we conclude that the law of conservation of nucleons can be used with considerable confidence in discussions of "practically observable" nuclear reactions. It proves very useful, for example, for hyperon reactions where it permits the conclusion that particles observed to decay into nucleons must be made from pre-existing nucleons or be produced in pairs (particles plus antiparticles). It also follows that nucleons must be found among the ultimate decay products of such hyperons; otherwise the decay of nucleons via virtual hyperon states would be observable. If nucleon pair production processes should be observed, the number of nucleons would only be conserved in an algebraic sense.

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[1] E. P. Wigner, Proc. Natl. Acad. Sci. **38**, 449 (1952). Conversion of neutron into proton and vice versa in the normal process of beta decay are excluded in these considerations because such processes merely represent the change of the nucleon from one state to another.

[2] Earlier unpublished considerations by one of the authors (M.G.) based on the observed spontaneous fission rate of Th^{232} [partial half-life $\sim 1.4\times 10^{18}$ yr; see E. Segrè, Phys. Rev. **86**, 211 (1952); Hollander, Perlman, and Seaborg, Revs. Modern Phys. **25**, 469 (1953)] leads to a nucleon lifetime for bound nucleons in excess of 10^{20} yr. This follows if one makes the plausible assumption that the decay of a nucleon leaves sufficient energy for fission to take place with high probability.

[3] Reines, Cowan, Harrison, and Carter, Rev. Sci. Instr. (to be published).

Variational Principle in Quantum Mechanics

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ALTHOUGH, strictly speaking, a Lagrangian formalism in quantum theory is not known, it has nevertheless very often been used as a starting point for quantized field theories. The usual procedure is to apply the Lagrangian formalism before the quantization in order to derive the field equations and the Poisson brackets for the field variables. The quantization is performed afterwards by replacing the Poisson brackets by commutators. In order to overcome the apparent disadvantages of this way of proceeding, Schwinger[1] has recently developed a quantum mechanical variational principle. It is the purpose of the present note to investigate how far this attempt has succeeded.

For simplicity, we shall not consider a field theory, but a mechanical problem with n degrees of freedom. Classically, this system is described by n second order differential equations in the variables $q_k(t)$ $(k=1,\cdots,n)$ or, if one introduces n extra variables, the conjugate momenta $p_k(t)$, by a set of $N=2n$ first order equations. These canonical equations of motion can be derived from the variational principle:

$$\delta\int_{t_1}^{t_2} L\,dt=0, \tag{1}$$

where

$$L=\sum_{k=1}^{n} p_k\dot{q}_k-H(p_k,q_k,t). \tag{2}$$

If one goes over to other variables $y_l(p_k,q_k)$, where $l=1,\cdots,N$, the equations of motion are again described by (1), where now L is expressed in the new variables. If the transformation is such that L expressed in the new variables has again the form (2), apart from an irrelevant time derivative, then one speaks of a canonical transformation.

Let us now compare this with Schwinger's variational formalism in quantum mechanics. Here, one starts with a Lagrangian of the form

$$L=\tfrac{1}{2}\sum_{k,\ l=1}^{N} a_{kl}(x_k\dot{x}_l-\dot{x}_kx_l)-H(x_k,t),$$

where the matrix a_{kl} is antisymmetric and has determinant $\neq 0$. The variables $x_k(t)$ are now operators in Hilbert space. The variations have to be restricted to c-number variations. Using Schwinger's prescription, one finds the equations of motion $2\sum_l a_{kl}\dot{x}_l=\partial H/\partial x_k$ and the commutation relations $(i/\hbar)[x_k,2\sum_m a_{lm}x_m]=\delta_{kl}$. With these commutation rules, one can write the equations of motion as $\dot{x}_k=(i/\hbar)[H,x_k]$. It appears that the commutation relations are determined by the matrix a_{kl}, and the equations of motion by the Hamiltonian operator H.

We shall now see what happens if we go over to new variables $y_k=Ux_kU^{-1}$ by means of a unitary transformation which does not depend explicitly on the time. For these variables, the commutation rules are the same and the equations of motion have again the form $\dot{y}_k=(i/\hbar)[H,y_k]$. The Lagrangian function must therefore, apart from a time derivative, have the form $L'=\frac{1}{2}\sum_{k,\ l} a_{kl}(y_k\dot{y}_l-\dot{y}_ky_l)-H$. The fundamental point is whether this expression L' differs from L only by a time derivative. We shall show with an example that this is not the case. We take a system with one degree of freedom, where a_{kl} has the simple form $a_{11}=a_{22}=0$ and $a_{12}=-a_{21}=1$. The Lagrangian is $L=\frac{1}{4}(p\dot{q}+\dot{q}p-\dot{p}q-q\dot{p})-H$; or, if we add $\frac{1}{4}(d/dt)(qp+pq)$, we have $L=\frac{1}{2}(p\dot{q}+\dot{q}p)-H$. We apply the unitary transformation $U(p,q)=\exp(\frac{1}{3}iq^3/\hbar)$, which gives $p'=p-q^2$ and $q'=q$. The new Lagrangian is $L'=\frac{1}{2}(p'\dot{q}'+\dot{q}'p')-H$. We can easily see that $L'-L=-\frac{1}{2}(q^2\dot{q}+\dot{q}q^2)$. This difference is not the time derivative of a function of p and q and thus the two Lagrangian functions are essentially

different. It appears therefore that one needs a different Lagrangian function for each set of canonical variables, which means that this method is actually not a physical principle as in classical physics, but rather a set of rules prescribing how to find the equations of motion and the commutation rules for a given set of variables. The Lagrangian function is known when the matrix a_{kl} and the Hamiltonian operator are given; in other words, one has to know the commutation rules and the Hamiltonian beforehand. The variational method seems then to be completely superfluous.

