

"fill in" the minima.<sup>20</sup> If this is the case, there seems to be a reasonable agreement between theory and experiment.

On the other hand, in the case of Bi<sup>210</sup> there does not seem to be much similarity between the experimental angular distribution and the distribution expected for either  $l_n=4$  or  $l_n=6$ , the values expected on the basis of the shell model. The fact that the distribution differs from the Pb<sup>208</sup> g.s. distribution in having a minimum at small angles would lead one to conclude that  $l_n \geq 2$ . Consistent with the interpretation of a high  $l_n$  value for the last neutron in the Bi<sup>210</sup> is the observation that no proton group occurs with an intensity of 1/30 the intensity of the  $Q=1.94$  Mev state at an angle of 45°. The nonsimilarity between the predicted and observed distributions may be because approximations of the stripping theory with respect to the Coulomb interaction are worse for larger angular momentum transfers or that for large  $l_n$  the reaction may not be Butler stripping. One unfortunately is therefore unable to assign an  $l_n$  value to the last neutron in Bi<sup>210</sup>.

### Odd-Even Effect

An explanation, as offered above for the low-intensity Bi<sup>210</sup> g.s., would be consistent with Harvey's<sup>2</sup> observation of low ( $d,p$ ) cross sections for odd  $N$ -even  $Z$  target

<sup>20</sup> W. Tobocman (private communication).

nuclei relative to isotopes of the same element and even  $N$ , if the matrix elements for nearby isotopes were similar. For example, Harvey gives, at an angle of 20°,

$$\frac{d\sigma}{d\omega}(\text{Zr}^{92}) / \frac{d\sigma}{d\omega}(\text{Zr}^{91}) = \frac{1}{40}.$$

If in both cases the last neutron is in a  $d_{5/2}$  state, the relative differential cross section would be

$$\left[ \frac{2I(\text{Zr}^{92})+1}{2J(\text{Zr}^{91})+1} \right] \left[ \frac{2J(\text{Zr}^{90})+1}{2I(\text{Zr}^{91})+1} \right] = \frac{1}{[2J_i+1]^2} = \frac{1}{36},$$

where  $J_i$  is the total angular momentum of the resultant nucleus for the reaction involving the lighter target nucleus.

In the case of the Ti isotopes, however, where the last neutron is probably in a  $f_{7/2}$  state, the expected ratio is  $1/[2J_i+1]^2=1/64$ , whereas the observed ratio is only about 0.10.

However, many of the nuclei given by Harvey have relative differential cross sections in agreement with the supposition that the matrix elements for nearby nuclei are similar to within a factor of 2 or 3. It may be that for those nuclei where there is serious disagreement, the  $l_n$  in each case is different. Factors other than the statistical weight would then be expected to cause such a difference.

## Determination of the Ranges and Straggling of Low-Energy Alpha Particles in a Cloud Chamber

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Ranges of 105 alpha particles with energies of 0.48, 0.545, and 0.615 Mev as fixed by a velocity selector have been measured by the use of a low-pressure cloud chamber. From the results of these measurements the average ranges  $R_A$  and the straggling coefficients in air at 15° and 760 mm were calculated for these three energies. The values of  $R_A$  were found to be respectively 0.299, 0.327 and 0.354 cm; and the values of the standard deviation were respectively 0.011, 0.0135, and 0.010 cm. After a correction to take into account the difference in definition, the  $R_A$ 's are two to three percent higher than those following from a range-energy curve given by Bethe.

### INTRODUCTION

THE theoretical expression obtained by Bethe for the energy loss of charged particles in a gas gives, if the effects of the binding of the  $K$ -shell electrons of the atoms of the gas are taken into account, quite accurate results for alpha particles down to about 5 Mev.<sup>1</sup> For energies below this, its usefulness is limited because of (1) the difficulty of calculating the  $L$ -shell

binding correction if the atoms of the gas are of low atomic number, (2) the breakdown of the Born approximation upon which Bethe's derivation is based, and finally, (3) the occurrence below about 1 Mev of an additional complication in the stopping process itself, namely, charge exchange, for which as yet no satisfactory theoretical treatment exists.<sup>2</sup> Hence, the range-

<sup>1</sup> M. S. Livingston and H. A. Bethe, *Revs. Modern Phys.* **9**, 261 (1937).

