



FIG. 3. Pulse-height distribution showing peak position for the Be  $2p \rightarrow 1s$   $\pi$ -mesonic line compared with other mesonic lines of known energy [P (M), 40.5 keV; F (L), 41.6 keV; Cl (M), 52.1 keV]. As a result of the comparison the Be energy is  $42.9 \pm 0.5$  keV corresponding to a negative shift of about  $1.0 \pm 0.5$  keV.

**Boron.**—The results obtained to date are not fully understood. Further studies with the separated isotopes will be made before reporting on this element.

**Carbon, nitrogen, oxygen, and fluorine.**—The  $K$ -mesonic lines of these elements cannot be studied with the critical absorption technique. We have measured their energies using the pulse-height selector and have found negative shifts varying roughly from 7 percent for carbon to about 13 percent for fluorine. The results of these measurements will be described in a later communication.

In conclusion we have observed that the  $2p \rightarrow 1s$  lines in the elements lithium to oxygen are shifted to lower energies with the possible exception of boron. This indicates an effective repulsive potential between the meson in the  $1s$  state and the nucleus.

\* Supported in part by the U. S. Atomic Energy Commission.

<sup>1</sup> Stearns, Stearns, DeBenedetti, and Leipuner, Phys. Rev. **95**, 1353 (1954); Stearns, Stearns, Leipuner, and DeBenedetti, Phys. Rev. **95**, 625(A) (1954).

<sup>2</sup> The  $K$ -absorption edge values were taken from a recent compilation of Lewis Slack, Naval Research Laboratory.

### Formula for Polarization in Nucleon-Nucleon Scattering\*

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(Received September 7, 1954)

THE importance assumed by experiments<sup>1</sup> on the polarization of nucleons makes it desirable to have available a general formula for the calculation of

polarization effects. The quantity under consideration is that usually denoted by  $P$  which is related to  $\bar{s}_y$ , the mean spin component normal to scattering plane in a single scattering experiment with unpolarized particles, by  $P = 2\bar{s}_y$ . An explicit formula in terms of the scattering matrix referred to weak-field spin states is available in several theoretical papers,<sup>2</sup> for example in Eq. (15) of Goldfarb and Feldman.<sup>2</sup> In principle, these formulas solve the problem but the work involved in the transition to special forms containing phase shifts is large at times. It appeared desirable therefore to reduce the general expressions to a point representing a compromise between compactness and explicitness. For  $p$ - $p$  scattering,

$$k^2(P\sigma)_{p-p} = 2 \sin\theta \cos\varphi \operatorname{Im}\{\alpha_1(\alpha_2 + \alpha_c)^* - (1 - \mu^2)\alpha_1\alpha_3^* + \alpha_4^*(\alpha_5 + \alpha_c)\}, \quad (1)$$

where the colatitude and azimuthal angles  $\theta$ ,  $\varphi$  are as in Swanson's paper,  $k = 2\pi$  times reciprocal of wavelength,  $\operatorname{Im}$  indicates the imaginary part of expression in braces. The six quantities  $\alpha_1, \alpha_2, \dots, \alpha_c$  are as follows:

$$\alpha_1 = -\Sigma[L(L+2)Q_{L+1}(L) - (2L+1)Q_L(L) - (L^2-1)Q_{L-1}(L)][P_L'/L(L+1)]e_{L0} - 2B[(l+1)P_{l+2}' - (l+2)P_l'], \quad (2)$$

$$\alpha_2 = \Sigma \frac{1}{2} e_{L0} [(L+2)Q_{L+1}(L) + (2L+1)Q_L(L) + (L-1)Q_{L-1}(L)]P_L - (l+1)(l+2)B(P_{l+2} + P_l), \quad (3)$$

$$\alpha_3 = \Sigma \frac{1}{2} e_{L0} [LQ_{L+1}(L) - (2L+1)Q_L(L) + (L+1)Q_{L-1}(L)][P_L''/L(L+1)] - B(P_{l+2}'' + P_l''), \quad (4)$$

$$\alpha_4 = \Sigma e_{L0} [Q_{L+1}(L) - Q_{L-1}(L)]P_L' - 2B[(l+1)P_{l+2}' - (l+2)P_l'], \quad (5)$$

$$\alpha_5 = \Sigma e_{L0} [(L+1)Q_{L+1}(L) + LQ_{L-1}(L)]P_L + 2B(l+1)(l+2)(P_{l+2} + P_l), \quad (6)$$

$$\alpha_c = \frac{1}{4}\eta[-s^{-2}\exp(-i\eta \ln s^2) + c^{-2}\exp(-i\eta \ln c^2)], \quad (7)$$

where

$$Q_J(L) = [-1 + \exp(2i\delta_J^L)]/(2i), \quad (8)$$

$$e_{L0} = \exp(2i\sigma_{L0}) = \exp[2i(\tan^{-1}(\eta/L) + \tan^{-1}(\eta/(L-1)) + \dots + \tan^{-1}\eta)]. \quad (9)$$

