energy release is $2.7+14+20+(300-125)+116 \cong 328$ Mev. If only one neutron was emitted from the star, the total energy of the secondary star would be greater than the above estimates. If it is assumed that more than two neutrons were ejected, the total estimated energy is not appreciably decreased. If track 4 is assumed to be a proton, the Z of the fragment would have to be 5 which is inconsistent with the observations, therefore it seems reasonable to assume that it was produced by a negative π meson and that the energy release from the secondary star is about $2.7+14+20+45+28 \cong 110$ Mev. Of course the energy could be considerably larger because two neutrons were assumed to be emitted in the same direction.

In any case the energy release is much greater than the expected value from the decay of a Λ^0 particle bound in the nuclear fragment (37 Mev for the mesonic decay and 176 Mev for the nonmesonic decay).

TABLE I. Characteristics of tracks from the secondary star.

Track	Range microns	Ionization	Identity	Energy in Mev
1	62	black	p,d,t	2.7 if p
2	109	black	α	14
3	>2000[a]	black	p,d,t	$20<E<24$ if p
4	>300[b]	gray	π or p	45 ± 18 if π 300 ± 125 if p

[a] The track left the emulsion after a range of 2000 microns. The total range is estimated to be less than 2500 microns.
[b] The track dips steeply and leaves the emulsion after 300 microns. The energy was determined from a comparison of the blob density along track 4 with minimum ionizing tracks of the same dip. The blob density is 1.5 ± 0.12 times minimum. The track is too straight for an electron with the observed ionization.

The high-energy release may be explained by assuming that a hyperon other than a Λ^0 was bound in the nuclear fragment. The decay energy from a bound[12] Y^+ would be about 117 Mev.[13] The observed energy release is somewhat higher than would be expected from a bound cascade hyperon $Y^-(Q_{Y^-} \cong 67$ Mev).[14] It is also possible that a θ^0 was bound in the fragment and decayed with the absorption of one of the π mesons. The expected decay energy is 360 Mev if only one π meson is emitted.[15] The possibility can not be excluded that a negative K meson was bound in a mesonic orbit around the fragment and absorbed after the fragment stopped.

The one unusual disintegration was found among 7000 cosmic-ray stars and 15 000 stars produced by 3-Bev protons. Among these stars 15 additional cases of the disintegration of a nuclear fragment have been observed which are consistent with the interpretation of a bound Λ^0 particle.

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Neutron-Proton Mass Difference*

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IT has recently been suggested[1] that the nucleon may be treated as a structured system having varying numbers of pions in bound states centered on a core particle of spin and isotopic spin $\frac{1}{2}$. An interesting question concerning this model is whether or not it is capable of accounting for the sign and order of magnitude of the neutron-proton mass difference on the basis of electromagnetic interactions alone.[2] The purpose of this note is to point out that the model does indeed account for the mass difference. Furthermore, its success in this respect depends directly on those peculiar features of the nucleon wave function that were found necessary[1] to account for the nucleon magnetic moments. The peculiarities are as follows:

(1) The total probability that pions occur with a total orbital angular momentum $L=1$ is small, of the order of 10 percent.

(2) The probability that a pair of pions occurs in the $L=1$ state is rather large, comparable to or possibly greater than the probability for the occurrence of a single pion (which must have $L=1$).

In order to isolate the effects that may contribute to the mass difference, we assume that the charged and uncharged core particles have the same mass.[2] It will also be assumed that the motion of the core may be neglected. On the basis of the charge independence hypothesis, neither the mass difference between the charged and neutral pions nor the electromagnetic interactions between the pions enters into the neutron-proton mass difference. The difference in the electromagnetic energies of the two systems then consists of two parts, the electrostatic interaction between the pions and charged core, and the interaction of the magnetic moment of the charged core with the magnetic field produced by the pion currents. It is assumed that the neutral core has no magnetic moment.

