

the equilibrium value of the angular momentum is  $L = I_0\omega$ , the classical value. Hence the equilibrium state of an ideal Bose-Einstein gas is not superfluid.

A superfluid state is reached if, for some reason, the "condensed" particles fail to shift over to the new ground state. Then the angular momentum falls along the dotted line in Fig. 1, and the observed moment of inertia is less than  $I_0$ . Order-of-magnitude estimates of the lifetime of this state are difficult to make, but do not disagree with a macroscopic relaxation time.

We suggest, therefore, that the actual liquid helium superfluid state is also metastable rather than an equilibrium state. A crucial experiment to decide whether the superfluid state is metastable or thermodynamically stable is the following: the vessel is set into rotation above the  $\lambda$  point, and then cooled down

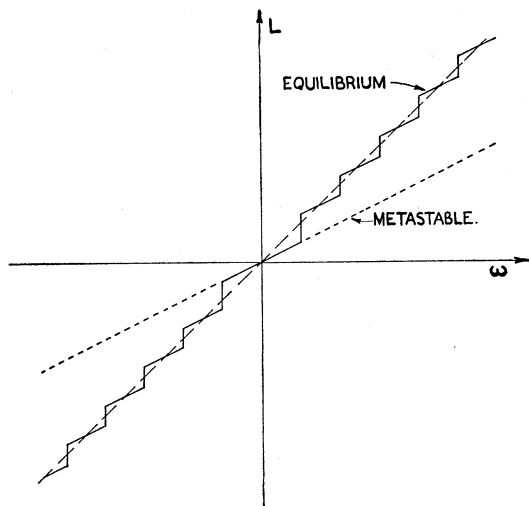


FIG. 1. The angular momentum  $L$  of an ideal Bose-Einstein gas below the condensation temperature, as a function of the angular velocity  $\omega$  of the container.

without application of torques. If the superfluid state is stable, the vessel's rate of rotation must speed up as it cools down. If our theory is right, the vessel will maintain its speed. The experiment has not been done so far.

In this connection, another experiment of Andronikashvili<sup>2</sup> might be thought to raise a difficulty; Andronikashvili showed, in essence, that as liquid helium is cooled down and more and more of the fluid becomes superfluid, this newly created superfluid does not take part in the rotation of the normal fluid. This looks at first sight as if the superfluid state were thermodynamically stable. However, Andronikashvili started rotating and cooling at a temperature below the  $\lambda$  point, so that there were already macroscopically many particles in the state with quantum number  $m=0$  at the time the experiment started. Now it is characteristic of Bose-Einstein statistics that the transition probability to a state  $i$  is proportional to  $N_i+1$ , where  $N_i$  is the

number of particles already occupying state  $i$ . In other words, Bose-Einstein statistics work according to the principle: "To him that hath shall be given." As the temperature drops, and more and more particles get pushed out of the high-energy states, the already highly populated state with  $m=0$  provides the center of condensation, rather than the true ground state. Of course, eventually equilibrium must be reached, but this presumably did not occur in the time interval Andronikashvili used for cooling down. This provides a lower limit for the relaxation time, of the order of minutes.

The only other experiment relevant to this discussion<sup>3</sup> employed angular velocities so high that the circumferential velocity near the wall of the vessel was higher than the critical velocity for superfluid flow. Hence it is not surprising that Osborne found all the fluid taking part in the rotation.

Once this explanation of the superfluid properties of liquid helium is accepted, the concentration of normal fluid can be computed by standard methods of equilibrium statistical mechanics. This is possible because, for angular velocities less than  $\hbar^2/MR^2$ , the metastable state is the same as the equilibrium state. This calculation has been done for the ideal Bose-Einstein gas, where it gives the expected result  $I = (T/T_\lambda)^{3/2} I_0$ , and it has been set up for the interacting gas, using methods developed for a study of the thermodynamic properties of liquid helium.<sup>4</sup> The approximation of a hard-sphere gas gives corrections in the right direction, but of insufficient magnitude. This is the same situation as occurs for the calculation of the specific heat.

A detailed paper on this subject is in preparation.

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## Color-Center Magnetic Resonance in LiF\*

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PARAMAGNETIC resonance phenomena at microwave frequencies in LiF crystals, colored by irradiation, have been studied by Hutchison<sup>1</sup> and by Schneider.<sup>2</sup> Hutchison observed a  $g$  value of 2.00 in neutron-bombarded crystals of LiF which he assumed was due to  $F$  centers. Schneider, on the other hand, reported observations on LiF crystals subjected to

180-kv x-rays and interpreted a broad intense peak ( $g=2.020$ ) as due to  $V$  centers, while ascribing a subsidiary resolved set of lines to the hyperfine structure of  $F$  centers.

