

Letters to the Editor

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Isotope Shift in $\lambda 4415$ Cd II

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THE isotope shifts in $\lambda 4415$ Cd II ($4d^{10}5p^2P_{3/2} - 4d^95s^2^2D_{5/2}$) have been reported by several workers¹⁻⁴ using either natural cadmium or separated isotopes, although all of the latter were not employed. In view of the complex structure of this line, it seemed desirable to study suitable mixtures of all the isotopes. By using a Schüler hollow-cathode discharge tube cooled with liquid nitrogen as a source, and a Fabry-Perot etalon in conjunction with a large prism spectrograph as the dispersive element, the following isotopes and their mixtures were studied: Cd¹¹⁰ and Cd¹¹⁴, Cd¹¹⁰ and Cd¹¹⁶, Cd¹¹² and Cd¹¹⁶, Cd¹¹³, Cd¹¹³ and Cd¹¹⁶, Cd¹¹¹, Cd¹¹¹ and Cd¹¹⁶, Cd¹⁰⁸ and Cd¹¹⁴, Cd¹⁰⁶ and Cd¹¹⁴, and Cd¹⁰⁶ and Cd¹¹⁰.

Table I lists the separations in millikaysers of the lines due to the various isotopes as compared to those of previous workers. A positive value indicates a shift toward the violet. The Cd¹¹⁰ line is taken as the origin.

From the consistency of the data the separations in the present work are accurate to ± 1.5 mK.

Both Cd¹¹¹ and Cd¹¹³ showed only two components in their hyperfine structure. The strong component lay

TABLE I. Separations with respect to Cd¹¹⁰ line, in millikaysers (1 kaysers = 1 cm^{-1}).

Cadmium isotope	Present work	Separation (mK)		
		Schüler and Westmeyer ^a	Murakawa ^b	Hindmarsh, Kuhn, and Ramsden ^c
106	-112.2	...	-106	...
108	-51.2	...	-56	...
111 strong component	-15.2	...	-20	...
110	0.0	0	0	0.0
111 weak component(s)	{ 38.8	...	33	...
	{	59	...
113 strong component	39.4	...	35	...
112	53.1	54	55	53.3
113 weak component(s)	{ 95.9	...	91	...
	{	118	...
114	99.9	105	109	101.2
116	135.6	...	147	135.6

^a See reference 1.

^b See reference 3.

^c See reference 4.

toward the red in each case. The ratio of intensities was measured by photographic photometry to be 1.35 ± 0.05 . Thus the hyperfine structure of both Cd¹¹¹ and Cd¹¹³ is due to two close-lying components in the $^2D_{5/2}$ state. This is analogous to the results of Ritschl⁵ on the spectrum of Cu I and of Schüler and Westmeyer⁶ on the spectrum of Zn II. Using the theoretical ratio of intensities of 1.40, the center of gravity of the Cd¹¹¹ pattern is located at +7.3 mK with respect to the Cd¹¹⁰ line, and the center of gravity of the Cd¹¹³ pattern is located at +62.9 mK with respect to the Cd¹¹⁰ line, giving a relative shift between Cd¹¹¹ and Cd¹¹³ of 55.6 mK. The anomalous shift of Cd¹¹³ reported by Woodward² was in error because of inadequate cleansing of the hollow cathode between use with different samples. The ratio of magnetic moments of Cd¹¹³ to Cd¹¹¹ was found to be 1.046 ± 0.027 , in good agreement with the value 1.0461 ± 0.0001 found by Proctor and Yu.⁷

¹ H. Schüler and H. Westmeyer, Z. Physik **82**, 685 (1933).

² E. C. Woodward, Phys. Rev. **93**, 948 (1954).

³ K. Murakawa, Phys. Rev. **93**, 1232 (1954).

⁴ Hindmarsh, Kuhn, and Ramsden, Proc. Phys. Soc. (London) **A67**, 478 (1954).

⁵ R. Ritschl, Z. Physik **79**, 1 (1932).

⁶ H. Schüler and H. Westmeyer, Z. Physik **81**, 565 (1933).

⁷ W. G. Proctor and F. C. Yu, Phys. Rev. **76**, 1728 (1949).

Quantum Theory of Cyclotron Resonance in Semiconductors

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WE have investigated in detail the quantum theory of "cyclotron" resonance of holes in semiconductors such as Si and Ge. In the case of electrons the quantum theory of cyclotron resonance is identical with the classical theory.¹ This is connected with the fact that the classical or W.K.B. approximation gives *exactly* the correct quantum levels for a simple harmonic oscillator. For holes the situation is very different because of complications arising from the degeneracy at the top of the valence band.² In this case we have shown that the quantum theory leads to different energy levels and selection rules for low quantum numbers, though of course for high quantum numbers the classical results are again valid.¹ Under the conditions of experiments such as those of Lax *et al.*³ conducted at liquid helium temperature, the mean quantum number is ~ 5 . By lowering the temperature further (to say 1 or 1.5°K) it should be possible to enhance the quantum effects and observe additional resonances. In fact there may be some evidence of these resonances in the structure observed by Dresselhaus, Kip, and Kittel.⁴

