Alpha-Decay Theory and a Surface Well Potential

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(Received August 28, 1953; revised manuscript received July 2, 1954)

A discussion is given of the escape of an alpha particle from a square-well potential which is in the form of a spherical shell surrounding the daughter nucleus. In the present approximation the alpha particle, as such, is excluded from the interior of the nucleus by making the one-body potential infinite in that region. The usual Coulomb barrier is used. It is found that the properties of this model differ in several important respects from those of the usual square-well one-body model, in which the well is located at the origin of coordinates. Arguments are presented which lead to the representation of the many-body decay constant as the product of a dimensionless "preformation factor" and a one-body decay constant, as opposed to the usual product of an intrinsic nonbarrier decay constant and a dimensionless barrier transmission coefficient. The preformation factor involves an "intrinsic" alpha-decay probability and, in addition, the probability per sec that an alpha particle at the nucleus will be absorbed as nucleons. The nuclear physics problem of deriving expressions for these latter two probabilities is not considered, but examples are given which can be used to show expected ranges of numerical values of the preformation factor when the one-body model is either the present one or the usual one.

I. INTRODUCTION

THE potential function traditionally used in the derivation of the alpha-decay constant consists of a square well which is located at the origin of coordinates and which is surrounded by the Coulomb interaction between the daughter nucleus and the alpha particle. The radius of the well is associated with that of the daughter nucleus or with that of the daughter plus an "effective" alpha-particle radius or nucleus-alpha interaction distance. The phrase "one-body decay constant," if not further modified, will be used in this paper in a general sense to mean a decay constant found by treating the motion of an *existing* alpha particle in *any* specified potential which is set up to represent the interaction between the alpha particle and the daughter atom. The treatment is generally of the nature of a straight exercise in quantum mechanics, although the care with which it is done varies. The result is such that if the parameters of the potential function and the relative angular momentum are specified, the corresponding decay energy and decay constant are always, in principle at least, calculable. Such a derivation made with the particular potential described above will be referred to here as the traditional one-body model.

The application of similar ideas and methods to the problem of nuclear cross sections is discussed in the text by Gamow and Critchfield,[1] for instance. It is shown there that the spacing of nuclear levels predicted with the use of the traditional type square-well one-body model for nucleons is much greater than is observed experimentally. Such results led long ago to the realization that these methods were inadequate.

The principal efforts to discuss the many-body problem involved in nuclear processes have been based on the method of Feshbach, Peaslee, and Weisskopf.[2] Briefly, it is considered in this method as presently developed that there is a radius R such that the usual one-body wave functions can be written when $r>R$. An equation is found for the logarithmic derivative of this outside function at $r=R$. Arguments are then made without the use of any particular model of the nuclear interior, i.e., without the use of any particular potential function in $r<R$, as to what should be the properties of the function to which this logarithmic derivative is to be equated. The property of particular interest is the dependence on energy. These arguments lead to a function whose form is the same as would be given by the logarithmic derivative of the traditional one-body inside wave function for zero relative angular momentum but with the argument of its periodic factor replaced by $(\pi/D^*)(E-E_r)$, in the neighborhood of the resonance energy E_r. Here D^* is an energy of the order of the average level spacing between levels of the same angular momentum and parity. The inside wave number at the nuclear surface remains in evidence as a multiplier of the periodic factor. Hence the nucleus is characterized by this wave number, the radius, and the level spacing.

The extension of this (resonance) theory into the higher energy region where the level widths are greater than their spacing and continuum theory should apply has been discussed by Feshbach and Weisskopf.[3] The method has been used in this range by Shapiro,[4] for instance, to calculate cross sections for formation of a compound nucleus by protons, deuterons, and alpha particles. It has been used in the resonance region by Devaney,[5] for instance, to discuss alpha decay. The method and, of particular interest here, Devaney's

[1] G. Gamow and C. L. Critchfield, *Theory of Atomic Nucleus and Nuclear Energy Sources* (Clarendon Press, Oxford, 1949), p. 217.

[2] Feshbach, Peaslee, and Weisskopf, Phys. Rev. **71**, 145 (1947).
[3] H. Feshbach and V. Weisskopf, Phys. Rev. **76**, 1550 (1949).
[4] M. M. Shapiro, Phys. Rev. **90**, 171 (1953).
[5] J. J. Devaney, Phys. Rev. **91**, 587 (1953).

application of it are also discussed in the text by Blatt and Weisskopf.[6]

A picture will be presented here which involves an additional step in the transition between the compound nucleus (or parent) and the separated particle and target (or daughter) nucleus. It is believed to be particularly suitable to the discussion of processes which involve the compound particles, deuterons [where the usual compound nucleus, $C(Z+1, A+2)$ in this case, is formed], and alpha particles, since neither of these are fundamental constituents of nuclei. Wider application is believed to be possible but only the process of alpha decay will be considered here.

A relation will be found between the many-body alpha decay constant, i.e., the experimentally observed decay constant, and a one-body decay constant. It will be shown that, granted the assumptions, the many-body decay constant should be represented as the product of a dimensionless "preformation factor" and a one-body decay constant. The implication is that the solution of the problem of alpha decay would be obtained by finding as independently as possible expressions for *each* of these two factors which most nearly fit the observations on alpha decay or any related observations that could be made. In particular, it is assumed that the traditional model is not necessarily the only, or even the best, representation of the one-body aspects of the problem of alpha decay.

The first part of this paper (Secs. II and III) will be concerned with the details of a possible alternative one-body model. This model will be found to have properties interestingly different from those of the traditional model. It is suggested, but not required, by the picture of the alpha decay process which will be used subsequently (Sec. IV) to discuss the relation between the many-body and a one-body decay constant.

The nature of the quantities which constitute the preformation factor will also appear in this latter discussion. No attempt will be made to solve the problem in nuclear physics which would lead to expressions for them, although some estimates will be made by relating them to quantities now in the literature. Some numerical examples which will be used in the comparisons between the present one-body model and the traditional model can also be used to show the range of values in which the total preformation factor ought to lie, when either of these two models is used as the basis of the accompanying one-body decay constant.

This one-body—many-body problem is certainly one of the more important problems in alpha-decay theory. Nevertheless it should not be forgotten that there are other important problems, none of which will be discussed here. Among these are the effects of nuclear shape associated with the presence of quadrupole moments,[7] a specific example of the effects of a non-central electrostatic interaction between the escaping alpha particle and the protons in the residue,[8] and the effects of appropriate changes in the shape of the barrier.[9] These aspects of the decay process will play a role in the determination of nuclear radii from alpha decay data. Of possibly greater importance, they will have a significant effect on the correlation of, for instance, alpha-decay constants and nuclear spin change, or, in general, on the explanation of the so-called "forbidden" alpha-particle groups.

II. THE SURFACE WELL MODEL

The one-body model to be discussed here can be based on the following arguments. It is presumed that: (a) an alpha particle does not exist *in* a nucleus; (b) in the absence of the external Coulomb field there would be an attractive interaction between the daughter nucleus and the alpha particle which would be very strong at the instant of the appearance of the latter at the nuclear surface, but which would approach zero very strongly as the two separated.

The existence not only of the particular Coulomb potential which applies in alpha decay, but also that of the potential in $r<R$, in other words the existence of a potential for the discussion of the motion of an alpha particle, depends on the existence of the alpha particle. Some different potential even in $r<R$ would be appropriate for neutrons, for instance. Thus, in a one-body interaction potential between the daughter nucleus and the existing alpha particle, a region from which the latter is excluded can be represented by an infinite potential in that region without implying any such peculiarities in the potential appropriate for fundamental particles. The potential near the nucleus indicated by the arguments of the preceding paragraph is a well at the nuclear surface. A potential of this type has been referred to as a Jastrow potential by Greenberg.[10] Jastrow[11] used it, with an exponential form for the nuclear interaction, to discuss nucleon-nucleon scattering. Its use in more general applications is apparently being made by Greenberg. In the present paper the net potential for the one-body alpha decay problem will be approximated by a square well at the nuclear surface surrounded by the usual sharply cut-off Coulomb interaction as illustrated in Fig. 1. In this figure, R is the channel radius as usually defined; that is, it is the location of the center of charge of the alpha particle when

[6] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952). For alpha decay see p. 568.

[7] See, for instance, D. L. Hill and J. A. Wheeler, Phys. Rev. **89**, 1102, 1133 (1953).

[8] See, for instance, M. A. Preston, Phys. Rev. **82**, 515 (1951); S. M. Dancoff, Atomic Energy Commission Unclassified Report AECD-2853 (unpublished).

