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Energy Distribution of Nuclear Reaction Products

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In a reaction involving two product particles the distribution in energy of either product particle measured in the laboratory system is the same as the distribution in angle in the zero-momentum system. This relation is valid relativistically and irrespective of the reaction energy. Some applications are discussed.

DIRECT measurements of the angular distribution of the products of nuclear reactions or of scattering processes are frequently more difficult to perform than measurements of the distribution in energy of product or recoiling particles. For this reason it is worth pointing out that the two distributions are related in a very simple manner, if there are only two product particles involved and if a single process occurs.

It is well known that, in single collisions between monoergic neutrons and protons, all neutron or proton energies up to the energy of the primary neutrons are produced with equal probability provided the neutron energy is sufficiently low that the scattering is isotropic in the c.m. system. This relation has been generalized to all nonrelativistic elastic collision processes.[1] It has been shown that the distribution in energy of the recoil particles as measured in the laboratory system is the same as the angular distribution with respect to $\cos\theta$ of the scattered particles in the c.m. system. It is the purpose of this note to point out that the relationship holds also for relativistic energies and for exoergic or endoergic collisions.

Assume that the product particles are numbered 1 and 2. Let $\mathbf{p}$ and E be the momentum and (relativistic) energy and let primed quantities refer to the laboratory system, while unprimed quantities are measured in the system of zero momentum: $\mathbf{p}_1+\mathbf{p}_2=0$. If the zero-momentum system moves with a velocity $\mathbf{v}$ with respect to the laboratory system, the energy of particle 1 in the laboratory system is

$$E_1'=(1-v^2/c^2)^{-\frac{1}{2}}(E_1-|\mathbf{p}_1||\mathbf{v}|\cos\theta),$$

where θ is the angle between $\mathbf{v}$ and $\mathbf{p}_1$, i.e., the scattering angle in the zero-momentum system if scattering is being considered. For a given reaction, the energy E_1' depends only on $\cos\theta$, and the energy distribution function $N(E_1')$ in the laboratory system and the angular distribution function $P(\cos\theta)$ in the zero-momentum system are related by

$$N(E_1')|dE_1'|=P(\cos\theta)|d(\cos\theta)|,$$

and therefore

$$N(E_1')=(1-v^2/c^2)^{\frac{1}{2}}P(\cos\theta)/|\mathbf{p}_1||\mathbf{v}|.$$

This shows that the angular distribution in the zero-momentum system expressed in terms of $\cos\theta$ is the same as the energy distribution in the laboratory system. The energy spectrum is contained in an energy range of width $2|\mathbf{p}_1||\mathbf{v}|(1-v^2/c^2)^{-\frac{1}{2}}$.

This theorem is especially useful for reactions induced by uncharged particles when it is possible to measure the ionization produced by one of the product particles, particularly in the cases of elastic and inelastic scattering of neutrons. Another example is the photodisintegration of the deuteron. By measuring the energy distribution of the photo-protons it is possible to determine directly the angular distribution of the disintegration processes.[2] For reactions induced by neutrons and photons some difficulty may arise in measuring the energy distribution of one of the reaction products separately, if both products are charged particles. If the two product particles have sufficiently different masses, it is, however, still possible to observe the distribution in ionization for each product.

Another problem to which this theorem applies is the disintegration of monoergic mesons in flight. Here $\mathbf{v}$ is the velocity of the meson before disintegration. In the reference system of the meson all directions of emission of the product particles are equally probable. Consequently the energy distribution of the product particles in the laboratory system is a constant between the limits given previously. Such an energy distribution has been observed by Panofsky, Aamodt, and Hadley[3] for the γ rays from the decay of neutral π mesons.

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[1] H. H. Barschall and M. H. Kanner, Phys. Rev. 58, 590 (1940).
[2] Bishop, Beghian, and Halban, Phys. Rev. 83, 1052 (1951).
[3] Panofsky, Aamodt, and Hadley, Phys. Rev. 81, 565 (1951).