<sup>2</sup> Further details and references are to be found in the review article by H. A. Bethe and J. Ashkin in *Experimental Nuclear Physics* (John Wiley and Sons, Inc., New York, 1953), Vol. I, Part II.

energy relation for alpha particles below 5 Mev must be based largely on experimental measurements.

Experimental range values are obtained, in general, either from direct measurements of track lengths observed in a cloud chamber or from various types of measurements with ionization chambers.<sup>2</sup> The range-energy relation in the low-energy region that has been generally adopted is based on the experiments of Holloway and Livingston<sup>3</sup> in which the specific ionization along the path of polonium alphas in air was measured. In order to obtain from their results a range-energy relation, it is necessary to know  $w$ , the average energy lost per ion pair formed in air as a function of the energy of the alpha particle. Recently,  $w$  was measured by Jesse *et al.*<sup>4,5</sup> for energies between 0.3 and 5.3 Mev. Jesse and Sadauskis<sup>5</sup> and Bethe<sup>6</sup> have used these results to construct the corresponding range-energy relation. This relation will be referred to as the "Bethe 1950" relation in order to distinguish it from an earlier one (1937).<sup>1</sup>

This relation is in good agreement with some well established range-energy points above 1.5 Mev obtained from cloud-chamber studies of alpha particles produced by nuclear reactions whose energy values have been quite precisely determined. Recently, Cook *et al.*<sup>7</sup> made some range determinations by measuring the Bragg ionization curves of artificially accelerated alphas of energies between 0.01 and 0.25 Mev. Their results are in good agreement with Bethe's 1950 relation, which in this range was obtained by an extrapolation of Jesse and Sadauskis' results to low energies. There is, however, a discrepancy between Bethe's 1950 relation and the results of some "absolute" range measurements carried out by Jesse and Sadauskis in the course of the above-mentioned investigation. Thus, for a range of 0.790 cm in air the direct measurements gave an energy value 0.04 Mev lower than the 1.611 Mev read from the curve.

The most widely quoted cloud-chamber investigation of low-energy alpha particles (0.01 to 0.3 Mev) is that of Blackett and Lees.<sup>8</sup> They investigated 38 tracks of low-energy alpha particles in a low-pressure cloud chamber, the particles originating in collisions between polonium alphas and helium atoms in the chamber. In order to obtain from the configuration of the tracks the energy of the slow alphas, these investigators had to estimate the energy of the other (faster) alphas emerging from the collision by measuring their track lengths and employing Briggs' range-energy relation. Later experiments showed Briggs' energy values to be

9 percent too low at 5.30 Mev, and consequently Bethe increased all of Briggs' and Blackett and Lees' energy values by this proportion in constructing his 1937 range-energy curve.<sup>1</sup> Nevertheless, the resulting curve still gives energies considerably below those obtained from the Bethe 1950 relation for a given range.

Furthermore, there is a much more recent cloud-chamber investigation by Mills<sup>10</sup> covering the energy range between 0.01 and 0.35 Mev. He employed a chamber which was filled with a mixture of forty percent water vapor and sixty percent helium and operated at a pressure of 45 mm of mercury before expansion. The slow alphas were obtained from elastic collisions of collimated monoenergetic neutrons with the helium in the chamber. In order to make his air equivalent range agree with that following from Bethe's 1950 range-energy curve<sup>6</sup> at about 0.3 Mev, Mills assumed that a large increase in the amount of water vapor took place throughout the chamber as a result of the expansion. Since the diffusion of water vapor under the conditions existing in the chamber is too slow to account for an appreciable increase in the vapor by the evaporation of water from the walls, he postulated the presence of small invisible droplets suspended throughout the body of the chamber. Mills' and Bethe's curves disagree, however, at other energies.

The lack of agreement between ionization and direct cloud-chamber measurements in the extreme low-energy region indicates a need for further measurements below 1 Mev. Since the results of the previous cloud-chamber investigations appear questionable, it would seem that any new experimental determinations would be most valuable if they were to be based on the cloud-chamber method. Furthermore, the error in the ionization measurements may be greatest below 1 Mev because complications arise in determining the end point of the range and because processes other than ionization (and excitation) begin to contribute appreciably to the energy loss.

