

equation:

$$\begin{pmatrix} Ak_x^2 + B(k_y^2 + k_z^2) & Ck_xk_y & Ck_xk_z \\ Ck_xk_y & Ak_y^2 + B(k_z^2 + k_x^2) & Ck_yk_z \\ Ck_xk_z & Ck_yk_z & Ak_z^2 + B(k_x^2 + k_y^2) \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = E \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}, \quad (1)$$

and the wave function is given by

$$\psi = a_1\varphi_1 + a_2\varphi_2 + a_3\varphi_3, \quad (2)$$

$\varphi_1, \varphi_2, \varphi_3$ being the p -like degenerate functions at the top of the valence band. The essential result of our investigation is that to obtain the level scheme with an external field one need only replace k_x by $(1/i)(\partial/\partial x) - (e/\hbar c)A_x$, etc., where \mathbf{A} is the vector potential of the field, and regard the a_i as functions of the coordinates. If there is any ambiguity in the order of two non-commuting factors, the symmetrized product is meant. If spin-orbit coupling is taken into account then, *mutatis mutandis*, the identical prescription still applies. Just what matrix replaces that of Eq. (1) depends on which model of the band one chooses. Kittel's model,⁴ which seems most reasonable at the present time, would lead to a four-by-four matrix, the elements of which are linear combinations of those of Eq. (1). The resulting system of coupled differential equations is at present being investigated. A more detailed publication is in preparation.

This work was carried out while we were guests of the Bell Telephone Laboratories, and we should like to take this opportunity of thanking the staff for their cooperation and friendliness.

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¹ The classical theory is due to W. Shockley, Phys. Rev. **79**, 191 (1950).

² The theory of the behavior of such bands near a degeneracy point is due to W. Shockley, Phys. Rev. **78**, 173 (1950). Shockley did not take into account the effect of spin-orbit coupling on the degeneracy, and its importance seems first to have been pointed out by R. J. Elliott (unpublished). As far as we know, there is no treatment of the spin-orbit case in print.

³ Dexter, Zeiger, and Lax, Phys. Rev. **94**, 557 (1954).

⁴ Dresselhaus, Kip, and Kittel, Phys. Rev. **95**, 568 (1954).

Redetermination of the Hyperfine Splitting in the Ground State of Atomic Hydrogen*

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A REDETERMINATION has been made of the zero-field hyperfine splitting of the ground state of atomic hydrogen. A microwave absorption technique was used employing a resonance line of ~ 3 -kc/sec width at half-maximum absorbed power. This width ($\frac{1}{8}$ of the normal Doppler breadth) was obtained through the mechanism of collision reduction of the Doppler effect.¹ Atomic hydrogen at a partial pressure of about 5×10^{-4} mm was mixed with very clean molecular hydrogen (~ 0.2 mm pressure), a medium with which

the atomic hydrogen can collide without disturbing its spin state. This provided a long diffusion time to the gas container walls where spin relaxation collisions may occur. The primary relaxation mechanism was electron exchange collisions² between two hydrogen atoms, the cross section for which is much greater than the geometrical cross section. Because of the paired electron spins in the molecule, electron exchange effects in the atom-molecule collisions are negligible. Furthermore, dipole-dipole and spin-orbit relaxation effects are very weak in such a collision. The production of narrower lines is presently limited by the rather poor noise figure of the detection equipment.

The resonant frequency, as measured, was shifted slightly by the following pressure-dependent mechanism: during an atom-molecule collision, although the molecular electric fields do not appreciably disturb the atom's hyperfine state (quantum numbers F, m_F), they mix some P state into the wave function. As the hyperfine interaction in P states is much weaker than in the $1S$ state, the result is a reduction in the time-averaged interaction energy, causing a small reduction of the measured splitting. The measured shift of about 100 cycles/sec is in agreement with a rough calculation.

The hyperfine transitions were induced in a cylindrical cavity excited in the TE_{112} mode, the hydrogen sample being confined in a glass bottle at the cavity center. The atomic hydrogen was produced in a Wood's discharge external to the cavity, and was pumped and diffused into the bottle. A triode oscillator, phase locked to the sum frequency of a harmonic of a very stable crystal-controlled oscillator and a stable variable-frequency oscillator (~ 433 kc/sec), supplied the microwave energy. A weak magnetic field of ~ 0.06 gauss parallel to the rf magnetic field led to the $\Delta M_F = 0$ transition. The direction of this weak field was varied from parallel to nearly perpendicular at 30 cycles/sec causing a 30-cycle modulation of the absorbed power. The resultant modulated signal was detected in a balanced barretter mixer, amplified at 30 cycles/sec, and fed into a lock-in amplifier. The locking signal was derived from the 30-cycle magnetic field modulation. A cavity-tuning technique was developed that kept the phase of the signal from the cavity, with respect to the carrier in the barretters, adjusted at all times to to produce a purely absorptive resonance. The line contour was determined experimentally to be Lorentz shaped.

Data were taken by measuring the signal strength at three discrete frequencies on the resonance line and fitting a Lorentz curve through the points. The line center was determined from this fitted curve. About 35 determinations were made at each of three pressures.

