Letters to the Editor

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Superconductivity of a Charged Boson Gas

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IT is the purpose of this note to point out that there exists a relatively simple physical system which exhibits the essential equilibrium features of a superconductor,[1] namely a phase transition of the second kind at a critical temperature T_c and the occurrence of a Meissner-Ochsenfeldt effect below it. This system is the ideal gas of charged bosons. The existence of a transition point is well known.[2]

If one establishes the relation between the Fourier components of the current density, $\mathbf{i}(\mathbf{q})$ and of the vector potential, $\mathbf{A}(\mathbf{q})$, of an applied weak inhomogeneous magnetic field in the form

$$\mathbf{i}(\mathbf{q})=K(q^2)\mathbf{q}\times(\mathbf{q}\times\mathbf{A}(\mathbf{q})), \tag{1}$$

then, as was shown elsewhere,[3] a pole in $K(q^2)$ for $q=0$ means the occurrence of the Meissner-Ochsenfeldt effect. The particular case,

$$K(q^2)=1/c\lambda q^2, \tag{2}$$

is equivalent to the London[1] equation,

$$\text{curl}(\lambda\mathbf{i}(x))=-(1/c)\mathbf{H}(x). \tag{3}$$

The thermal average current density for our model is easily evaluated by perturbation theory on the distribution function in a manner described before[3] and yields below the transition point:

$$K(q^2)=n_s\frac{e^2}{mc}\frac{1}{q^2}+K_0(q^2), \tag{4}$$

where n_s is the density of condensed bosons and $K_0(q^2)$ has a singularity of order $1/q^1$ only. This proves our assertion.

The fact that $K_0(q^2)$ still has a singularity shows that (3) is not valid but has to be replaced by an integral relationship. (This was already proposed by Pippard[4] for real superconductors.) In particular the penetration depth is not determined by n_s alone.

The very uniqueness of the phenomenon of Bose-Einstein condensation might be taken as a clue that this model is essentially the only one which exhibits the phenomenon of superconductivity. One would then have to show that in a metal at low temperatures charge-carrying bosons occur, e.g., because of the interaction of electrons with lattice vibrations.[5] In this connection it might be worthwhile investigating the possibility of existence of bound two-electron states. A more detailed discussion of this work will appear in a forthcoming paper.

I am greatly indebted to Dr. J. M. Blatt and Dr. S. T. Butler for stimulating discussion.

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Superfluidity of a Boson Gas

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WE have made a theoretical analysis of an idealized version of Andronikashvili's[1] experiment establishing the superfluid character of liquid helium below the λ point. A vessel is filled with liquid helium and set into rotation. The apparent moment of inertia is less than the classical value $I_0=\frac{1}{2}NMR^2$ (N = number of atoms, M = mass of atom, R = radius of vessel). The ratio between the observed moment of inertia and I_0 is by definition the "concentration of normal fluid."

We now make a calculation of the equilibrium value of the angular momentum L of an ideal Bose-Einstein gas as a function of the angular velocity ω. The result is shown in Fig. 1. Above the λ point, L is given by the dashed straight line, with slope equal to I_0. Below the λ point, L is a discontinuous function of ω, as shown by the full lines. The discontinuities occur whenever the angular velocity increases by an amount $\Delta\omega=\hbar^2/MR^2$, for then a new value of the angular momentum quantum number m gives rise to the lowest state of an atom in the vessel. Hence at this point a macroscopic number of atoms shifts from the lowest state with quantum number $m-1$ to the lowest state with quantum number m, and this accounts for the sudden jump in the equilibrium angular momentum L.

If equilibrium is reached, the observed moment of inertia is the mean slope of the solid curve, averaged over these quantum mechanical fluctuations; that is,

the equilibrium value of the angular momentum is $L=I_0\omega$, the classical value. Hence the equilibrium state of an ideal Bose-Einstein gas is not superfluid.

A superfluid state is reached if, for some reason, the "condensed" particles fail to shift over to the new ground state. Then the angular momentum falls along the dotted line in Fig. 1, and the observed moment of inertia is less than I_0. Order-of-magnitude estimates of the lifetime of this state are difficult to make, but do not disagree with a macroscopic relaxation time.