[9] See, for instance, M. L. Chaudhury, Phys. Rev. **88**, 137 (1952).

[10] J. M. Greenberg, Phys. Rev. **87**, 209 (1952).

[11] R. Jastrow, Phys. Rev. **79**, 389 (1950); **81**, 165 (1951); **91**, 749 (1953). See also M. M. Levy, Phys. Rev. **86**, 806 (1952) and reference 6, pp. 170, 186, 187, and 300.

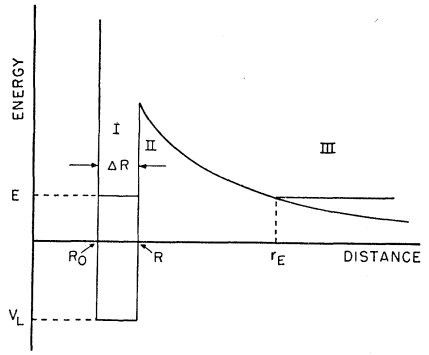

FIG. 1. The potential function for the surface well model. R_0 is taken to be the radius of the alpha daughter, R is the channel radius and $\Delta R=R-R_0$. The exterior classical turning point is at r_e.

the nucleus and the alpha particle just begin to overlap in the sense that nuclear interaction begins.

This "overlap" situation is usually particularized by thinking of the nucleus and the alpha particle as being well defined spheres which, if they approach each other, do not interact until their surfaces are in contact. If it is also supposed that the center of charge of the alpha particle is at the center of the alpha particle, then this highly idealized picture requires that the channel radius be the sum of the radii of the two spheres. Further, it must be considered that the alpha particle is entirely outside the nucleus when the center of charge of the alpha particle is at a vanishingly small distance outside the channel radius R.

The present object is the investigation of a model in which wave functions are written for a well formed alpha particle only in those regions where the alpha particle is entirely outside the nucleus, but to write them everywhere in those regions. One can continue with the idealized picture described above by supposing that the alpha particle is said to be absorbed as nucleons into the nucleus when more than half of the alpha particle sphere has penetrated the nuclear surface. Then in Fig. 1, R_0 and ΔR would be designated as the radius of the daughter nucleus and the alpha particle, respectively. The potential for a well formed alpha particle in $r<R$ is drawn as a well at the nuclear surface to indicate the assumptions that a well formed alpha particle does not exist in $r<R_0$ and that there is a Coulomb barrier for well formed alpha particles, i.e., that the potential energy of a well formed alpha particle close to the nuclear surface is less than $(2Ze^2/R)$.

The description entailing the assignment of such sharply defined radii to the two particles and the specification of the location of the center of charge of the alpha particle is more detailed than is warranted or probably necessary. If the assumptions mentioned in the previous paragraph are granted, it should be sufficient to retain the definitions of R and R_0 and to designate ΔR and the well depth simply as parameters which describe the attractive nuclear interaction in that approximation which consists of a square well at the nuclear surface.

A relation between the observed (many-body) alpha decay constant and one derived with any one-body model will be proposed in Sec. IV. One aspect of the mechanism to be used, the possible reabsorption of a well formed alpha particle into the nucleus as nucleons rather than its emission to the outside region, has been indicated already. The assumption of the possibility of such a process means that the potential function for a well formed alpha particle cannot be conservative. It will be argued later, however, that the mean life of an existing well formed alpha particle prior to emission or reabsorption is sufficiently long to allow one to write alpha particle wave functions in that part of the region $r<R$ which is available to it, but only for those periods during which it is in existence. Thus the potential might be called "quasi-conservative."

An additional question arises with specific regard to the potential function of Fig. 1. One might consider an approach to the problem of deriving the preformation factor, as opposed to the phenomenological arguments used in Sec. IV to show its existence, by setting up equations to represent a smooth outward flow of probability at a rate appropriate to the observed decay constant. The net current of well formed alpha particles out of the region in which they do not exist would be obtained by the use of an imaginary term in the potential in that region. Such a treatment might possibly, but not certainly, indicate that a large imaginary potential rather than a large real potential and a small imaginary potential, would be more appropriate in $r<R_0$. Either would give a small net current; that is, the limiting case of an infinite imaginary potential also leads to a node in the wave function at $r=R_0$, so that the discussion to be given here would not be altered by such a change.

During those periods when the potential of Fig. 1 applies, and if these periods are sufficiently long, one has the conventional picture of a well formed alpha particle attempting to penetrate a potential barrier. The only difference during such periods between this and the traditional one-body model, for instance, is in the size and location of the region available to the alpha particle prior to decay; the barrier is the same.

The principle object of this section, then, is to examine the characteristics of this phase of the process by discussing a one-body model based on the potential function of Fig. 1. Its inclusion in a many-body model of alpha decay will be discussed thereafter. The one-body equations to be derived here have been identically reproduced by the method of complex eigenenergies introduced by Gamow[12] and by Bethe's quasi-stationary state method.[13] The former method is faster (although

[12] See reference 1, p. 156. Also see E. C. Kemble, *The Fundamental Principles of Quantum Mechanics* (McGraw-Hill Book Company, Inc., New York, 1937), p. 192.

[13] H. A. Bethe, Revs. Modern Phys. **9**, 69, 161 (1937).

more formal than instructive) and will be used here. It has been used by Devaney[5] and by Preston[14] and, of course, much of the content of the derivation is contained in their work. It is convenient to reproduce it in its entirety, however, for ease of continuity and because it is desired to keep a term which is somewhat more important in this model than it was in their work. It also seems desirable to use, where convenient, the nomenclature of the Coulomb functions[15] which, when multiplied by $(1/r)$, are the solutions of the radial wave equation for this problem in that region where the Coulomb interaction applies.[16]

Devaney[5] has pointed out that previous alpha decay models are included as special cases of his discussion. In general, his procedure will be followed so that this will be true here also. Thus the first results will be in terms of the logarithmic derivative at the place $r=R$.

Let the complex energy be given by

$$W=E-(i\lambda\hbar)/2. \tag{1}$$

The radial wave function must be of the form $(1/r)\phi(r)$. Far out in the region beyond the barrier, Region III (see Fig. 1) $\phi(r)$ must have the form

$$\phi^{\mathrm{III}}(r)\propto A\,\exp(i\rho), \tag{2}$$

where

$$\rho=kr, \tag{3}$$

$$k=[(2MW)^{\frac{1}{2}}/\hbar], \tag{3a}$$

and M is the reduced mass of the alpha particle. The expression on the right side of Eq. (2) must be that approached at large distances by

$$\phi^{\mathrm{III}}(r)=PG_L+QF_L, \tag{4}$$

where G_L and F_L are the irregular and regular Coulomb functions, respectively, for relative angular momentum quantum number L.

From consideration of the asymptotic expressions for the Coulomb functions,[15] it is seen that

$$Q=iP. \tag{5}$$

Hence, in the barrier, Region II,

$$\phi^{\mathrm{II}}(r)=P(G_L+iF_L). \tag{6}$$

It is shown in the Appendix that in this region, provided that r is not too near the classical turning point, r_E (see Fig. 1), the Coulomb functions can be accurately represented by

$$G_L=|\Phi(\rho)|^{-\frac{1}{4}}\exp[\omega(\rho)], \tag{7}$$

$$F_L=(1/2)|\Phi(\rho)|^{-\frac{1}{4}}\exp[-\omega(\rho)], \tag{8}$$

where

$$|\Phi(\rho)|=2\eta/\rho+(L+\tfrac{1}{2})^2/\rho^2-1, \tag{9}$$

$$\omega(\rho)=\int_{\rho}^{\rho_E}|\Phi(\xi)|^{\frac{1}{2}}d\xi, \tag{9a}$$

$$\eta=(z'Ze^2/\hbar)[M/(2W)]^{\frac{1}{2}}, \tag{9b}$$

and z' and Z are the atomic numbers of the alpha particle and daughter nucleus, respectively.