The determination of ranges in the cloud chamber has the further advantage that one obtains quite directly the amount of range straggling. This latter quantity is of especial interest at low energies because the relative straggling should tend to increase as a larger proportion of the energy loss is due to nuclear collisions. At a somewhat higher energy (1.2 Mev) than those considered here, Bøggild<sup>11</sup> found a relative range straggling for alphas amounting to about 2 percent, although that due to electronic processes alone can be expected to be only about 1 percent.<sup>12</sup> Some doubt is cast on Bøggild's results, however, because of the fact that his experiments were done in such a way that the quantity measured was the combined straggling of

<sup>3</sup> M. G. Holloway and M. S. Livingston, *Phys. Rev.* **54**, 18 (1938).

<sup>4</sup> Jesse, Forstat, and Sadauskis, *Phys. Rev.* **77**, 782 (1950).

<sup>5</sup> W. P. Jesse and J. Sadauskis, *Phys. Rev.* **78**, 1 (1950).

<sup>6</sup> H. A. Bethe, *Revs. Modern Phys.* **22**, 213 (1950).

<sup>7</sup> Cook, Jones, and Jorgensen, *Phys. Rev.* **91**, 1417 (1953).

<sup>8</sup> P. M. S. Blackett and D. S. Lees, *Proc. Roy. Soc. (London)* **A134**, 658 (1932).

<sup>9</sup> G. H. Briggs, *Proc. Roy. Soc. (London)* **A114**, 341 (1927).

<sup>10</sup> R. G. Mills, *Rev. Sci. Instr.* **24**, 1041 (1953). See also: R. G. Mills, University of California Radiation Laboratory Report UCRL 1815, 1952 (unpublished).

<sup>11</sup> J. K. Bøggild, *Nature* **161**, 810 (1948).

<sup>12</sup> N. Bohr, *Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd.* **18**, 8, 128 (1948).

alpha particles and lithium ions which had to be divided somewhat arbitrarily into the contributions of the two particles.

We have decided therefore to make a study of the range and straggling of alpha particles for energies below one Mev by the use of a low-pressure cloud chamber. Since at least a part of the difficulties encountered in the previous cloud-chamber studies arises from the fact that the initial energy of the low-energy alpha particles was always determined in a somewhat indirect fashion, we arranged the experiment so that the energy of the particles was known when they entered the chamber. For this purpose we made use of a natural alpha emitter (polonium) as a source of low-energy particles. The alphas were first slowed down and then entered an energy selector of sufficiently good resolution which permitted the particles having the desired energy to enter the chamber.

## EXPERIMENTAL METHOD

### Description of Apparatus

The experimental arrangement is shown in the diagram in Fig. 1. The alpha particles emitted by the polonium source were slowed down in passing through the (adjustable) air gap and the aluminum window before entering the magnetic analyzer. The particles of the desired energy were selected by the analyzer and were admitted to the cloud chamber through a thin nylon window.

The details were as follows. The source,<sup>13</sup> which had an average strength of 5 to 10 millicuries, was plated on a metal disk mounted on a micromanipulator which, in turn, was rigidly attached to the analyzer. The analyzer was constructed as shown in the figure. The six baffle plates, which were staggered in order to facilitate pumping, defined an annular channel of a mean curvature of 16.5 cm, a width of 0.6 cm, and a length of 14 cm. For a given magnetic field, the disposition of these plates essentially determined the mean energy of the particles traversing the spectrograph while the aperture and the resolving power depended also on the location and size of the exit window. In order to facilitate the process of aligning the instrument, the exit tube, to which the window is attached, was joined to the rest of the spectrograph by a sylvon.

A geometrical analysis of the allowed trajectories was carried out which showed that for the energies of interest to us (0.40 to 0.65 Mev) the resolving power of the velocity selector amounted to about 1.5 percent in momentum for incident particles with a uniform distribution in magnitude and direction of momentum. The pressure in the analyzer was kept below  $6 \times 10^{-6}$  cm, so that there was presumably no appreciable contribution from scattering in the gas. Since the particles after passing through the aluminum window had a larger