As is well known, the electrostatic terms have the wrong sign; the neutron may divide into a positive core associated with a π^- or associated with a (π^-, π^0) pair while the corresponding state of the proton has a neutral core, hence no electrostatic energy of this kind. This effect alone would lead to a neutron mass smaller than the proton mass, but the effect is a small one because the probability for the occurrence of the pions is small according to condition (1) above. Even if the radius of the pion orbit is assumed to be as small as twice the nucleon Compton wavelength, the electrostatic energy amounts to about -250 kev as compared to the 1.3-Mev mass difference.

On the other hand, the magnetic coupling with the core moment leads to a large positive contribution. The reason for its being large is that the pion current operator involves a pion pair creation and a pair annihilation term.[3] Hence, according to condition (2), there is an important contribution to the average current due to a cross term between the two pion state and the zero pion state. This contribution is proportional to the *amplitude* of the two pion state whereas the other terms are proportional to the square of the amplitude which is relatively small. The same cross term in the current is responsible for the greater part of the magnetic moment anomaly.[1] The cross term always involves a (π^+, π^-) pair, so for the neutron (the core is neutral) there is no magnetic energy of this kind. On the other hand, in the proton the current loop formed by the pair must have such a direction as to add to the magnetic moment of the core. Then the magnetic field at the core due to the loop is parallel to the core moment, and the magnetic energy is negative. Hence the proton mass is reduced.

It is interesting to note that the calculation of this effect is quite analogous to the calculation of atomic hyperfine structure and that the sign of the effect is fixed just as in the hyperfine splitting.

The magnetic energy of the proton due to this cross term between the zero and two pion state is given by

$$\Delta E_p = -6\mu_c \Delta\mu_p \rho^{-3}, \tag{1}$$

where μ_c is the magnetic moment of the charged core, $\Delta\mu_p$ is the contribution to the proton moment due to the cross term between the zero and two pion states and ρ is a distance depending on the extent of the pion wave function. If the wave function of the two pions has the form $f(\mathbf{r}_1,\mathbf{r}_2)\mathbf{r}_1\times\mathbf{r}_2$, the quantity ρ is defined in terms of the function $G(r)$ given by

$$r^{-2}\mathbf{r}G(r) = \frac{1}{2}\int d^3r_1 \int d^3r_2 K(\mathbf{r}-\mathbf{r}_1)K(\mathbf{r}-\mathbf{r}_2) \times f(\mathbf{r}_1,\mathbf{r}_2)(\mathbf{r}_1+\mathbf{r}_2), \tag{2}$$

where

$$K(\mathbf{r}) = (2\pi)^{-3} m_\pi^{\frac{1}{2}} \int d^3k (k^2+m_\pi^2)^{-\frac{1}{4}} \exp(i\mathbf{k}\cdot\mathbf{r}), \tag{3}$$

and m_π is the reciprocal Compton wavelength of the pion. Then ρ is defined by

$$\rho^{-3} = \int_0^\infty r^{-3}G(r)r^2dr \Big/ \int_0^\infty G(r)r^2dr.$$

An experimental estimate of ρ may be obtained directly from Eq. (1). If ΔE_p is set equal to the total $n-p$ mass difference, $\Delta\mu_p$ to the magnetic moment anomaly and μ_c is taken to be one nuclear magneton, we find

$$\rho = 2.4\hbar/Mc, \tag{4}$$

where M is the nucleon mass. The electrostatic effect and the other contributions to the moment anomaly will lead to a slight reduction in the value of ρ.

It is interesting that the result, Eq. (4), is in reasonable agreement with the estimate of the cutoff momentum for the bound pion function obtained from the pion-nucleon scattering.[4] It was found there that the cutoff in the bound p function occurs at a momentum of $0.6Mc$ corresponding to a cutoff radius $1.6\hbar/Mc$. In view of the spreading effect of the kernels in Eq. (2) and the reduction in the coefficient in Eq. (4) to be expected when all the small effects are included, the agreement appears to be satisfactory.

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[3] The occurrence of these operators in the expression for the current is also responsible for pion pair creation or annihilation with the absorption or emission of gamma radiation. The contribution to the proton mass may be described in terms of the emission of virtual radiation by pair annihilation followed by absorption of the radiation by the charged core.

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