If by definition

$$f^{\mathrm{II}}/(kR)\equiv[(\phi^{\mathrm{II}})^{-1}(d\phi^{\mathrm{II}}/d\rho)]_{kR}, \tag{10}$$

then

$$f^{\mathrm{II}}=-kR|\Phi|^{\frac{1}{2}}\frac{(1+\gamma)-(i/2)(1-\gamma)\exp(-2\omega)}{1+(i/2)\exp(-2\omega)}, \tag{11}$$

where

$$\omega=\omega(kR), \tag{12}$$

$$|\Phi|=|\Phi(kR)|, \tag{12a}$$

and

$$\gamma=\tfrac{1}{4}|\Phi|^{-\frac{3}{2}}[d|\Phi(\rho)|/d\rho]_{kR}. \tag{12b}$$

The number γ which arises from the differentiation of $|\Phi(\rho)|^{-\frac{1}{4}}$ in Eqs. (7) and (8) was omitted by Preston[14] and by Bethe.[13] It would be contained in Devaney's[5] v'. Its value will generally be about -0.02.

Equation (1) is now to be used to replace W wherever it occurs in Eq. (11) and separation is then made into real and imaginary parts to the first power of $\exp(-2\omega)$ or of λ. The term which arises from keeping the first power of λ is the one mentioned previously, which was

[14] M. A. Preston, Phys. Rev. **71**, 865 (1947).

[15] For references and discussion see Bloch, Hull, Broyles, Bouricius, Freeman, and Breit, Revs. Modern Phys. **23**, 147 (1951).

[16] Actually the Coulomb functions would apply to the problem of the decay of a bare nucleus. Nevertheless they will be used here, although a correction will be made for the potential produced at the nucleus by the atomic electrons, when calculations are made. G. Ambrosino and H. Piatier, Compt. rend. **232**, 400 (1951), suggested that the potential energy should be corrected as above and that the total energy should be corrected for the difference in total electron binding energy between the parent and the daughter. Rasmussen, Thompson, and Ghiorso, University of California Radiation Laboratory Report UCRL-1473, September, 1951 (unpublished), quoted (as then) unpublished work of Serber and Snyder as showing that if one makes both of these corrections, he is correcting for the same thing twice. In their own subsequent publication, R. Serber and H. S. Snyder, Phys. Rev. **87**, 152 (1952), did not use this particular phraseology but their arguments show that it is so. In the case of alpha decay of a heavy element it is found that even were the alpha-particle velocity very high compared to any orbital electron velocity (which it is not), so that the electrons would undergo no rearrangement during the passage of the alpha particle, the excitation energy of the residue would be only about 500 ev. The principal part of the electron binding energy difference between the parent and daughter atoms is due to the charge change at the nucleus, and only a negligible part of it is due to the rearrangement of the electrons. Since the residue is (essentially) the daughter *atom* in a very nearly unexcited state, the observed alpha-particle kinetic energy plus the recoil energy is the total energy as generally defined. Further it is the proper total energy to use in calculations regarding the (experimentally observed) decay of *atoms*, provided that the potential used is also that in the *atom*, i.e., the *screened* nuclear-alpha Coulomb potential. In *effect*, the screening correction to the potential energy can be made, it turns out, by adding the electron binding energy difference to the observed decay energy in those terms arising from the region $r>R$ and then using the unscreened potential. For references and discussion of this problem as it applies to beta decay, see Freedman, Wagner, and Engelkemeir, Phys. Rev. **88**, 1155 (1952).

omitted by Devaney and Preston but which is of somewhat greater importance in the surface well model. A further remark on the source of this term will be made later, in Sec. III, when more details will be given on exactly what is meant here by the traditional model.

The result of the separation of Eq. (11) into real and imaginary parts is

$$f^{\mathrm{II}}=-kR|\Phi|^{\frac{1}{2}}(1+\gamma)+i\{kR|\Phi|^{\frac{1}{2}}\exp(-2\omega) -[MR\lambda(1-\gamma)]/[2\hbar k|\Phi|^{\frac{1}{2}}(1+\gamma)]\}. \quad (13)$$

The nomenclature used in Eq. (11) is repeated in Eq. (13), but it is to be understood that where the energy appears in the latter it is the real part E. A factor $(1-2\gamma)$ in the last term on the right side of Eq. (13) has been replaced by the ratio $(1-\gamma)/(1+\gamma)$ for convenience. Not only is γ small, but the term itself is small.

It is now necessary to discuss the function to which f^{II} is to be equated, i.e., the corresponding function f^{I} in Region I at $r=R$. Following Feshbach *et al.*,[2] this function is written as

$$f^{\mathrm{I}}=f^{\mathrm{I}}(E)-[(i\lambda\hbar)/2](\partial f^{\mathrm{I}}/\partial W)_E, \quad (14)$$

since the second term in Eq. (1) is, in absolute value, so small compared to the first. Upon equating f^{I} and f^{II} it is found that

$$f^{\mathrm{I}}(E)=-kR|\Phi|^{\frac{1}{2}}(1+\gamma), \quad (15)$$

and

$$\lambda=(2\hbar/M)\frac{k|\Phi|^{\frac{1}{2}}\exp(-2\omega)}{-[\hbar^2/(MR)](\partial f^{\mathrm{I}}/\partial W)_E+(1-\gamma)/[k|\Phi|^{\frac{1}{2}}(1+\gamma)]}. \quad (16)$$

Equation (15) is not to be construed to mean that the right side is an expression for $f^{\mathrm{I}}(E)$, i.e., Eq. (15) is not an identity. Rather, once an expression for $f^{\mathrm{I}}(W)$ has been chosen, and if other necessary parameters such as R and L were to be assigned, then the decay energy for the corresponding (idealized) one-body model would be found from Eq. (15) and the decay constant would be found from Eq. (16). In practice, with the traditional model, say, and data for an alpha particle group for which L is known, Eqs. (15) and (16) would be solved simultaneously for R and the depth of the well.

For the potential function illustrated in Fig. 1, the logarithmic derivative of the one-body wave function in Region I is determined from

$$\phi^{\mathrm{I}}(r)=(\kappa r)^{\frac{1}{2}}[J_{L+\frac{1}{2}}(\kappa r)J_{-(L+\frac{1}{2})}(\kappa R_0) -J_{-(L+\frac{1}{2})}(\kappa r)J_{L+\frac{1}{2}}(\kappa R_0)], \quad (17)$$

where κ is the wave number in the well and R_0 is to represent the *nuclear* radius only. The functions J are Bessel functions of the indicated order. From this equation it is found that

$$f^{\mathrm{I}}(E)\equiv L+1-\kappa R(v/u) \quad (18)$$

and[17]

$$-[\hbar^2/(MR)](\partial f^{\mathrm{I}}/\partial W)_E =R\{1-[(2L+1)/(\kappa R)](v/u) +(v/u)^2-[2/(\pi\kappa Ru)]^2\}, \quad (19)$$

where

$$v=J_{L+\frac{1}{2}}(\kappa R)J_{-(L+\frac{1}{2})}(\kappa R_0)+J_{-(L+\frac{1}{2})}(\kappa R)J_{L+\frac{1}{2}}(\kappa R_0), \quad (20)$$

$$u=(\kappa R)^{-\frac{1}{2}}\phi^{\mathrm{I}}(R). \quad (20a)$$

[17] The quantity $[J_{L+\frac{1}{2}}(x)J_{-(L+\frac{1}{2})}(x)+J_{-(L+\frac{1}{2})}(x)J_{L+\frac{1}{2}}(x)]$ appears, during the derivation, on the right side of Eq. (19). This quantity can be shown to be the Wronskian $W[J_{L+\frac{1}{2}}(x),J_{-(L+\frac{1}{2})}(x)]$; further, it can be shown that xW is a constant. Evaluation of the constant by letting x be large gives $W=(-1)^{L+1}[2/(\pi x)]$. Also see E. T. Whittaker and G. N. Watson, *A Course of Modern Analysis* (Macmillan Company, New York, 1946), p. 380, example 13.

After a little manipulation, the results of using Eq. (18) in Eq. (15) and Eq. (19) in Eq. (16) can be put in the forms

$$\kappa Rv=(g_L+L+1)u \quad (21)$$

and

$$\lambda_0{}^L=[2\hbar/(MRS_s)]k|\Phi|^{\frac{1}{2}}\exp(-2\omega), \quad (22)$$

where

$$g_L=kR|\Phi|^{\frac{1}{2}}(1+\gamma) \quad (23)$$

and

$$S_s=1+(\kappa R)^{-2}\{g_L{}^2+g_L-L(L+1) -[2/(\pi u)]^2\}+(1-\gamma)/g_L. \quad (23a)$$

The superscript L has been added to λ to show that the result is for other than $L=0$, and the subscript zero is used to emphasize that this is a one-body decay constant.