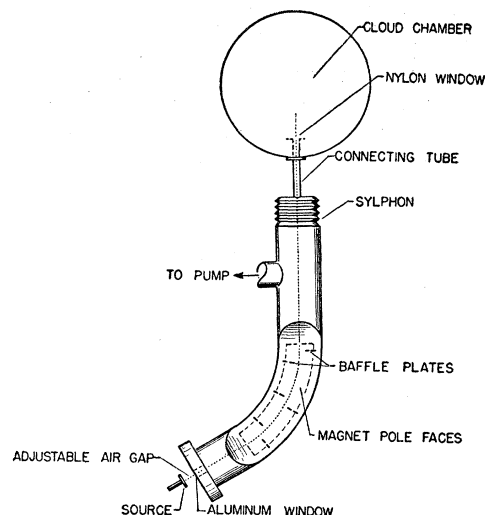


FIG. 1. Schematic diagram of the experimental arrangement.

spread in momentum, the energy dispersion of those entering the cloud chamber was essentially determined by the velocity selector. The magnetic field which was varied in strength between 8200 and 7300 gauss, was produced by an electromagnet with pole pieces of the shape indicated in the figure and a pole gap of 1.4 cm. In order to prevent the magnet from overheating, it was excited for only about ten percent of the cycle of the cloud chamber. For reasons stated above, it is desirable that most of the range of the particles leaving the velocity selector be in the cloud chamber. Therefore, the window between the chamber and the analyzer should be just thick enough to withstand the pressure difference between the chamber and the analyzer (between 13 and 20 cm of Hg). It was found that a nylon film about 0.04 mg/cm<sup>2</sup> thick, backed up by a perforated plate was satisfactory for this purpose. The film consisted of 8 layers prepared in the usual fashion.<sup>14</sup>

The cloud chamber was of the horizontal type and cylindrical in shape, with a height of 3.2 cm unexpanded, and a diameter of 25 cm. The chamber was filled with a mixture of hydrogen gas and water and ethyl alcohol vapor. At the operating temperature of 20.5°C the pressure before expansion amounted to 12.25 cm for the hydrogen, 1.50 cm for the water, and 2.24 cm for the alcohol, the last two pressures being the equilibrium values above a 50-50 volume mixture of the corresponding liquids. The expansion ratio employed was 1.25, and the cycling time was one minute, the sequence of operations being governed by an electronic controller.

We had investigated previously<sup>15</sup> the question of whether it is necessary to limit the natural sensitive time of the chamber for an experiment of this sort. We found that, under the conditions specified above, this

<sup>13</sup> This source was obtained from the Canadian Radium-Uranium Corporation.

<sup>14</sup> Brown, Febber, Richards, and Saxon, *Rev. Sci. Instr.* **19**, 818 (1948).

<sup>15</sup> S. Barile and R. Webeler, *Rev. Sci. Instr.* **25**, 389 (1954).

time (for the formation of sharp tracks) is 0.014 sec during which period the expansion goes from 90 to 99 percent of completion. The change of conditions in the chamber during this period is too small to introduce any appreciable errors, and therefore no special arrangement was required.

The cloud chamber was arranged so that two stereoscopic views could be obtained on a single negative, one by the use of a mirror. The direction of the direct view was at an angle of  $3^\circ$ , and that of the other, at an angle of  $15^\circ$  with the axis of the chamber. The direction of entry of the particles into the chamber was essentially perpendicular to the direction of both views. From the two views of a track any large deflections or curvature in the path of the particle could be detected.

### Experimental Procedure

Before taking cloud-chamber pictures of particles with a given energy entering the chamber, the position of the source and the position of the exit window were adjusted as follows. The nylon window was replaced by a glass plate coated with silver-activated zinc sulfide which served as a scintillation counter in conjunction with a photomultiplier tube mounted inside the cloud chamber. The air gap between the source and the entrance window was adjusted with the aid of the micromanipulator, and the position of the exit window was varied by moving the cloud chamber with respect to the spectrograph, so as to give a maximum counting rate with the desired magnetic field.

The same arrangement was also used to determine the air equivalent of the nylon exit window at 0.615 Mev (which was one of the energies investigated). The source position and the alignment of the selector were adjusted as described, and a nylon film of the correct thickness was interposed between the entrance window and the baffle plate system of the analyzer by means of a vacuum-tight seal. The air gap was then varied by moving the source with the micromanipulator and the new source position corresponding to a maximum

counting rate located. The change in the air gap thus produced was found to be 0.046 cm which was taken as the air equivalent of the film. This value agrees within 5 percent with the calculated air equivalent given in a later section.