We have studied the magnetic resonance in LiF single crystals, with some samples irradiated by x-rays at 50 kv and others bombarded by a cyclotron proton beam at approximately 360 Mev. The sample, usually less than 0.4 cc in volume, is placed on one end plate of rectangular cavity, oscillating in the  $TE_{011}$  mode and resonating at the frequency of a stabilized klystron around 9130 Mc/sec. The signal showing the magnetic resonance is derived from a modulation of the static field at 400 cps. The resonance is exhibited either in the form of the derivative of the absorption curve or in the form of the dispersion effect shown by the shift in the resonance frequency of the cavity. The steady magnetic field, measured by a proton resonance probe, is varied slowly through the resonance value while the signal is being recorded automatically. All our experiments have been carried out at room temperature.

In each of our samples a single, broad, approximately Gaussian, resonance peak has been observed. The signal-to-noise ratios ranged from about 30 for an x-rayed sample to 500 for a proton-bombarded sample. No resolved hyperfine structure lines have been detected in any of our recordings. The  $g$  factor calculated from a large number of recorded curves is  $g=1.999 \pm 0.001$  for the x-rayed samples, and  $g=2.002 \pm 0.001$  for the proton-bombarded ones. As a simple measure of the absorption line width, we use the field separation between two points of maximum slope, designated here as  $\Delta H_m$ . The value of  $\Delta H_m$  for a given resonance curve is obtained by extrapolating to zero field modulation. The observed values of  $\Delta H_m$  are 65 oersteds for the x-rayed samples and 110 oersteds for the proton-bombarded samples. Both the  $g$  factor and the line width have been shown experimentally to be independent of the concentration of the color centers for samples subjected to the same kind of radiation. Consequently, the effects of dipolar interaction appear to be negligible.

Using the approximate theory of Kahn and Kittel,<sup>3</sup> one can show that the  $g$  factor for an  $F$ -center resonance in LiF should be smaller than the free electron  $g$  by not more than about  $10^{-5}$ . This prediction agrees with the  $g$  value obtained for the proton-bombarded sample, but not with the  $g$  value of the x-rayed sample.

Kip *et al.*<sup>4</sup> assumed the line width to be caused by hyperfine interactions and predicted a nearly Gaussian shape for the envelope of the hyperfine components. They showed that the width of the envelope at half-maximum ( $\Delta H_{\frac{1}{2}}$ ) was  $1.12\xi A \text{ cm}^{-1}$  for a nuclear spin of  $3/2$ . Here  $\xi$  is the fractional effective  $s$  character of the valence electron and  $A$  is the hyperfine interaction constant. For  $\text{Li}^7$  (95 percent abundant), we have  $I=3/2$  and  $A=0.0134 \text{ cm}^{-1}$ .<sup>5</sup> This gives  $\Delta H_{\frac{1}{2}}=161\xi$

oersteds. Our experimental values of  $\Delta H_m$  give  $\Delta H_{\frac{1}{2}}=76.5$  oersteds for the x-rayed sample and 130 oersteds for the bombarded sample if Gaussian line shape is assumed. This would require  $\xi \approx 0.5$  for the x-rayed sample if the resonance is to be attributed to  $F$  centers. Such a value for  $\xi$  seems to be reasonable for  $\text{Li}^7$ . The value  $\xi \approx 0.8$  required to account for the line width of the proton-bombarded sample appears to be somewhat high. It is to be noted, however, that Hutchison has also observed a line width of approximately 160 oersteds for his neutron-bombarded sample.

The results on the x-rayed LiF strongly favor the  $F$  center interpretation. The  $g$  factor is less than the free electron value and the line width agrees well with the theory of Kip *et al.*<sup>4</sup> for  $F$  centers. While this evidence does not exclude the presence of other one-electron color centers, it does eliminate any sizable contribution from  $V$  centers. The case of the proton-bombarded LiF is far less clear cut. It may be that the resonance is still due to  $F$  centers with their environment drastically changed by heavy particle bombardment. It is also conceivable that the contributions of  $M$  centers,  $R_1$  centers, and possibly other one-electron centers are large enough to affect the line shape.

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## Measurement of the Complex Tensor Permeability of Ferrites

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ARTMAN and Tannenwald<sup>1</sup> have described a method for determining the components of the permeability tensor of a ferrite by measuring the frequency shift and change in  $Q$  produced by the insertion of a small spherical sample in a degenerate-mode cavity. The cylindrical  $TE_{111}$  mode cavity permits splitting of the cavity resonance into two frequencies corresponding to the resonance of the cavity for each of the counter-rotating circular components of the incident linear polarization. We have found that there are several advantages to be gained by using a very thin disk in a similar cavity. Not only is it possible to pro-