III. COMPARISON WITH THE TRADITIONAL MODEL

The traditional potential energy function has been used as the basis of derivations of the alpha decay constant which have led to many different results. This situation will be discussed briefly to show what will be meant here by "the traditional model" and why.

In an unpublished investigation of the one-body model of alpha decay[18–20] now being made, it has been shown[19] that the most accurate derivation based on the traditional potential function, which is now in the literature, is that by Preston.[14] Comparison of Preston's result with what is called here, as a consequence of that investigation, the "exact" result showed that the differences are just those terms which Preston pointed out

[18] G. H. Winslow and O. C. Simpson, Argonne National Laboratory Report ANL-4841, June, 1952 (unpublished).

[19] G. H. Winslow and O. C. Simpson, Argonne National Laboratory Report ANL-4901, December, 1952 (unpublished).

[20] G. H. Winslow and O. C. Simpson, Argonne National Laboratory Report ANL-4910, January, 1953 (unpublished).

that he was neglecting. The numerical effect of these terms is small. Indeed, as far as calculated radii are concerned, the numerical effect of the approximations made by Bethe[13] and Rasetti[21] is not large. These latter two results are identical; they yield calculated radii which differ by only about 0.4 percent[22] from those given by the exact result, whereas the radii from some of the other equations can differ by as much as ten percent for the same input data.

This "exact" result was determined by showing that it was identically reproduced by the method of complex eigenenergies used in Sec. II, by Bethe's quasi-stationary state method, and by Rasetti's method of examining the change in time of the nonstationary wave function.[23] These latter two methods give somewhat greater insight into that term in Eq. (13) which is proportional to λ. It arises as a result of recognizing that there is a small part of the wave function which extends into the barrier. This recognition must be made during the normalization process when Bethe's method is used or, equivalently, when, with Rasetti's method, the probability remaining in the source is calculated. Thus this term is, logically, a necessary adjunct of the recognition that the phase of the interior wave function is less than π at $r=R$. The recognition of this latter fact is the principal difference between Preston's or the exact result and the Bethe-Rasetti approximation. Since the phase of the interior wave function at $r=R$ turns out to be smaller for the shell potential than for the traditional potential, that part of the wave function which extends into the barrier plays a more important role in the surface shell model than in the traditional model.

It is the so-called exact result referred to above which is meant here by "the result of the traditional model." Its properties have been discussed by Preston; they will be reviewed here in the process of comparison with the properties of the surface shell model. The equations, in the notation used here, can be quickly obtained from

$$\phi^{\mathrm{I}}(r)=(\kappa r)^{\frac{1}{2}}J_{L+\frac{1}{2}}(\kappa r). \tag{24}$$

After determination of the corresponding f^{I} and $(\partial f^{\mathrm{I}}/\partial W)_E$ and substitution into Eqs. (15) and (16), the results are that

$$\kappa R J_{L+\frac{3}{2}}(\kappa R)=(g_L+L+1)J_{L+\frac{1}{2}}(\kappa R) \tag{25}$$

and that $\lambda_0{}^L$ has the same form as in Eq. (22), except that S_e is to be replaced by S_T, where

$$S_T=1+(\kappa R)^{-2}[g_L{}^2+g_L-L(L+1)]+(1-\gamma)/g_L. \tag{26}$$

It is now convenient to write

$$\lambda_\alpha{}^L=p_L\lambda_0{}^L, \tag{27}$$

where $\lambda_\alpha{}^L$ is an observed alpha decay constant and, for the moment, p_L is to be considered only as a device by which the traditional model and the surface well model can be compared. It is also convenient to write for both models

$$R=R_0(=r_0A^{\frac{1}{3}})+\Delta R, \tag{28}$$

where R_0 is considered to be the radius of the daughter; hence A is the mass number of the daughter. In the surface well model ΔR is one of the parameters in the square well approximation to the nuclear potential at the nuclear surface. It plays the role of the "effective" alpha particle radius, i.e., it corresponds to the ρ_α used by Blatt and Weisskopf.[6] It is apparent, however, that $\lambda_0{}^L$ calculated with the surface well model will depend separately on R_0 and ΔR but that the value calculated with the traditional model, or with Devaney's[5] model, will depend only on the sum R.

Decay data[24] for Po^{212} have been used as an example to calculate values of $\log p_0$ with the two models. Various values of R and ΔR were used; the results with the traditional model are shown in Fig. 2(a) and those with the surface well model are shown in Fig. 2(b). Apart from the difference in the way the values of $\log p_0$ for the two models depend on R_0 (or r_0) and on ΔR, it can be seen that for a given r_0 and ΔR, the decay constant that would be calculated with the surface well model would be much larger than the one that would be calculated with the traditional model. The explanation for this is seen by examining the depths of the wells as given in Table I. In classical language, the greater kinetic energy in the surface well would mean a higher frequency of impact with the barrier wall.

The next comparison to be made is that of the way in which the two one-body decay constants change with L. The calculations were made with $\eta=20$ and $kR=10$ at each value of L, as an example. Calculated

TABLE I. Comparison of well depths (in Mev) in Po^{212} decay for different radii (in 10^{-13} cm).

R \ ΔR	Surface well model 0.5	1.5	2.5	Traditional Any
7.75	−78.13	−5.23	2.98	8.17
8.25	−76.88	−5.02	3.05	8.26
8.75	−75.69	−4.81	3.12	8.34
9.25	−74.56	−4.61	3.20	8.40

[21] F. Rasetti, *Elements of Nuclear Physics* (Prentice-Hall, Inc., New York, 1936), p. 100.

[22] If the derivation is made with WKB connections throughout, as was done by Kemble (see reference 12, p. 178), the result again yields calculated radii which differ by only about 0.4 percent from those given by the exact result. When following any method in which probability currents are used specifically, as Kemble did, it must be recognized that the amplitude of the outgoing wave in the well, from which wave the current incident on the barrier wall is calculated, is, to the required accuracy, just half of the amplitude of the (nearly stationary) total wave function in the well.

[23] Rasetti's method has been carried out in great detail only for $L=0$. It seems certain, however, that agreement with other methods would be obtained for any L.

[24] The data used in the examples were taken from the new table of isotopes: Hollander, Perlman, and Ghiorso, Revs. Modern Phys. 25, 469 (1953). The author wishes to emphasize that reference to this very convenient table should not decrease the credit due to the original observers to whom the table refers.

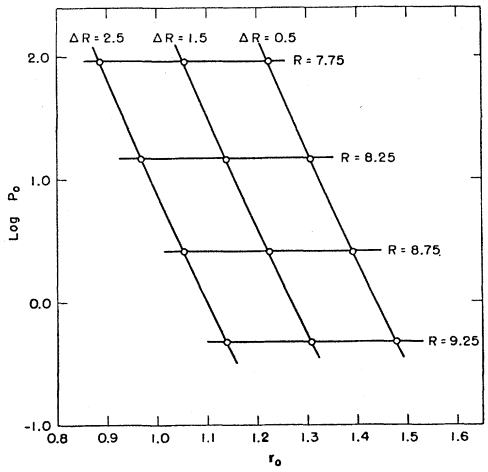

FIG. 2(a). Values of ratios p_0 of experimentally observed decay constant of Po^{212} to those calculated with the traditional model for various values of R and ΔR as a function of $r_0[=(R-\Delta R)(208)^{-\frac{1}{3}}]$. All distances are in units of 10^{-13} cm.

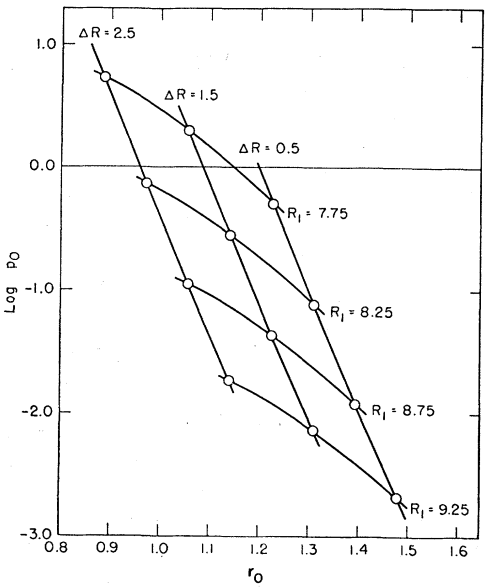

FIG. 2(b). Values of ratios, p_0, of experimentally observed decay constant of Po^{212} to those calculated with the surface well model for various values of R and ΔR as a function of $r_0[=(R-\Delta R)(208)^{-\frac{1}{3}}]$. All distances are in units of 10^{-13} cm.

values of $\log(\lambda_0{}^L/\lambda_0{}^0)$ are shown in Fig. 3, where Curve *a* is for the traditional model and Curve *b* is for the surface well model. It is seen that the former goes through a maximum at $L\doteq 2$, whereas the latter decreases monotonically. The formal explanation can be made briefly. If the first nonzero root of Eq. (25) is found at successively increasing values of L, it is found that the size of these roots increases rather rapidly. As a result S_T initially decreases more rapidly than does $\exp(-2\omega)$ and it is only after $L=2$ that the decrease in $\log(\lambda_0{}^L/\lambda_0{}^0)$ begins. This is possibly the most startling result of a consistent application of the traditional model and was first pointed out, as already mentioned, by Preston.