Cloud-chamber photographs were obtained in this fashion with three settings of the analyzer corresponding to alpha particle energies  $E_0$  of 0.615, 0.545, and 0.48 Mev and were analyzed as follows. The stereoscopic pictures were projected to full size by using the camera with which they were taken as a projector and adjusting its position so that the image of a ruler photographed alongside the chamber appeared in its natural size. Measurements of the total track length on the projection of the direct view were made by the use of dividers set at 3 mm. Since near the window the alpha particles did not have any visible droplet trail, the length of the missing portion of the track was simply taken as being equal to the length of a straight line from the window to the beginning of the visible portion of the track, drawn so as to be tangent at this point. Tracks having kinks were excluded from consideration because it was difficult to obtain their lengths with any accuracy. Since such tracks were in any case few in number, neglecting them cannot have affected the results appreciably. However, tracks which showed sudden changes in droplet density as a consequence of charge exchange were used if otherwise normal.

The range usually quoted is simply the projection  $x$  of the total path on a line parallel to the direction of the particle at the beginning of the path. Therefore, in order to compare our results with this conventional range, we measured this quantity for each track. From  $x$  and  $r'$ , the average length of the track in two dimensions, the (most probable) true length  $r$  in three dimensions can be estimated. A simple consideration shows that on the average  $r = 2r' - x$ , if  $(r' - x)$  is small enough as was the case here. The resulting distributions for  $r'$  which are based on 105 tracks, are shown in Fig. 2 for the three energies mentioned.

### DISCUSSION OF RESULTS

#### (a) General

The results of an investigation of the present sort are usually presented in terms of the parameters of the distribution of ranges of individual particles in air. The range itself is ordinarily represented by the arithmetic mean range in air,  $R_{Av}$ ; the median range,  $R_m$ ; or the extrapolated range  $R_e$ . To define the extrapolated range, it is necessary to consider the number-distance curve, which represents the number of particles with a range equal to or larger than a given distance as a function of this distance. If now a line is drawn tangent to this curve at the point of steepest slope, the abscissa of the point at which it intersects the range axis is equal to  $R_e$ . The straggling is usually specified in terms of the range straggling coefficient which is equal to  $1/\sqrt{2}$  times the

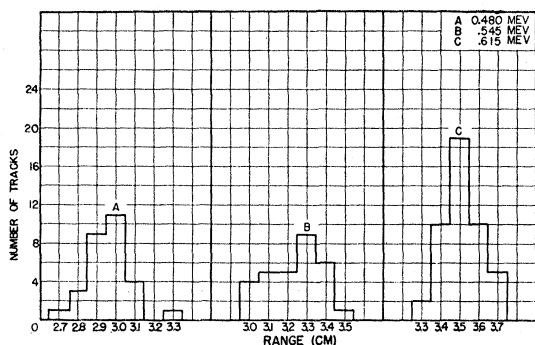


FIG. 2. Distribution of track lengths of alpha particles in a cloud chamber filled with hydrogen, water vapor, and ethyl alcohol vapor at respective pressures of 12.25 cm, 1.50 cm, and 2.24 cm.

TABLE I. Average range  $R_{Av}$ , median range  $R_m$ , extrapolated range  $R_e$ , and straggling coefficient  $\sigma$  (standard deviation of  $R_{Av}$ ) for low-energy alpha particles in air at a temperature of 15°C and 760-mm pressure as a function of the initial energy  $E_0$ .

$E_0$ (Mev)	No. of tracks	$R_{Av}$ (cm)	$R_m$ (cm)	$R_e$ (cm)	$\sigma$ (cm)	$\sigma/R_{Av}$
0.480	29	0.299	0.304	0.317	0.011	0.037
0.545	30	0.327	0.334	0.350	0.0135	0.042
0.615	46	0.354	0.358	0.370	0.010	0.029

reciprocal of the standard deviation  $\sigma$  of the distribution, or in terms of the so-called "straggling" which we take as  $s = R_e - R_m$ .

### (b) Ranges in Air

In order to obtain the parameters for air from the experimental results, it is necessary to know the composition of the gas in the chamber at the time the measured tracks are produced and the stopping powers of its constituents and of those of the nylon window relative to air. While the range of pressures of the hydrogen in the chamber during the sensitive time can be inferred from the position of the piston (the expansion being 90 to 99 percent completed), the variation of the vapor pressure of the alcohol-water mixture during the motion of the piston cannot. It seems plausible, however, that up to the end of the sensitive time, the number of atoms which constitute the vapor will remain essentially the same as that which would prevail if no evaporation took place—at least in the parts of the chamber away from the walls.