$\delta_J^L$  is the phase shift for total angular momentum  $J\hbar$  and orbital angular momentum  $L\hbar$ , while the other symbols have the following meaning:  $\eta = e^2/\hbar v$ ,  $v$  = relative velocity;  $s = \sin\theta/2$ ,  $c = \cos\theta/2$ ,  $\theta$  = scattering angle in center of mass system;  $\mu = \cos\theta$ ,  $P_L = P_L(\mu)$ ,  $P_L' = dP_L/d\mu$ ,  $P_L'' = d^2P_L/d\mu^2$ . If there is no coupling between states of different  $L$  for the same  $J$  the constant  $B = 0$ . If states with  $J = l+1$  having values  $L = l$  and  $L = l+2$  are coupled to each other, then

$$B = \frac{1}{2}C_{l,l+2}(l+1)^{-\frac{1}{2}}(l+2)^{-\frac{1}{2}}\exp[i\sigma_{l,0} + i\sigma_{l+2,0}], \quad (10)$$

where  $C_{l, l+2}$  is as in Thaler, Bengston, and Breit and  $Q_{l+1}(l)$ ,  $Q_{l+1}(l+2)$  are modified by replacing them by  $Q_{l+1}(l) + C_{l, l+2}$ ,  $Q_{l+1}(l+2) + C_{l+2, l+2}$ . The matrix

$$(T) = \begin{pmatrix} Q_{-+} + C_{-+} & C_{-+} \\ C_{+-} & Q_{++} + C_{++} \end{pmatrix}, \quad (11)$$

can then be related to  $(S)$  by

$$(T) = [(S) - 1]/(2i), \quad (12)$$

and  $(S)$  is a general unitary symmetric matrix expressible by means of 3 parameters. The subscripts  $-$  and  $+$  are used to indicate the smaller and larger of the two  $L$ . The  $Q$  as used in Thaler, Bengston, and Breit have a generic meaning in connection with the special model used by them. In calculations, however, the diagonal elements of  $(T)$  enter instead in the equations for  $\alpha_1, \dots, \alpha_5$ .

Substitution of Eqs. (2), (3)  $\dots$ , (7) into Eq. (1) enables one to collect coefficients of a product involving two  $Q$ 's such as  $Q_1, Q_2^*$ . These combine with terms in  $Q_2 Q_1^*$  which can be converted under the  $\text{Im}$  sign to the order  $Q_1 Q_2^*$  by taking the complex conjugate and changing the sign. For uncoupled terms,

$$\text{Im}(Q_2 Q_1^*) = \sin \delta_2 \sin \delta_1 \sin(\delta_2 - \delta_1)$$

then gives readily the forms needed for numerical work.

In Eq. (1) the sums over  $L$  are supposed to be taken over odd values only. The formula can be used for the calculation of  $(P\sigma)_{p-n}$  provided a coefficient  $\frac{1}{4}$  is inserted on the right side, thus changing the 2 in front of  $\sin \theta$  to  $\frac{1}{2}$  and provided the sum is made to extend over odd and even  $L$  employing all triplet states. In both cases the singlet states do not enter.

\* Assisted by the Office of Ordnance Research, U. S. Army.

<sup>1</sup> Oxley, Cartwright, Rouvina, Baskir, Klein, Ring, and Skillman, Phys. Rev. **91**, 419 (1953); L. F. Wouters, Phys. Rev. **84**, 1069 (1951); Marshall, Marshall, Nagle, and de Carvalho, Phys. Rev. **93**, 1431 (1954); Chamberlain, Donaldson, Segrè, Tripp, Wiegand, and Ypsilantis, Phys. Rev. **95**, 850 (1954); Kane, Stallwood, Sutton, Fields, and Fox, Atomic Energy Commission Report NYO-6569, July 6, 1954 (unpublished), to cite some of the many recent papers.

<sup>2</sup> J. Schwinger, Phys. Rev. **69**, 681 (1946); **73**, 407 (1948); L. Wolfenstein, Phys. Rev. **75**, 1664 (1949); **76**, 541 (1949); L. J. B. Goldfarb and D. Feldman, Phys. Rev. **88**, 1099 (1952); D. R. Swanson, Phys. Rev. **89**, 749 (1953); R. H. Dalitz, Proc. Phys. Soc. (London) **A65**, 175 (1952); A. Simon and T. A. Welton, Phys. Rev. **90**, 1036 (1953).

### Formula for Polarization in $p$ - $p$ Scattering for $P$ and $F$ Waves\*

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(Received September 7, 1954)

AN expression for the calculation of polarization of protons produced by single scattering taking into account  $P$  and  $F$  waves has been worked out and is

given below. Coupling between  $^3P_2$  and  $^3F_2$  is neglected, but otherwise the most general condition describable by a set of phase shifts for states of definite orbital as well as total angular momentum is considered. The result is written in a form convenient for numerical work.