These roots of Eq. (21), on the other hand, increase so slowly with L that S_s is essentially constant. Indeed, it has been found with this example that S_s changes only by about three percent between $L=0$ and $L=6$. Thus the Gamow method[25] of treating the effect of changes in L by only including the changes in the exponential term, which represents an arbitrary departure from the traditional model, is the exact result of the surface well model.

It is of interest to examine the dependence of the roots of Eqs. (21) and (25) on L a little more closely. The squares of these roots are proportional to the kinetic energies in the wells; it is these energies which will be considered. With the same example as used to discuss $\log(\lambda_0{}^L/\lambda_0{}^0)$ and with $R=9\times10^{-13}$ cm and (for the surface well model) $\Delta R=1.2\times10^{-13}$ cm, it is found that the kinetic energies in the wells depend on L in the manner shown in Fig. 4. Here, again, Curve *a* is for the traditional model and Curve *b* is for the surface well model. It is seen that this energy increases linearly with $L(L+1)$ for the latter model but not for the former. Further, as indicated previously, these energy changes are a much smaller fraction of the total kinetic energy in the surface well than in the traditional well where the kinetic energy at $L=0$ is of the order of 0.5 Mev.

Since the total energy was held constant for this example, these kinetic energy changes correspond to increases in well depth. The fact that the changes are a small part of the total kinetic energy in the surface well model can be used to find an expression for them with the aid of Eq. (21). Since the result was found to agree with the exact calculation for this case of constant total energy, the same procedure was used to examine

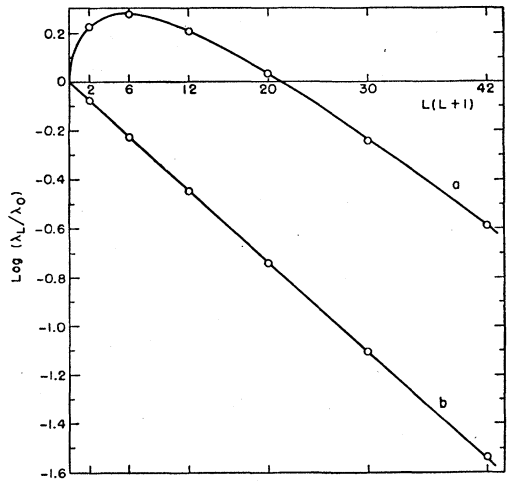

FIG. 3. Comparison of $\log(\lambda_0{}^L/\lambda_0{}^0)$ for constant energy and radii. Curve *a* is for the traditional model and Curve *b* is for the surface well model.

[25] See reference 1, p. 173.

TABLE II. ($\tau_{\text{experimental}}/\tau_{\text{theoretical}}$) for the first weak group.[a]

Parent	a	b	c	d
Cm^{244}	3.32	1.05	1.73	1.8
Cm^{242}	3.22	1.01	1.68	1.7
Pu^{240}	3.14	0.99	1.66	1.7
Pu^{238}	3.41	1.07	1.80	1.9
Pu^{236}	4.33	1.36	2.28	2.2
U^{238}	2.70	0.85	1.45	1.5
U^{236}	2.07	0.65	1.11	1.1
U^{234}	2.48	0.78	1.33	1.3
U^{232}	1.89	0.60	1.01	1.1
U^{230}	2.78	0.88	1.48	1.5
Th^{232}	1.85	0.58	1.00	1.0
Th^{230}	1.80	0.57	0.97	1.0
Th^{228}	1.57	0.50	0.84	0.9
Th^{226}	1.98	0.63	1.06	1.1
Ra^{226}	1.54	0.49	0.84	0.9
Ra^{224}	2.28	0.73	1.24	1.25

[a] Column a: Traditional model, $L=2$.
Column b: Surface well model, $L=2$. These numbers would be essentially those obtained from the traditional model if only the effect of L on the exponential term were included.
Column c: The results if no L terms are included. They were obtained by elimination of such terms from the results in Column b and are to be compared with the results obtained by the use of Kaplan's approximation to Preston's formulation of the traditional model, Column d.
Column d: F. Asaro, reference 27.

the energy changes with a constant well depth. This case is more difficult to solve exactly since the total energy appears in both κ and $(k|\Phi|^{\frac{1}{2}})$. The result[26] is that

$$(\Delta E)_L \doteq [\hbar^2/(2MRR_0)][L(L+1)] \times \frac{[1+R_0/(g_0\Delta R)]}{[1+R/(g_0\Delta R)]}. \quad (29)$$

Finally, some numerical calculations pertaining to the main group and first weak group of alpha particles emitted by even-even isotopes have been made. It was assumed that $L=0$ for the main group and $L=2$ for the weaker group. The values $r_0=\Delta R=1.2\times10^{-13}$ cm were picked arbitrarily, and the ratio (p_0/p_2) was calculated. Some preliminary calculations showed that not only was S_s insensitive to changes in L, but it was also insensitive to the small energy differences involved here and to changes in the values of r_0 and ΔR. This last result has been shown, actually, only for $r_0=\Delta R$. Thus S_s was assumed to be the same for each group from the same isotope, which made it unnecessary to calculate any values of κ for the surface well model.

[26] This result should be compared with the discussion of the relation between the energies and spins of nuclear states given by A. Bohr and B. R. Mottleson in Phys. Rev. **89**, 316 (1953). The association of the values of $(\Delta E)_L$ given by Eq. (29) with states of the daughter nucleus has been suggested by O. C. Simpson. It would be argued that there are many possible nucleon configurations corresponding to a given state and that one such configuration should be that of an alpha particle at the surface, i.e., in the surface well. Numerically speaking, the values of $(\Delta E)_L$ calculated from Eq. (29) are too large, particularly for the heavier alpha emitters. Nevertheless the result is attractive and the answer to the numerical difficulty may be an incomplete potential function, i.e., it may not be sufficiently correct to consider the depth of the well as fixed (apart from the angular momentum term) when L is changed.

It was, of course, necessary to find κ in order to calculate (p_0/p_2) for the traditional model, since it is just the large changes in S_T with changes in L which cause that numerical difference between the models which is to be demonstrated here. The same values of r_0 and ΔR were used as with the surface shell model. Insensitivity of the traditional model to values of R was not shown here by direct calculation, but it can be shown by comparison with some numbers calculated by Asaro.[27]

Asaro, with the use of Kaplan's[28] approximation to Preston's[14] result, calculated radii with data for the main groups of several even-even isotopes and then used these radii to find values of a so-called $\tau_{\text{theoretical}}$ (theoretical half-life) for the weaker group emitted by each isotope. This second calculation was again made with Kaplan's equation, i.e., no term which is explicitly dependent on L was included. Asaro then tabulated values of $(\tau_{\text{experimental}}/\tau_{\text{theoretical}})$.

Because of the insensitivity of (p_0/p_2) to the radii (provided, of course, that the same radii are used with each group from a given isotope) it is reasonable, for the moment at least, to refer to these ratios as $(\tau_{\text{experimental}}/\tau_{\text{theoretical}})$ for the weaker group, in the sense used by Asaro. The results are given in Table II. No arguments can be given for the preference of one model over the other on the basis of the values of these ratios themselves. The drop by a factor of two (roughly) as one proceeds down the table can probably not be explained by any model of this type, i.e., a *one-body* model with complete spherical symmetry, with a barrier having a vertical inner face, and in which the distances are given literal meanings, i.e., in which the same radii are used for each group. It is suggested, however, that when the principal interest is alpha-decay theory as opposed to the empirical use of alpha-decay systematics, the effect of L on the exponential term, at least, should be included in such ratios wherever possible. That is, it should be expected that such terms must appear in a

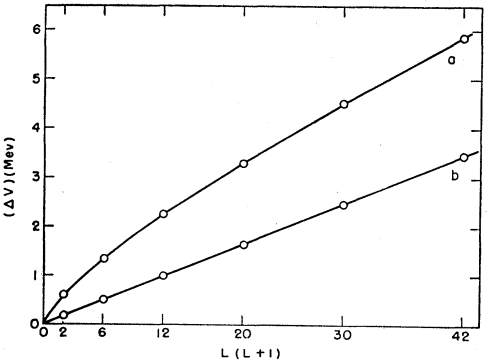

FIG. 4. The increase in kinetic energy in the well as L is increased. Curve a is for the traditional model and Curve b is for the surface well model. Since E was held constant the curves represent the increase in the depths of the wells.

