expected to be almost isotropic. Besides, any reasonable level density function[9] will predict a cross section for this scattering process which is several orders of magnitude smaller than the observed value of about 40 mb. Further work will be carried out to improve the separation of the elastic and inelastic scattering at small angles in order to test the theory of Austern, Butler, and McManus.

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Conservation of the Number of Nucleons*

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It has often been surmised that there exists a conservation law of nucleons, i.e., that they neither decay spontaneously nor are destroyed or created singly in nuclear collisions.[1] In view of the fundamental nature of such an assumption, it seemed of interest to investigate the extent to which the stability of nucleons could be experimentally demonstrated.[2]

To investigate the possible decay of a free proton, the large scintillation detector developed for the neutrino search[3] was employed. The detector was partially shielded from cosmic rays by placing it in an underground room with about 100 feet of rock above. The counting rate and pulse spectrum as seen by the detector may be used in arriving at a lower limit for the proton lifetime for certain postulated modes of decay. For fast particles the output of the detector is proportional to the energy deposited in the scintillator and hence for minimum ionizing particles to the track length in the scintillator.

The spectrum expected from any hypothetical proton decay depends then on the geometrical disposition of the protons as well as on the decay scheme assumed. As to the decay, we are free to assume any scheme which is consistent with the laws of conservation of charge, energy, momentum, and angular momentum. In view of the proton rest energy of 0.9 Bev and the known lighter particles into which it might conceivably decay, it seems reasonable to expect that these charged products would have a kinetic energy of the order of 100 Mev. If this decay occurred within the scintillator, the spectrum would essentially reflect the detector geometry because the decay-particle ranges would probably exceed the maximum detector dimension ($\sim$100 cm, equivalent for minimum ionizing particles to $\approx$140 Mev). There are 1.5×10^{28} protons in the scintillator (approximate chemical formula C_7H_8). The scintillator was surrounded by paraffin walls 2 ft thick. This effectively doubled the source of protons, giving $\sim3\times10^{28}$ protons. The pulse-height distribution observed is shown in Fig. 1. The integrated counting rate

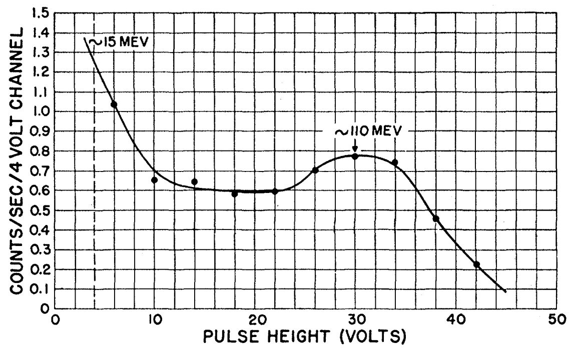

Fig. 1. Pulse-height spectrum in 300-liter liquid scintillator located underground. Time of run = 1000 sec per point. The integrated area for pulses larger than the cut-off bias, which corresponds to $\sim$15 Mev, is 6.6 counts/sec.

for pulses >15 Mev is 6.6 counts/sec. This corresponds to a lower limit of 1.5×10^{20} yr for the mean lifetime of an unbound proton. However, most of the counts in our experiment can be attributed to cosmic rays for two reasons: (1) The counting rate is in fair agreement with that expected from cosmic-ray μ mesons; (2) the spectral shape with its characteristic maximum at $\sim$110 Mev is consistent with what would be expected for cosmic-ray mesons underground traversing our scintillator. It seems, therefore, safe to conclude that at most $\sim\frac{1}{5}$ of the observed counts could be due to proton decay, and hence the lifetime of free protons is $>10^{21}$ yr. Lifetimes for some specific decay schemes which might be assumed can be shown to be even greater.

In our scintillator bound nucleons are an order of magnitude more numerous than hydrogen atoms. This yields a lifetime for bound nucleons $>10^{22}$ yr, a result which can also be interpreted as indicating the absence of "nucleon-destroying" collisions within nuclei.

It is clear that the technique here employed is capable of considerably higher sensitivity, but we believe that the values already obtained are of sufficient interest to be put on record. Higher sensitivity could be obtained both by using larger counters and by going deeper underground or in the ocean to eliminate cosmic rays.

