# Letters to the Editor

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## Superconductivity of a Charged Boson Gas

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IT is the purpose of this note to point out that there exists a relatively simple physical system which exhibits the essential equilibrium features of a superconductor,[1] namely a phase transition of the second kind at a critical temperature $T_c$ and the occurrence of a Meissner-Ochsenfeldt effect below it. This system is the ideal gas of charged bosons. The existence of a transition point is well known.[2]

If one establishes the relation between the Fourier components of the current density, $\mathbf{i}(\mathbf{q})$ and of the vector potential, $\mathbf{A}(\mathbf{q})$, of an applied weak inhomogeneous magnetic field in the form

$$\mathbf{i}(\mathbf{q})=K(q^2)\mathbf{q}\times(\mathbf{q}\times\mathbf{A}(\mathbf{q})), \tag{1}$$

then, as was shown elsewhere,[3] a pole in $K(q^2)$ for $q=0$ means the occurrence of the Meissner-Ochsenfeldt effect. The particular case,

$$K(q^2)=1/c\lambda q^2, \tag{2}$$

is equivalent to the London[1] equation,

$$\mathrm{curl}(\lambda\mathbf{i}(x))=-(1/c)\mathbf{H}(x). \tag{3}$$

The thermal average current density for our model is easily evaluated by perturbation theory on the distribution function in a manner described before[3] and yields below the transition point:

$$K(q^2)=n_s\frac{e^2}{mc}\frac{1}{q^2}+K_0(q^2), \tag{4}$$

where $n_s$ is the density of condensed bosons and $K_0(q^2)$ has a singularity of order $1/q^1$ only. This proves our assertion.

The fact that $K_0(q^2)$ still has a singularity shows that (3) is not valid but has to be replaced by an integral relationship. (This was already proposed by Pippard[4] for real superconductors.) In particular the penetration depth is not determined by $n_s$ alone.

The very uniqueness of the phenomenon of Bose-Einstein condensation might be taken as a clue that this model is essentially the only one which exhibits the phenomenon of superconductivity. One would then have to show that in a metal at low temperatures charge-carrying bosons occur, e.g., because of the interaction of electrons with lattice vibrations.[5] In this connection it might be worthwhile investigating the possibility of existence of bound two-electron states. A more detailed discussion of this work will appear in a forthcoming paper.

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[5] H. Fröhlich, Phys. Rev. **79**, 845 (1950).

## Superfluidity of a Boson Gas

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WE have made a theoretical analysis of an idealized version of Andronikashvili's[1] experiment establishing the superfluid character of liquid helium below the λ point. A vessel is filled with liquid helium and set into rotation. The apparent moment of inertia is less than the classical value $I_0=\frac{1}{2}NMR^2$ ($N$ = number of atoms, $M$ = mass of atom, $R$ = radius of vessel). The ratio between the observed moment of inertia and $I_0$ is by definition the "concentration of normal fluid."

We now make a calculation of the equilibrium value of the angular momentum $L$ of an ideal Bose-Einstein gas as a function of the angular velocity $\omega$. The result is shown in Fig. 1. Above the λ point, $L$ is given by the dashed straight line, with slope equal to $I_0$. Below the λ point, $L$ is a discontinuous function of $\omega$, as shown by the full lines. The discontinuities occur whenever the angular velocity increases by an amount $\Delta\omega=\hbar^2/MR^2$, for then a new value of the angular momentum quantum number $m$ gives rise to the lowest state of an atom in the vessel. Hence at this point a macroscopic number of atoms shifts from the lowest state with quantum number $m-1$ to the lowest state with quantum number $m$, and this accounts for the sudden jump in the equilibrium angular momentum $L$.

If equilibrium is reached, the observed moment of inertia is the mean slope of the solid curve, averaged over these quantum mechanical fluctuations; that is,