[27] F. Asaro, University of California Radiation Laboratory Report UCRL-2180, June, 1953 (unpublished), Table 19.
[28] I. Kaplan, Phys. Rev. **81**, 962 (1951).

complete expression for the alpha-decay constant. Thus, here, the assignment of $L=0$ to the main group and of $L=2$ to the first weak group seems reasonable.

IV. THE MANY-BODY DECAY CONSTANT

Suppose a channel radius is calculated in the traditional way by using, for example, data for the main group of an even-even isotope; that is, the calculation is made by equating the experimental decay constant directly to the expression for the one-body decay constant derived with the traditional potential function. It was shown in the previous section that if this radius, after some reasonable separation into R_0 and ΔR [see Eq. (28)], is now used to calculate a decay constant with the surface well model, then the result will be considerably larger than the experimental decay constant. Nevertheless, the radius used in the latter calculation seems reasonable in the sense that it will be very nearly the same as might be found by summing a radius, taken from some other consideration, for the alpha daughter and a reasonable value for the nuclear interaction distance between the daughter and the alpha particle.

Two remarks immediately suggest themselves. First, the procedure by which this radius was obtained is inconsistent with the idea that alpha decay is *not* a one-body process. Although several of the remarks on this topic[29–31] imply that the traditional model is complete when applied to the main group from an even-even nucleus, nevertheless it should not be expected *a priori* that any one-body model would be complete for any case. Second, if it is desired to continue the use of such values of the channel radii because of their general applicability but if it is also considered that, because of its nature and properties, the surface well model should have some relation to actual alpha decay, then consideration of what might cause an appropriate reduction in decay probability is forced.

Further, when any particular one-body model is used, it is found that the channel radii calculated for most groups of alpha particles emitted by noneven-even nuclei, or even for the main groups of those even-even nuclei which have less than 128 neutrons, are considerably smaller than those calculated for the main groups of the heavier even-even nuclei. On the basis of comparison with the latter channel radii, the former are customarily said to be "abnormally small." The radius so calculated with decay data of Po^{212} itself (128 neutrons) is nearly "normal." Further, no *reasonable* assignments of relative angular momentum quantum number L will allow the calculation of "normal" radii for many of the groups from noneven-even nuclei.[29,30] If these small channel radii are treated directly as nuclear radii, or if a constant interaction distance is used to obtain the nuclear radii from them, then the latter become abnormally small.

This problem is common. In addition to their occurrence with the one-body models, these abnormally small radii are also to be found in a table published by Blatt and Weisskopf,[32] credited to Devaney and calculated with his many-body model in which a constant value of $\rho_\alpha(=\Delta R)=1.2\times10^{-13}$ cm was used. When the parents are, say, Bi^{212}, Po^{210}, and Po^{212}, the values calculated for r_0 are, respectively, 1.14, 1.21, and 1.31, all in units of 10^{-13} cm. These numbers when calculated with Bethe's[13] approximation to the traditional model and $\Delta R=1.2\times10^{-13}$ cm are 1.05, 1.22, and 1.32, respectively. Presumably there should be a relative angular momentum quantum number involved in the case of Bi^{212}. If $L=4$, say,[14] then Devaney's value for r_0 would be raised to about 1.23×10^{-13} cm. Blatt and Weisskopf also consider values of r_0 as low as 1.2×10^{-13} cm to be spurious, as compared to the "normal" values of 1.3×10^{-13} cm to 1.4×10^{-13} cm. They again suggest the reason commonly quoted in discussions of a one-body model, i.e., an inexactitude in the equations such that they do not take into account an expected increased difficulty of assembling an alpha particle when it is necessary to dip into the Pb^{208} core for some of the constituents.

Other estimates of nuclear radii—from total cross sections for fast neutrons, for instance—do not show such a pronounced effect in this region. As an example, the "radii" given by Hildebrand and Leith[33] for 42-Mev neutrons would correspond to r_0 for lead or bismuth, being about three percent less than that for thorium rather than the 10 to 20 percent drop noted in the values deduced from alpha decay data. Further, it has been suggested[30] that the radii of noneven-even nuclei should be larger, if anything, than those of neighboring even-even nuclei.

The arguments which will be made in this section concerning a factor which will reduce the decay probability below that predicted by a one-body model will lead to a result which involves two quantities. One of these will be an "intrinsic alpha-decay constant" which is not, however, directly equal to Devaney's intrinsic alpha-decay constant. The second will be the probability per second that an alpha particle at the nucleus (or at the nuclear surface) will be absorbed into the nucleus as nucleons, i.e., that a compound nucleus will be formed. The appearance of the second of these two quantities might allow the explanation of the abnormally small radii, whereas the first quantity alone might fail in this respect. In conjunction with estimates of these probabilities, to be made in the next section, it will be suggested on the other hand that small *channel* radii might not be abnormal.

[29] Perlman, Ghiorso, and Seaborg, Phys. Rev. **75**, 1096 (1949).
[30] Perlman, Ghiorso, and Seaborg, Phys. Rev. **77**, 26 (1950).
[31] See reference 8, M. A. Preston.
[32] See reference 6, p. 578. The exact source of the values of level spacing which were used does not appear to be given.
[33] R. H. Hildebrand and C. E. Leith, Phys. Rev. **80**, 842 (1950).

Although such estimates will be made, no attempt will be made to derive expressions for these probabilities in terms of such quantities as nuclear spins, relative angular momentum, level widths and spacings, etc. Rather, the way in which they would be expected to appear in an equation for the alpha-decay constant will be shown by a sort of phenomenological argument. The following presumptions are added to those of Sec. II: (c) in the absence of the external Coulomb field there would still be a noninfinite probability per second for the emission of alpha particles; (d) the presence of the barrier, which is the net effect of the Coulomb and nuclear interactions, makes it possible that an alpha particle, once formed, might become unformed and reabsorbed as nucleons rather than be emitted through the barrier. In addition, in order to apply the concept to a model of the traditional type in which the alpha particle is not excluded from the region occupied by the nucleus, it is necessary to amplify somewhat—essentially to modify—the previous presumption (a) that alpha particles do not exist *in* nuclei. To do this use is made of a suggestion by Bethe[13] that "there might be some slight tendency for the preformation of alpha particles in nuclei due to their great stability." This suggestion is applied literally where the corresponding one-body model is of the traditional type. In the case of a surface well model it is considered that there is a tendency for the "preformation" of alpha particles at the nuclear surface. In either case it is assumed here,[34] in order to advance the argument, that there are periods during which one-body wave functions apply in some region behind the Coulomb barrier; that is, some one-body potential (not necessarily, of course, any sort of square well potential) is to be used during these periods, but only during these periods.