The following equations hold for  $R_{Av}$ , the equivalent average range in air at atmospheric pressure and 15°C of a particle having an energy  $E_0$ ; for  $t$ , the thickness of the nylon window traversed by a particle of incident energy  $E_0$  and emergent energy  $E_f$ ; and for  $r_{Av}$ , the average length of a track in the cloud chamber:

$$R_{Av} = \int_{E_0}^0 \frac{dE}{(dE/dx)_a}, \quad t = \int_{E_0}^{E_f} \frac{dE}{(dE/dx)_n}, \quad (1)$$

$$r_{Av} = \int_{E_f}^0 \frac{dE}{(dE/dx)_c},$$

where the denominators in the integrands of Eqs. (1) are the stopping powers of air, nylon, and the cloud-chamber mixture, respectively. The stopping power of any mixture or compound may be expressed in terms of that of air by

$$dE/dx = (dE/dx)_a \sum_i p_i (N_i/N_a) \equiv (dE/dx)_a P, \quad (2)$$

where  $p_i$  and  $N_i$  are, respectively, the relative atomic stopping power and number of atoms per unit volume for the  $i$ th element present, and  $N_a$  is the number of atoms per unit volume for air. The expressions  $\sum_i p_i (N_i/N_a)$  for nylon and for the cloud-chamber gas will be referred to as  $P_n$  and  $P_c$ , respectively.

In order, then, to determine the equivalent ranges in air from our results, one needs the values of  $p$  for the elements H, C, O, and N. For the first two elements Livingston and Bethe<sup>1</sup> give curves for  $p$  against  $E$  for  $E$  greater than 2 Mev and also list separately values of  $p$  for several energies. In the case of hydrogen the value read from the curve differs slightly from that given in the text. In order to estimate  $p$  in the energy range of interest to us, the curve for hydrogen was altered so as to go through the tabulated point and both curves were then extrapolated to zero energy taking into account the tendency of the curvature to increase slightly with decreasing energy. The values of atomic stopping power for oxygen were obtained by extrapolating to zero energy some recent measurements of the molecular stopping power of water<sup>16</sup> and subtracting the contribution of the hydrogen. The values for oxygen so obtained were, in turn, used to compute the atomic stopping power for nitrogen from the known composition of air.

Over the small energy interval between  $E_0$  and  $E_f$  the change in the quantity  $P_n$  is insignificant and may be neglected without further consideration. The change in  $P_c$  amounts to two percent in the energy interval from 0.1 to 0.6 Mev. Thus if the value of  $P_c$  is taken as constant in this range and equal to that at  $E_0/2$ , only a small error is introduced. With the above approximations, the first of Eqs. (1) becomes

$$R_{Av} = P_n t + P_c r_{Av}. \quad (3)$$

The values computed for  $P_n$  are practically the same for alpha particles of the three energies studied and result in a value of 0.044 cm for  $P_n t$ . This value is five percent lower than that following from the direct experimental investigation described above. Because the contribution of the corresponding term in Eq. (3) to  $R_{Av}$  is small relative to that of the other term, the corresponding error in  $R_{Av}$  is negligible. The value obtained for  $P_c$  was 0.103 under our conditions of operation. The average and median ranges  $R_{Av}$  and  $R_m$  are given in columns 3 and 4 of Table I.

In order to make a comparison with the ranges following from Bethe's 1950 curve, it is necessary to express our results in terms of projected ranges. For this purpose, we give in Table II, together with Bethe's mean values  $\bar{X}$ , the quantities  $X_{Av}$  and  $X_m$ , which represent, respectively, the average and median of the

TABLE II. Projected average and median ranges  $X_{Av}$  and  $X_m$  of low-energy alpha particles in air at a temperature of 15°C and 760-mm pressure as a function of the initial energy  $E_0$ ;  $X_{Av}$  and  $X_m$ , present investigation;  $\bar{X}$ , Bethe's 1950 curve.

