γ_d spectrum. The quoted value of M is based on the "best" fit normalization (see above). The two other normalization factors resulted in M values differing by $\Delta M=+0.18$ Mev (using background fit to high-energy signal) and $\Delta M=-0.13$

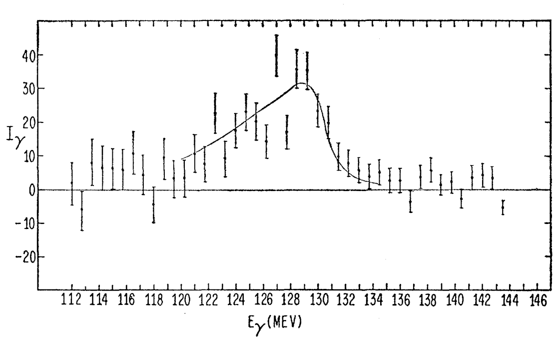

FIG. 9. Gamma-ray spectrum from $\pi^-+d\rightarrow 2n+\gamma_d$ with background subtracted. The data have been corrected for energy-dependent factors (Sec. II, F). The statistical probable error is shown. The solid line is the theoretical spectrum for $\hbar c\alpha=-15$ Mev. Of the calculated theoretical spectra, this is the one for which $\hbar c\alpha$ is closest to the experimentally measured $\hbar c\alpha=-12.4$.

Mev (using background fit to low-energy signal). The final error was evaluated as

$$[(\delta M)^2+(\Delta M)^2]^{\frac{1}{2}}=\begin{matrix}+0.43\\-0.41\end{matrix}\text{ Mev.}$$

III. RESULTS AND CONCLUSIONS

The parameter chosen for comparison of the experimental data with the theory is the weighted first moment as defined above. The position of Watson and Stuart's theoretical spectra[6] on the energy scale, relative to the observed γ_d spectrum, is established by the observed γ_p spectrum as described in Sec. II, A. The most probable value of $\hbar c\alpha$ corresponding to the observed γ_d spectrum is then determined in the following manner. The weighted first moments of all the folded theoretical curves are plotted in Fig. 10 relative to that of the spectrum for $\hbar c\alpha=0$ Mev as a function of $\hbar c\alpha$. A smooth curve is drawn through these points. The weighted first moment of the experimental γ_d spectrum relative to that of the spectrum for $\hbar c\alpha=0$ Mev is -0.43 Mev with a probable error of $+0.43$ Mev and -0.41 Mev. This value is plotted in Fig. 10 and the corresponding value of $\hbar c\alpha$ is determined by inspection as $\hbar c\alpha=-12.4\begin{matrix}+12.4\\-10.9\end{matrix}$ Mev. This leads to a "scattering length" of $a=-15.9\times10^{-13}$ cm with limiting values based on the probable errors of $a=8.5\times10^{-13}$ and $a=-\infty$, the latter value separating the bound and unbound states.

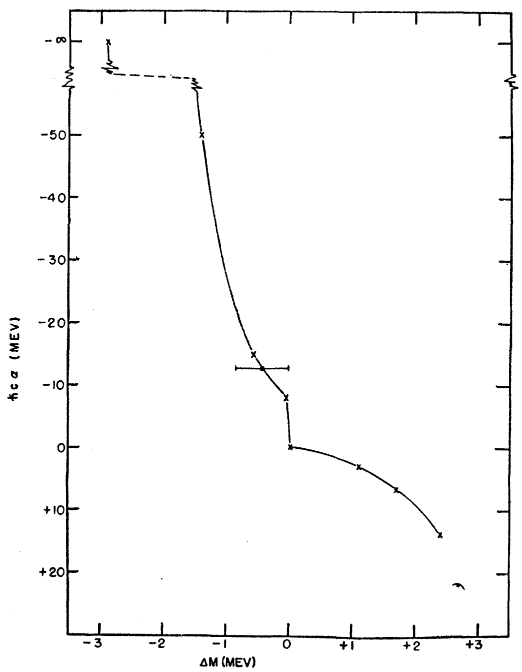

FIG. 10. Plot of the shift of the weighted first moment of the theoretical gamma-ray spectra from $\pi^-+d\rightarrow 2n+\gamma_d$ relative to the theoretical spectrum for $\hbar c\alpha=0$ Mev. The position of the weighted first moment of the experimental data relative to that of the theoretical spectrum for $\hbar c\alpha=0$ Mev is shown with the calculated probable error. $\Delta M=M_i-M_0$, where M_i is the moment for the theoretical spectra for which $\hbar c\alpha=+13.73$, $+6.59$, $+3.00$, -8, -15, -50, $-\infty$ Mev, and M_0 is the moment for the theoretical spectrum for which $\hbar c\alpha=0$.

It is seen that the results essentially rule out the "no interaction" case ($\hbar c\alpha=-\infty$) which lies outside the measured value by six probable errors. The probable error is still too large, however, to conclude definitely that there is no bound state of the n-n system. The probability is less than 0.1 percent that the dineutron is bound by more than 50 kev.

The results are consistent with the p-p and n-p scattering data and are thus in agreement with the theory of charge independence of nuclear forces.

IV. ACKNOWLEDGMENTS

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