Suppose there is only one alpha particle in existence behind the Coulomb barrier at a time, i.e., suppose that there is one species with no alpha particle behind the barrier (hence, actually, no well and no barrier) and a second species with one alpha particle behind the barrier. Let $\lambda_1{}^L$ be the decay probability of the first species to the second, i.e., let it be (or at least very nearly be) the decay constant in the absence of the external Coulomb field. Let $\lambda_2{}^L$ represent the decay constant of the second species to the first, i.e., let it be the probability per second that an alpha particle in the nucleus or at the nuclear surface, as the case may be, will break up and its constituents be reabsorbed. Thus the second species branches by decaying to the first with decay constant $\lambda_2{}^L$ and by emitting alpha particles with decay constant $\lambda_0{}^L$. If the generic relationships between these two species are set up, the effective alpha decay constant can be determined. It turns out that, if $\lambda_0{}^L$ is much smaller than either $\lambda_1{}^L$ or $\lambda_2{}^L$, a reasonable expectation, then there is a net decay by alpha emission with the decay constant

$$\lambda_\alpha{}^L=\lambda_1{}^L\lambda_0{}^L/(\lambda_1{}^L+\lambda_2{}^L). \tag{30}$$

Thus Eq. (27) is now given additional significance beyond that given it at the time it was written in that the many-body alpha decay constant is, according to this picture, to be written as the product of a dimensionless "preformation factor" and a one-body decay constant. The preformation factor is given by

$$p_L=\lambda_1{}^L/(\lambda_1{}^L+\lambda_2{}^L). \tag{31}$$

The reciprocal of p_L is, except for the effects of L in the one-body decay constant, just the "departure factor" (for the traditional model) as used by Perlman, Ghiorso, and Seaborg.[30]

The solution of the problem of deriving expressions for $\lambda_1{}^L$ and $\lambda_2{}^L$ is not to be attempted here, as was stated previously. However, it is considered that the probability $\lambda_1{}^L$ is to be associated with an intrinsic, non-Coulomb barrier, alpha-decay constant and that the probability $\lambda_2{}^L$ is conceptually equivalent to the sticking coefficient ξ as used by Bethe,[35] Peaslee,[36] Weisskopf, and Ewing,[37] for instance, or, more particularly, to the probability P discussed by Feshbach, Porter, and Weisskopf.[38] Thus according to this argument the problem of the probability of formation of a compound nucleus, once the particle involved is at the nucleus, should also appear in the discussion of the theory of alpha decay.

V. DISCUSSION

The many-body model is distinguished in this paper from the one-body model by the requirement that it be recognized in the former that an alpha particle is not always present behind its accompanying Coulomb barrier during the interval between the formation of the alpha active parent and its decay. This requirement is considered to be separate from that of finding the potential which best represents the interaction between an existing alpha particle and the residue during those intervals when there might be an alpha particle present behind its Coulomb barrier.

The method of Feshbach, Peaslee, and Weisskopf,[2] which was used by Devaney[5] to discuss the many-body theory of alpha decay, has been referred to as the use of an "equivalent two-body potential" by Francis and Watson.[39] (The phrase "one-body" used in this paper and generally in discussions of alpha decay has the same meaning as "two-body.") The model has also been used in this way by Blatt and Weisskopf,[40] for instance, to estimate a normalization constant for the interior wave function of an escaping particle. It would

[34] Supporting arguments are given in Sec. V.

[35] See reference 13, Part D of §54, p. 96.
[36] D. C. Peaslee, Phys. Rev. **74**, 1001 (1948).
[37] V. F. Weisskopf and D. H. Ewing, Phys. Rev. **57**, 472 (1940).
[38] Feshbach, Porter, and Weisskopf, Phys. Rev. **87**, 188 (1952).
[39] N. C. Francis and K. M. Watson, Phys. Rev. **92**, 291 (1953).
[40] See reference 6, p. 421.

certainly seem reasonable to adopt this attitude where the "particle" involved is a fundamental particle, although some arguments to be raised later suggest that this attitude should be modified even in such a case.

When one comes to consider the case of a compound particle, such as an alpha particle, reference should be made to the derivation by Weisskopf[41,42] of P (not the P of Feshbach, Porter, and Weisskopf[38]) $=(2\pi\hbar)/D$, where D is an appropriate level spacing, as a nuclear period. It was derived by showing (under the specified assumptions) that P is the repetition rate of a $|\psi(t)|^2$ which corresponds to a particular grouping of nucleons. Such a grouping might correspond to an alpha particle which, however, is "nonexistent" between repetitions of this $|\psi(t)|^2$. Thus the method should meet the principal requirement of a many-body model which was set forth above.

On the other hand, further reference to the derivation of this period P shows that it should be interpreted in this case as the interval between successive appearances of an alpha-particle grouping at, but within, the surface of the parent nucleus. Now, according to Feshbach, Porter, and Weisskopf,[38] there is evidence that the compound nucleus in a nuclear reaction, even when the bombarding particle is a neutron, is formed less rapidly than had been hitherto assumed. This would mean that the formation of a compound nucleus is inhibited by something more—by some other "barrier"—than by the ordinary centripetal barrier, by the partial reflection at a potential jump, or, for charged particles, by the Coulomb barrier. It would be expected that this inhibition would be even more pronounced for an alpha particle and, further, that it would operate in both directions. In Devaney's model it is the Coulomb barrier which permits the existence of the well-defined state whose period is P. It is argued here, however, that this state would still be reasonably well defined in the absence of the Coulomb barrier because of the existence of the further "barrier" discussed above.

The decay constant $\lambda_1{}^L$, then, might be obtained by multiplying $D/(2\pi\hbar)$ by some number less than unity; that is, $\lambda_1{}^L$ is used here to represent the probability per second for the actual division of the compound nucleus (parent) into two entities, the daughter nucleus and a well formed alpha particle. Were there no Coulomb barrier these entities would usually move apart immediately after formation. The decay constant would be $\lambda_1{}^L$ except for the low but not zero reflection at a potential "jump" caused by the rapidly varying attractive nuclear interaction potential. It is this complete separation which is held in check by the Coulomb barrier. Since the latter is so broad and high, the return of the system to its original condition is more likely to occur than is alpha-particle emission.

Expression (30) is to be considered as an approximate expression of these ideas. The goodness of the approximation depends on the life of the divided but unseparated system. Again, however, if the probability of reformation of the compound nucleus (parent) is inhibited as described above, the approximation should be reasonably good, even to the extent of using a strict one-body decay constant for $\lambda_0{}^L$, since many traversals of the region allowed to the alpha particle would occur, on the average, before either reformation of the compound nucleus (parent) or escape of the alpha particle through the barrier. Further, by the very nature of the expression for p_L [Eq. (31)] its value will be less than unity, and so a factor has been found which will reduce the decay probability below that predicted by a one-body model.

It has been stated that Eq. (31), which involves the two quantities $\lambda_1{}^L$ and $\lambda_2{}^L$, might provide a means for explaining the so-called abnormally small radii even if $\lambda_1{}^L$ alone could not do so. If a proper derivation of these two quantities should show that $\lambda_2{}^L$ is relatively much greater in those cases where the small radii occur, then the explanation could be effected even with the same interaction distance for all cases. That is, a decrease of p_L of the proper magnitude, caused by an increase of $\lambda_2{}^L$, could lead to the calculation of normal channel radii.

An attempt to make a rough estimate of $\lambda_1{}^0$ and $\lambda_2{}^0$ has led, on the other hand, to a somewhat different explanation. This suggestion is equally applicable to Devaney's model, to the traditional model, or to expression (30) with the result of the surface well model used for $\lambda_0{}^0$. Suppose in the latter case that $\lambda_2{}^0$ is written as

$$\lambda_2{}^0=[v/(2\Delta R)]\xi, \tag{32}$$

where ξ is the probability of formation of the compound nucleus (parent) per impact of the alpha particle on the surface of the daughter nucleus and v is the velocity in the well. At least for the main groups from even-even nuclei it might be possible to write also that

$$\lambda_1{}^0=[D/(2\pi\hbar)]\xi. \tag{33}$$

If, for Po^{212} as an example, D is taken from Devaney,[32] then one can find the values of r_0 and ΔR which will satisfy Eq. (30) for this model. It is found that the corresponding values of the channel radius R are not exactly constant as they would be in Devaney's or the traditional model, but they are nearly so. The value $r_0=1.31\times10^{-13}$ cm corresponds to $\Delta R=1.2\times10^{-13}$ cm. Generally, for α decay and with distances of this size, $[v/(2\Delta R)]\gg[D/(2\pi\hbar)]$.

If now one proceeds to Po^{210} the same effect is observed here as with the other models, i.e., the channel radius is small. The value of $[v/(2\Delta R)]$ is very nearly the same for every nucleus if ΔR is the same for every nucleus. But the question is now asked, why should one necessarily expect ΔR to be constant and associate the small r_0 which then accompanies a small channel radius,

[41] See reference 6, p. 386.
[42] V. F. Weisskopf, Helv. Phys. Acta **23**, 187 (1950).

with an error in the theory? In the surface well model, at least, ΔR is a parameter in a square-well approximation to the nuclear potential at the nuclear surface. It might very well be the case that such a parameter would be different between the two cases where the daughter nucleus is the closed shell nucleus Pb^{208} and where the daughter lacks two neutrons of being the closed shell nucleus. If this position is taken, then it is found for Po^{210} that $r_0=1.31\times10^{-13}$ cm corresponds to $\Delta R\doteq0.6\times10^{-13}$ cm. If one goes back to Devaney's result[32] for this nucleus, for instance, and uses 0.6×10^{-13} cm for his ρ_α, then his result for r_0 becomes 1.32×10^{-13} cm. This agreement is not surprising, of course. The leading term in Eq. (23a) is, for $L=0$, just $(\Delta R/R)\times[1+g^2/(\kappa R)^2]$. Thus, if only this term is used along with the above expressions for λ_1^0 and $\lambda_2^0(\lambda_2^0\gg\lambda_1^0)$, the expression for λ_α^0 becomes identical with Devaney's Eq. (2.21) with $L=0$.