The quantum result may be formulated as follows. Consider first the case of no spin-orbit coupling. Then it is known² that in the absence of an external magnetic field the energy surfaces are found by solving the secular

equation:

$$\begin{pmatrix} Ak_x^2 + B(k_y^2 + k_z^2) & Ck_xk_y & Ck_xk_z \\ Ck_xk_y & Ak_y^2 + B(k_z^2 + k_x^2) & Ck_yk_z \\ Ck_xk_z & Ck_yk_z & Ak_z^2 + B(k_x^2 + k_y^2) \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = E \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}, \quad (1)$$

and the wave function is given by

$$\psi = a_1\varphi_1 + a_2\varphi_2 + a_3\varphi_3, \quad (2)$$

$\varphi_1, \varphi_2, \varphi_3$ being the p -like degenerate functions at the top of the valence band. The essential result of our investigation is that to obtain the level scheme with an external field one need only replace k_x by $(1/i)(\partial/\partial x) - (e/\hbar c)A_x$, etc., where \mathbf{A} is the vector potential of the field, and regard the a_i as functions of the coordinates. If there is any ambiguity in the order of two non-commuting factors, the symmetrized product is meant. If spin-orbit coupling is taken into account then, *mutatis mutandis*, the identical prescription still applies. Just what matrix replaces that of Eq. (1) depends on which model of the band one chooses. Kittel's model,⁴ which seems most reasonable at the present time, would lead to a four-by-four matrix, the elements of which are linear combinations of those of Eq. (1). The resulting system of coupled differential equations is at present being investigated. A more detailed publication is in preparation.

This work was carried out while we were guests of the Bell Telephone Laboratories, and we should like to take this opportunity of thanking the staff for their cooperation and friendliness.

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¹ The classical theory is due to W. Shockley, Phys. Rev. **79**, 191 (1950).

² The theory of the behavior of such bands near a degeneracy point is due to W. Shockley, Phys. Rev. **78**, 173 (1950). Shockley did not take into account the effect of spin-orbit coupling on the degeneracy, and its importance seems first to have been pointed out by R. J. Elliott (unpublished). As far as we know, there is no treatment of the spin-orbit case in print.

³ Dexter, Zeiger, and Lax, Phys. Rev. **94**, 557 (1954).

⁴ Dresselhaus, Kip, and Kittel, Phys. Rev. **95**, 568 (1954).

Redetermination of the Hyperfine Splitting in the Ground State of Atomic Hydrogen*

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A REDETERMINATION has been made of the zero-field hyperfine splitting of the ground state of atomic hydrogen. A microwave absorption technique was used employing a resonance line of ~ 3 -kc/sec width at half-maximum absorbed power. This width ($\frac{1}{8}$ of the normal Doppler breadth) was obtained through the mechanism of collision reduction of the Doppler effect.¹ Atomic hydrogen at a partial pressure of about 5×10^{-4} mm was mixed with very clean molecular hydrogen (~ 0.2 mm pressure), a medium with which

the atomic hydrogen can collide without disturbing its spin state. This provided a long diffusion time to the gas container walls where spin relaxation collisions may occur. The primary relaxation mechanism was electron exchange collisions² between two hydrogen atoms, the cross section for which is much greater than the geometrical cross section. Because of the paired electron spins in the molecule, electron exchange effects in the atom-molecule collisions are negligible. Furthermore, dipole-dipole and spin-orbit relaxation effects are very weak in such a collision. The production of narrower lines is presently limited by the rather poor noise figure of the detection equipment.

The resonant frequency, as measured, was shifted slightly by the following pressure-dependent mechanism: during an atom-molecule collision, although the molecular electric fields do not appreciably disturb the atom's hyperfine state (quantum numbers F, m_F), they mix some P state into the wave function. As the hyperfine interaction in P states is much weaker than in the $1S$ state, the result is a reduction in the time-averaged interaction energy, causing a small reduction of the measured splitting. The measured shift of about 100 cycles/sec is in agreement with a rough calculation.

The hyperfine transitions were induced in a cylindrical cavity excited in the TE_{112} mode, the hydrogen sample being confined in a glass bottle at the cavity center. The atomic hydrogen was produced in a Wood's discharge external to the cavity, and was pumped and diffused into the bottle. A triode oscillator, phase locked to the sum frequency of a harmonic of a very stable crystal-controlled oscillator and a stable variable-frequency oscillator (~ 433 kc/sec), supplied the microwave energy. A weak magnetic field of ~ 0.06 gauss parallel to the rf magnetic field led to the $\Delta M_F = 0$ transition. The direction of this weak field was varied from parallel to nearly perpendicular at 30 cycles/sec causing a 30-cycle modulation of the absorbed power. The resultant modulated signal was detected in a balanced barretter mixer, amplified at 30 cycles/sec, and fed into a lock-in amplifier. The locking signal was derived from the 30-cycle magnetic field modulation. A cavity-tuning technique was developed that kept the phase of the signal from the cavity, with respect to the carrier in the barretters, adjusted at all times to to produce a purely absorptive resonance. The line contour was determined experimentally to be Lorentz shaped.

Data were taken by measuring the signal strength at three discrete frequencies on the resonance line and fitting a Lorentz curve through the points. The line center was determined from this fitted curve. About 35 determinations were made at each of three pressures.