We suggest, therefore, that the actual liquid helium superfluid state is also metastable rather than an equilibrium state. A crucial experiment to decide whether the superfluid state is metastable or thermodynamically stable is the following: the vessel is set into rotation above the λ point, and then cooled down without application of torques. If the superfluid state is stable, the vessel's rate of rotation must speed up as it cools down. If our theory is right, the vessel will maintain its speed. The experiment has not been done so far.

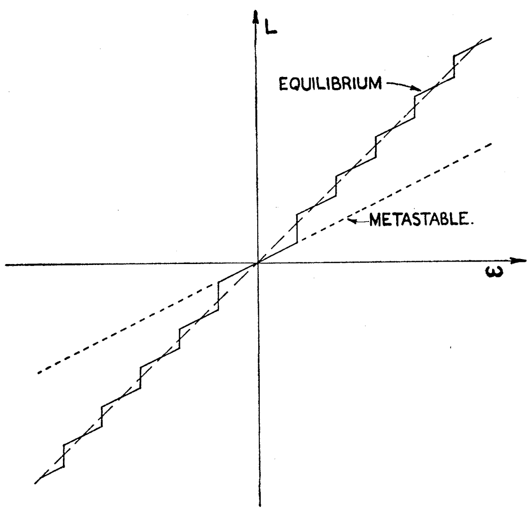

FIG. 1. The angular momentum L of an ideal Bose-Einstein gas below the condensation temperature, as a function of the angular velocity ω of the container.

In this connection, another experiment of Andronikashvili[2] might be thought to raise a difficulty; Andronikashvili showed, in essence, that as liquid helium is cooled down and more and more of the fluid becomes superfluid, this newly created superfluid does not take part in the rotation of the normal fluid. This looks at first sight as if the superfluid state were thermodynamically stable. However, Andronikashvili started rotating and cooling at a temperature below the λ point, so that there were already macroscopically many particles in the state with quantum number $m=0$ at the time the experiment started. Now it is characteristic of Bose-Einstein statistics that the transition probability to a state i is proportional to N_i+1, where N_i is the number of particles already occupying state i. In other words, Bose-Einstein statistics work according to the principle: "To him that hath shall be given." As the temperature drops, and more and more particles get pushed out of the high-energy states, the already highly populated state with $m=0$ provides the center of condensation, rather than the true ground state. Of course, eventually equilibrium must be reached, but this presumably did not occur in the time interval Andronikashvili used for cooling down. This provides a lower limit for the relaxation time, of the order of minutes.

The only other experiment relevant to this discussion[3] employed angular velocities so high that the circumferential velocity near the wall of the vessel was higher than the critical velocity for superfluid flow. Hence it is not surprising that Osborne found all the fluid taking part in the rotation.

Once this explanation of the superfluid properties of liquid helium is accepted, the concentration of normal fluid can be computed by standard methods of equilibrium statistical mechanics. This is possible because, for angular velocities less than $\hbar^2/MR^2$, the metastable state is the same as the equilibrium state. This calculation has been done for the ideal Bose-Einstein gas, where it gives the expected result $I=(T/T_\lambda)^{3/2}I_0$, and it has been set up for the interacting gas, using methods developed for a study of the thermodynamic properties of liquid helium.[4] The approximation of a hard-sphere gas gives corrections in the right direction, but of insufficient magnitude. This is the same situation as occurs for the calculation of the specific heat.

A detailed paper on this subject is in preparation.

We would like to thank Dr. M. R. Schafroth for many stimulating discussions.

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Color-Center Magnetic Resonance in LiF*

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PARAMAGNETIC resonance phenomena at microwave frequencies in LiF crystals, colored by irradiation, have been studied by Hutchison[1] and by Schneider.[2] Hutchison observed a g value of 2.00 in neutron-bombarded crystals of LiF which he assumed was due to F centers. Schneider, on the other hand, reported observations on LiF crystals subjected to