TABLE I. Results of the measurement of ν_0 .

Pressure, mm Hg	No. of observations	Frequency,* Mc/sec
0.14	35	1420.40572 ± 0.00003
0.18	35	1420.40575 ± 0.00003
0.27	33	1420.40569 ± 0.00002
0 (extrapolated) ...		1420.40580 ± 0.00005

* A 10-cycle/sec correction to the frequencies caused by the 0.06-gauss field is included.

The results, together with the value obtained by extrapolation to zero pressure, are given in Table I.

These results are not likely to be changed by more than ± 20 cycles/sec by the astronomical time correction to the nominal *WWV* frequencies, which, however, will not be available for several months. The listed uncertainties are probable errors. The present determination is in disagreement with the value of the hyperfine splitting using a molecular beam technique,³ which gave

$$\nu_0 = 1420.4051 \pm 0.0002 \text{ Mc/sec.}$$

Both of the above results are in agreement with theory within the uncertainties introduced by high-order radiative corrections and proton structure effects.

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¹ R. H. Dicke, *Phys. Rev.* **89**, 472 (1953).

² First proposed to E. M. Purcell by N. F. Ramsey and V. F. Weisskopf.

³ A. G. Prodel and P. Kusch, *Phys. Rev.* **88**, 184 (1952).

Hall Effect in Positive Column

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HALL voltage was observed in the positive column of dc gas discharge tubes, and was measured as a function of magnetic field, tube current, distance between probes, and gas pressure by the floating double probe method.

With a Helmholtz coil having a radius of 20 cm, a homogeneous magnetic field was applied perpendicular to the axis and to the line joining two probes which were placed perpendicular to the tube axis.

A hot cathode tube was constructed with a Pyrex cylinder 37 mm in inside diameter and 70 cm in total length, and four probes were inserted in the positive column near the middle part of this tube as shown in Fig. 1. Probes 1 and 2 were used to measure the Hall voltage and variations of the plasma densities at these points, and probes 3 and 4 were used for determining the electric field and density on the axis of the column. Argon and neon were used, with pressures ranging from 0.2 to 15 mm Hg, and the tube current ranged from 0.04 to 4 amperes.

An external magnetic field deflected the column in the same direction as electrons would be bent in vac-

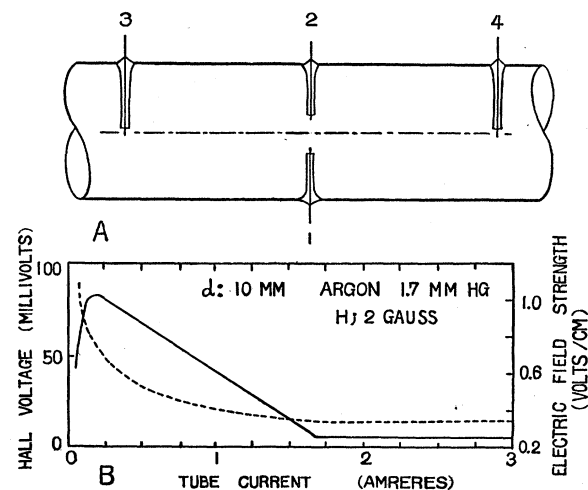


FIG. 1. A: The part of the tube where four probes are inserted. B: The Hall voltage and the electric field in the column vs tube current, in a magnetic field of 2 gauss. The tube contains argon at a pressure of 1.7 mm Hg. The solid and dotted lines indicate the Hall voltage and electric field, respectively.

uum, but the sign of the Hall voltage was opposite to that of an *n*-type semiconductor.

Figure 1 indicates a general relation between the Hall voltage and the tube current at constant magnetic field and pressure. When the tube current was increased, the Hall voltage first increased to a maximum, then decreased, and finally reached a constant value, whereas the electric field in the column decreased monotonically. Therefore three regions were distinguished, corresponding to the character of the Hall voltage.

When the gas pressure was increased, the second region became narrow and the constant value in the third region became larger. It should also be noted that at large tube current and very low pressure the sign of the Hall voltage was reversed.

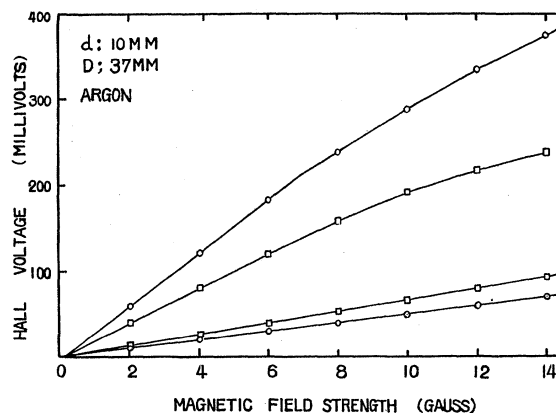


FIG. 2. Hall voltage vs magnetic field. The open circles and squares are data taken with the tube containing argon at pressures of 0.7 mm Hg and 1.4 mm Hg, respectively. The upper two lines are taken in the second region (at 1.0 amp) and the lower two lines in the third region (at 2.7 amp).