The above estimates are only based on a suggested possibility. To make them does not exclude the possibility that detailed considerations leading to proper expressions for λ_1^L and λ_2^L would show that p_L changes from alpha emitter to alpha emitter in such a way that $R=r_0A^{\frac{1}{3}}+\Delta R$, with r_0 and ΔR constant, even in those cases where present methods yield abnormally small radii. It must be remembered in other connections also that λ_0^L is only one factor in an expression for λ_α^L. For instance, Fig. 3 shows the way in which λ_0^L depends on L for two possible descriptions of the one-body phase of the alpha decay mechanism; the way in which p_L might be expected to depend on L has not been shown here.

The two solutions of the one-body phase of the problem which have been used here might be considered as extreme cases. Not only variations of each of them but also combinations are possible. For instance, one could use a square well with the potential in $r<R_0$ large but not infinite. As between the two extremes, it is believed that a model of the surface well type is to be preferred. If there is a well-defined *nuclear* surface, as is certainly implied by the subtraction of an "effective" alpha particle radius from the channel radius to obtain the radius of such a nuclear surface,[6] then it is more logical to introduce this surface as a factor to be considered during the derivation of the alpha decay constant. The surface-well model allows one to combine: (a) the well-defined *nuclear* surface; (b) the exclusion of alpha particles, as such, from the interior of a nucleus; (c) the "slow" formation of a compound nucleus[38]; (d) the improbability of a pure Coulomb potential at the *nuclear* surface. Along with this combination of ideas it yields, as an exact result, a linear dependence of $\log\lambda_0^L$ on $L(L+1)$; thus this is the dependence of the *barrier penetrability* term in $\log\lambda_\alpha^L$. The deep surface (square) well is consistent with a large attractive interaction between an alpha particle and a nucleus at close distances. The nature of the dependence of the energy levels in the surface well on $L(L+1)$ is considered to be preferable to the nature of that dependence in the traditional well, in spite of the present difficulty with its magnitude. Finally, it is much easier to use what are presently considered to be normal channel radii, together with an expected $p_L<1$, with the surface well model than with the traditional model. These remarks about the content of the surface well model still apply after one has made those specific assumptions and approximations which lead to Devaney's expression for the alpha-decay constant.

I wish to express my appreciation of many instructive and searching discussions to Dr. O. C. Simpson of this laboratory, who first introduced me to the problem of alpha-decay systematics and theory.

APPENDIX

The question of the accuracy of the WKB wave functions when applied to the solution of the alpha decay problem has been discussed by many others. Among these are Devaney[5] and Preston,[14] to whose work frequent reference has been made here. The present author would like to add some brief remarks to this discussion. They will be developed more fully elsewhere.[43]

The WKB functions for large ρ are trigonometric functions as are the asymptotic expressions for the Coulomb functions. If, in the evaluation of the argument of the WKB functions, reciprocal powers of ρ are neglected compared to ρ, to $\ln\rho$, and to constants, and if $L(L+1)$ is replaced by $(L+\frac{1}{2})^2$, then this argument differs from that which appears in the asymptotic expressions for the Coulomb functions[15] only by

$$\epsilon=1/(24\eta)-[L(L+1)]/(24\eta^3)+\cdots.$$

It is to be remembered that $\eta=20$ is typical for this problem.

In addition, with the typical value $\rho(R)=10$, one can examine the behavior of the WKB functions near the nuclear surface. For L in the range up to $L=4$ it has been found[43] that: (a) The usual WKB functions, i.e., those containing $L(L+1)$ in the "centrifugal" term, obey certain Coulomb function recursion relations to about 10^{-3} percent; (b) The WKB functions with $L(L+1)$ replaced by $(L+\frac{1}{2})^2$, the functions used in this paper, obey these recursion relations to about 0.06 percent; (c) The ratio of the irregular WKB functions modified as in (b) to the usual irregular WKB functions (ratio $\doteq1.01$) is the same as the ratio of the irregular Coulomb functions to the usual irregular WKB functions as given by Bloch *et al.*[44] to about 0.1 percent.

[43] G. H. Winslow and O. C. Simpson, continuation of references 18–20, now in preparation. This report has been issued as Argonne National Laboratory Report ANL-5277, April, 1954 (unpublished).

[44] Bloch, Hull, Broyles, Bouricius, Freeman, and Breit, Phys. Rev. **80**, 553 (1950).

Thus, since one of the recursion relations used contains both the regular and irregular functions, the modified WKB functions are the same as the Coulomb functions to about 0.1 percent *in this range of the variables* ρ and η. In other ranges of these variables the modified functions might or might not be preferred.[45]

The notation which has been used in this paper is a mixture of that used by Bloch *et al.*,[15] Bethe,[13] and Preston.[14] Bethe's C has been replaced by Preston's ω on the grounds that C is generally used to represent a constant of some sort. Bethe wrote the quantity $|\Phi|$ as $|\Phi(\rho)|$ in certain places but did not bring η into evidence. It is believed to be preferable to use the ρ as a notation for kr, and to reserve the symbol x, which was used for kr by Preston[14] and for kR by Devaney,[5] for its older meaning as the ratio of the energy to the height of the barrier.

Examination of Eq. (16) shows that it could be written as

$$\lambda=4E(\hbar kRS')^{-1}[G_L(kR)]^{-2},$$

with

$$S'=-[h^2/(MR^2)](\partial f^I/\partial W)_E+(1-\gamma)g_L^{-1},$$

but, depending on the model used, it might be more convenient in computation to include one factor of $[G_L(kR)]^{-2}$, $|\Phi|^{\frac{1}{2}}$, in the expression of S'. The exponential term will be common, however, to a model to which Eq. (16) applies. To the first power of $(L+\frac{1}{2})^2$,

$$\log_{10}\exp(-2\omega)=(D/E_\alpha^{\frac{1}{2}})f(x)+0.75445(D/E_\alpha^{\frac{1}{2}})^{-1}[(1-x)/x]^{\frac{1}{2}}(L+\tfrac{1}{2})^2,$$

where[46]

$$D=1.09449[ZA/(A+4)],$$

$$f(x)=\cos^{-1}x^{\frac{1}{2}}-[x(1-x)]^{\frac{1}{2}},$$

E_α is the alpha-*particle* energy in Mev "corrected" as described in reference 16, and, as before, Z and A are the atomic and mass numbers of the daughter. The first term in the expansion in powers of $(L+\frac{1}{2})^2$ contributes to the third decimal place in $\log_{10}\exp(-2\omega)$ already when $L=0$, where the value of the term is about 0.009. Expansion to higher powers shows that the term in $(L+\frac{1}{2})^4$ does not contribute to the third decimal place until $L=3$, where its value is about 0.002, and the term in $(L+\frac{1}{2})^6$ does not contribute until $L=6$, where its value is about 0.001.

The ratio x is conveniently written as

$$x=E_\alpha R/K,$$

where, with R in units of 10^{-13} cm,

$$K=2.8797_2[ZA/(A+4)].$$

Further convenient formulas related to the g_L of Eq. (23) are

$$k^2R^2|\Phi|=1.32547(D/E_\alpha^{\frac{1}{2}})^2x(1-x)+(L+\tfrac{1}{2})^2$$

and

$$\gamma=-[4kR|\Phi|^{\frac{1}{2}}(1-x)]^{-1}\times[1+(k^2R^2|\Phi|)^{-1}(1-2x)(L+\tfrac{1}{2})^2].$$

It was mentioned in the text that γ is generally about -0.02 (and, hence, is frequently omitted); the correction term in the equation for γ is only about 0.05 or so at $L=5$.

[45] Yost, Wheeler, and Breit, Phys. Rev. **49**, 174 (1936).

[46] Values for the necessary atomic constants have been taken from J. W. M. DuMond and E. R. Cohen, Revs. Modern Phys. **25**, 691 (1953) and Li, Whaling, Fowler, and Lauritsen, Phys. Rev. **83**, 512 (1951).