$E_0$	$X_{Av}$ (cm)	$X_m$ (cm)	$\bar{X}$ (cm)	$(X_{Av} - \bar{X})/X_{Av}$	$(X_m - \bar{X})/X_m$
0.480	0.293	0.298	0.285	0.027	0.044
0.545	0.321	0.328	0.312	0.028	0.049
0.615	0.348	0.352	0.340	0.023	0.034

<sup>16</sup> H. G. de Carvalho and H. Yagoda, Phys. Rev. **88**, 273 (1952).

sum of the air equivalents of the projected track lengths in the gas of the chamber and the nylon window. While, by making a comparison in this way, one can assure that the ranges are defined in the same way as far as geometrical considerations are concerned, it should be pointed out that the definition of the mean range (Bethe) is based on the "specific ionization curve of a single alpha particle" while our definitions  $X_{Av}$  and  $X_m$  are based on the number range distribution.

It can be seen that our  $X_{Av}$ 's are about two to three percent higher than the ranges  $\bar{X}$  following from Bethe's curve, and that our values of  $X_m$  are still higher.

There were three chief potential sources of systematic errors in our range determinations. In the first place, the (extrapolated) stopping powers relative to air which were used to convert the measured ranges to ranges in air may have been inaccurate. Secondly, there may be some doubt as to the locations of the end points of the tracks. Thirdly, the total amount of water and alcohol vapor present in the chamber may have changed during the expansion due to evaporation of some of the excess liquid either from the walls of the chamber or from small invisible droplets dispersed through the gas (Mills' process).

No estimate can be made of the errors resulting from any inaccuracies in the values of the relative stopping powers used. Any errors of the last two types mentioned, however, would necessarily tend to make the ranges which were obtained for air at atmospheric pressure smaller than the true ranges. Thus, the difference between our results and Bethe's 1950 curve, which is in the opposite direction, cannot be explained on this basis.

### (c) Straggling

Columns 6 and 7 of Table I contain the standard deviation  $\sigma$  and the coefficient of variation  $\sigma/R_{Av}$ , respectively. In order to determine the straggling one has to estimate the effect of the energy spread of the alpha particles leaving the velocity selector. As mentioned before, geometrical considerations show that the maximum spread is 1.5 percent in the momentum and is, therefore, 3 percent in the energy (independent of the setting of the analyzer). It follows from our results that the slope of the range energy curve is about 0.4 cm/Mev for the energies in question and, hence, the resulting spread in the measured ranges due to this cause alone amounts to about two percent of the range. For any distribution the standard deviation is by definition less

than half the spread; and in the present case, on physical grounds, the standard deviation in the ranges associated with the finite resolution of the spectrograph is very much less than half the corresponding spread in the ranges. Even if the standard deviation from this cause was as large a fraction as 0.01 of the range it would make a contribution of only about ten percent to the measured values of  $\sigma$  and  $\sigma/R_{Av}$ . Therefore the straggling due to the finite resolution of the analyzer can be neglected and the coefficients  $\sigma$  listed in the table may be taken as a measure of the true straggling. In view of the number of tracks measured, one may consider the individual  $\sigma$ 's given in the table to be correct to about fifteen or twenty percent.

According to Bohr,<sup>12</sup> the value of  $\sigma/R$  which one would expect on the basis of electronic collisions would amount to somewhat less than 0.01, independent of energy and stopping material. It follows therefore that the straggling at these energies is mostly due to charge exchange and nuclear collisions, processes which become relatively more important near the end of the track. Thus the true value of  $\sigma$  should be nearly independent of the energy for short range alpha particles.

Therefore it is possible to compute this quantity more accurately from a properly weighted rms average of the  $\sigma$ 's for the three energies. The resulting value of  $\sigma = 0.012$  cm agrees well with the value of 0.014 cm obtained by Bøggild<sup>11</sup> for alpha particles of a range of 0.7 cm in air (about twice our ranges), which verifies the assertion that most of the straggling occurs at the end of the path.

We can also compute the extrapolated range  $R_e$  and the straggling  $s$  from the number range curves following from Fig. 2; however, it is rather difficult to locate the points of inflection in the individual curves. Consequently, we assumed that  $s$  is the same for all three energies just as  $\sigma$  proved to be and constructed from them a single number range curve constructed by superposing the three curves so that the points corresponding to their averages all coincided. The value of  $s$  thus obtained is 0.016 cm; the extrapolated ranges given in column 5 of Table I were computed from the relation  $R_e = R_m + s$  using this value of  $s$ .

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