If  $P$  is the polarization, defined as twice the expectation value of the  $y$  component of the spin,<sup>1</sup> then

$$\begin{aligned} k^2(P\sigma)_{p-p} = & (a + b \cos^2 \theta + c \cos^4 \theta) \sin \theta \cos \theta \cos \varphi \\ & - 2 \text{Im} \{ \alpha_c^* [\frac{3}{2} e_{10} (Q_2(P) - Q_1(P)) \\ & - \frac{1}{8} e_{30} (-8Q_2(F) - 7Q_3(F) + 15Q_4(F))] \\ & - \alpha_c [e_{10} (Q_2(P) - Q_0(P))^* - \frac{3}{2} e_{30} (Q_4(F) - Q_2(F))^*] \} \\ & \times \sin \theta \cos \varphi - 2 \text{Im} \{ \alpha_c^* [\frac{5}{8} e_{30} (-8Q_2(F) - 7Q_3(F) \\ & + 15Q_4(F))] - \alpha_c [(15/2) e_{30} (Q_4(F) - Q_2(F))^*] \} \\ & \times \sin \theta \cos^2 \theta \cos \varphi, \quad (1) \end{aligned}$$

where  $\sigma$  is the single scattering cross section,  $k$  the wave number of the incident protons. Other quantities are:  $e_{LO} = \exp(2i\sigma_{LO})$ , where  $\sigma_L$  is the Coulomb phase shift and  $\sigma_{LO} = \sigma_L - \sigma_O$ ;  $Q_J(L) = \exp(i\delta_J^L) \sin \delta_J^L$ , where  $\delta_J^L$  is the phase shift in a state of orbital angular momentum  $L\hbar$  and total angular momentum  $J\hbar$ ;

$$\begin{aligned} \alpha_c = & (\eta/4) [-s^{-2} \exp(-i\eta \ln s^2) \\ & + c^{-2} \exp(-i\eta \ln c^2)], \quad (2) \end{aligned}$$

where  $\eta = e^2/\hbar v$ ,  $s = \sin(\theta/2)$ ,  $c = \cos(\theta/2)$ ,  $\theta$  is the scattering angle in the center-of-mass system,  $v$  the relative velocity;

$$\begin{aligned} a = & 9 \{ (P_1, P_2) + \frac{2}{3} (P_0, P_2) + (7/12) (F_3, F_2) \\ & - (77/12) (F_4, F_2) - (35/24) (F_4, F_3) + (F_2, P_1, 31) \\ & + \frac{2}{3} (F_2, P_0, 31) - (10/3) (F_2, P_2, 31) - (7/12) (F_3, P_2, 31) \\ & + (5/3) (F_4, P_0, 31) + (5/2) (F_4, P_1, 31) \\ & - (23/12) (F_4, P_2, 31) \}; \quad (3) \end{aligned}$$

$$\begin{aligned} b = & (1/16) \{ -420 (F_3, F_2) + 6020 (F_4, F_2) + 1540 (F_4, F_3) \\ & + 1200 (F_2, P_2, 31) - 840 (F_4, P_1, 31) + 420 (F_3, P_2, 31) \\ & - 220 (F_4, P_2, 31) - 560 (F_4, P_0, 31) \}; \quad (4) \end{aligned}$$

$$c = (25/8) \{ 140 (F_2, F_4) + 49 (F_3, F_4) \}, \quad (5)$$

where

$$\begin{aligned} (L_J, L_{J'}) = & \sin \delta_J^L \sin \delta_{J'}^{L'} \sin(\delta_J^L - \delta_{J'}^{L'}), \\ (L_J, L_{J'}, 31) = & \sin \delta_J^L \sin \delta_{J'}^{L'} \sin(\delta_J^L - \delta_{J'}^{L'} + 2\sigma_{31}), \quad (6) \end{aligned}$$

and

$$\sigma_{LL'} = \sigma_L - \sigma_{L'}, \quad \text{with} \quad \sigma_L - \sigma_{L-1} = \tan^{-1}(\eta/L).$$

This result agrees with that of Goldfarb and Feldman<sup>1</sup> up to second order in  $\cos \theta$  if coupling is neglected in their work and Coulomb terms are neglected in Eq. (1). It has been checked throughout with the zero-coupling limit of the result of Breit and Ehrman.<sup>2</sup>

\* Assisted by the Office of Ordnance Research, U. S. Army.

<sup>1</sup> L. J. B. Goldfarb and D. Feldman, Phys. Rev. **88**, 1099 (1952).

<sup>2</sup> G. Breit and J. B. Ehrman, preceding Letter [Phys. Rev. **96**, 805 (1954)].