Instead of taking a new Lagrangian function after a change of variables, it is also possible to keep the same Lagrangian expressed in the new variables. It is clear that the allowed variations for these new variables will now not be c numbers. Therefore, this procedure is just as unsatisfactory as the first.

[1] J. Schwinger, Phys. Rev. **91**, 713 (1953).

Electron Spectrum from μ-Meson Decay*

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In a recent Letter to the Editor,[1] Sagane *et al.* state that our measurements[2] of the μ-meson decay electron spectrum give a value $\rho\sim 0$, corresponding to a zero intercept at the upper-energy end. Unfortunately, the distortions due to end effects and bremsstrahlung are severe in the detector described, and we concluded that it was not possible for us to say anything about the value of ρ. The measured spectrum was shown in our article to illustrate a method of energy calibration using the end point, which we believe is not significantly affected by the above-mentioned effects.

* Work performed under the auspices of the U. S. Atomic Energy Commission.
[1] Sagane, Dudziak, and Vedder, Phys. Rev. **95**, 863 (1954).
[2] Harrison, Cowan, and Reines, Nucleonics **12**, No. 3, 44 (1954)

Nonuniform Charge Distributions and μ-Meson Capture

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Extensive calculations have been carried out to evaluate the transition probability for μ-meson capture in terms of several electric charge distributions, suggested by theoretical analyses of observations on μ-mesonic x-rays and high-energy electron-nucleus scattering.

By means of the density law $\rho(x)=\rho_0 f(x)\,(x=r/a)$ we have derived the maximum momentum $p(x)=P_0 f^{\frac{1}{3}}(x)$ of a proton at a distance x from the nucleus center: $P_0=[3Z\pi/4I(\infty)]^{\frac{1}{3}}(\hbar/a)$ is the maximum momentum at $x=0$ and $I(\infty)$ is the second order moment of the shape function $f(x)$.

According to a nuclear model based on this relationship between the density of particles $\rho(x)$ and the maximum momentum $p(x)$, the capture probability of the meson is:

$$\Lambda=g^2\frac{M^2c}{2\pi\hbar^4}\int\psi^*\psi\left\{\frac{\nu_{\max}{}^2-\nu_{\min}{}^2}{q}\right\}_{\text{Av}}dZ,$$

where $\psi(x)$ is the normalized meson wave function, $q=|\mathbf{N}+\boldsymbol{\nu}|$ ($\mathbf{N},\boldsymbol{\nu}$ neutron and neutrino momenta measured in units Mc). The quantity in brackets, averaged over the neutron-neutrino angle, is a sensitive function of the neutron excess: $\nu_{\max}$ and $\nu_{\min}$ are, respectively, the maximum and minimum neutrino momentum derived as functions of the proton momentum p in accordance with the exclusion principle and the conservation of energy and momentum.

It follows that the capture of the meson is forbidden for all protons having momentum $p<p_0(x)$; therefore, the summation over protons must be carried out in momentum space from p_0 to P_0, where

$$p_0(x)=[1+2N_0f^{\frac{1}{3}}(x)+P_0{}^2f^{\frac{2}{3}}(x)+2(\mu^2/3N_0)f^{-\frac{2}{3}}(x)-2(M_\mu/M)+2\epsilon_0]^{\frac{1}{2}}-1,$$

N_0 being the maximum neutron momentum at $x=0$, μ the meson momentum and ϵ_0 the meson binding energy on the K orbit. Consequently, the conservation of momentum requires that the integration in the ordinary space be extended from 0 to x_0, defined by the equation $p_0(x_0)=0$.

For simplicity's sake it is still possible to determine the *average* behavior of the capture probability according to the formula[1] $\Lambda=(1/\tau_0)(Z_{\text{eff}}/\bar{Z}_0)^4$ ($\tau_0=2.22$ μsec) provided the effective atomic number is defined through the relation $Z_{\text{eff}}=\{(\hbar^2/M_\mu e^2)^3\pi\rho_0\phi(Z)\}^{\frac{1}{4}}$, where

$$\phi(Z)=\int_0^{x_0}\{f(x)-[p_0(x)/P_0]^3\}(x\psi)^2dx\Big/\int_0^\infty(x\psi)^2dx.$$

The Schrödinger equation, with Coulomb potentials derived from the assumed charge distributions, has been solved by usual nonlinear variational methods.

Numerical inspection of the equation defining Z_{eff} shows that while the exclusion principle and the energy-momentum conservation tend to reduce the effective atomic number, an opposite effect arises from nonuniform charge distributions allowing larger central concentrations. For this reason the values of Z_{eff} for