We cannot conceive of an experiment which would prove the absolute stability of nucleons, but judging from the demonstrated "practical" stability of nucleons

we conclude that the law of conservation of nucleons can be used with considerable confidence in discussions of "practically observable" nuclear reactions. It proves very useful, for example, for hyperon reactions where it permits the conclusion that particles observed to decay into nucleons must be made from pre-existing nucleons or be produced in pairs (particles plus antiparticles). It also follows that nucleons must be found among the ultimate decay products of such hyperons; otherwise the decay of nucleons via virtual hyperon states would be observable. If nucleon pair production processes should be observed, the number of nucleons would only be conserved in an algebraic sense.

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Variational Principle in Quantum Mechanics

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ALTHOUGH, strictly speaking, a Lagrangian formalism in quantum theory is not known, it has nevertheless very often been used as a starting point for quantized field theories. The usual procedure is to apply the Lagrangian formalism before the quantization in order to derive the field equations and the Poisson brackets for the field variables. The quantization is performed afterwards by replacing the Poisson brackets by commutators. In order to overcome the apparent disadvantages of this way of proceeding, Schwinger[1] has recently developed a quantum mechanical variational principle. It is the purpose of the present note to investigate how far this attempt has succeeded.

For simplicity, we shall not consider a field theory, but a mechanical problem with n degrees of freedom. Classically, this system is described by n second order differential equations in the variables $q_k(t)$ $(k=1,\cdots,n)$ or, if one introduces n extra variables, the conjugate momenta $p_k(t)$, by a set of $N=2n$ first order equations. These canonical equations of motion can be derived from the variational principle:

$$\delta\int_{t_1}^{t_2} Ldt=0, \tag{1}$$

where

$$L=\sum_{k=1}^{n} p_k\dot{q}_k-H(p_k,q_k,t). \tag{2}$$

If one goes over to other variables $y_l(p_k,q_k)$, where $l=1,\cdots,N$, the equations of motion are again described by (1), where now L is expressed in the new variables. If the transformation is such that L expressed in the new variables has again the form (2), apart from an irrelevant time derivative, then one speaks of a canonical transformation.

Let us now compare this with Schwinger's variational formalism in quantum mechanics. Here, one starts with a Lagrangian of the form

$$L=\tfrac{1}{2}\sum_{k,\ l=1}^{N} a_{kl}(x_k\dot{x}_l-\dot{x}_kx_l)-H(x_k,t),$$

where the matrix a_{kl} is antisymmetric and has determinant $\neq 0$. The variables $x_k(t)$ are now operators in Hilbert space. The variations have to be restricted to c-number variations. Using Schwinger's prescription, one finds the equations of motion $2\sum_l a_{kl}\dot{x}_l=\partial H/\partial x_k$ and the commutation relations $(i/\hbar)[x_k,2\sum_m a_{lm}x_m]=\delta_{kl}$. With these commutation rules, one can write the equations of motion as $\dot{x}_k=(i/\hbar)[H,x_k]$. It appears that the commutation relations are determined by the matrix a_{kl}, and the equations of motion by the Hamiltonian operator H.

We shall now see what happens if we go over to new variables $y_k=Ux_kU^{-1}$ by means of a unitary transformation which does not depend explicitly on the time. For these variables, the commutation rules are the same and the equations of motion have again the form $\dot{y}_k=(i/\hbar)[H,y_k]$. The Lagrangian function must therefore, apart from a time derivative, have the form $L'=\frac{1}{2}\sum_{k,l}a_{kl}(y_k\dot{y}_l-\dot{y}_ky_l)-H$. The fundamental point is whether this expression L' differs from L only by a time derivative. We shall show with an example that this is not the case. We take a system with one degree of freedom, where a_{kl} has the simple form $a_{11}=a_{22}=0$ and $a_{12}=-a_{21}=1$. The Lagrangian is $L=\frac{1}{4}(p\dot{q}+\dot{q}p-\dot{p}q-q\dot{p})-H$; or, if we add $\frac{1}{4}(d/dt)(qp+pq)$, we have $L=\frac{1}{2}(p\dot{q}+\dot{q}p)-H$. We apply the unitary transformation $U(p,q)=\exp(\frac{1}{3}iq^3/\hbar)$, which gives $p'=p-q^2$ and $q'=q$. The new Lagrangian is $L'=\frac{1}{2}(p'\dot{q}'+\dot{q}'p')-H$. We can easily see that $L'-L=-\frac{1}{2}(q^2\dot{q}+\dot{q}q^2)$. This difference is not the time derivative of a function of p and q and thus the two Lagrangian functions are essentially