

different. It appears therefore that one needs a different Lagrangian function for each set of canonical variables, which means that this method is actually not a physical principle as in classical physics, but rather a set of rules prescribing how to find the equations of motion and the commutation rules for a given set of variables. The Lagrangian function is known when the matrix a_{kl} and the Hamiltonian operator are given; in other words, one has to know the commutation rules and the Hamiltonian beforehand. The variational method seems then to be completely superfluous.

Instead of taking a new Lagrangian function after a change of variables, it is also possible to keep the same Lagrangian expressed in the new variables. It is clear that the allowed variations for these new variables will now not be c numbers. Therefore, this procedure is just as unsatisfactory as the first.

¹ J. Schwinger, Phys. Rev. **91**, 713 (1953).

Electron Spectrum from μ -Meson Decay*

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IN a recent Letter to the Editor,¹ Sagane *et al.* state that our measurements² of the μ -meson decay electron spectrum give a value $\rho \sim 0$, corresponding to a zero intercept at the upper-energy end. Unfortunately, the distortions due to end effects and bremsstrahlung are severe in the detector described, and we concluded that it was not possible for us to say anything about the value of ρ . The measured spectrum was shown in our article to illustrate a method of energy calibration using the end point, which we believe is not significantly affected by the above-mentioned effects.

* Work performed under the auspices of the U. S. Atomic Energy Commission.

¹ Sagane, Dudziak, and Vedder, Phys. Rev. **95**, 863 (1954).

² Harrison, Cowan, and Reines, Nucleonics **12**, No. 3, 44 (1954)

Nonuniform Charge Distributions and μ -Meson Capture

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EXTENSIVE calculations have been carried out to evaluate the transition probability for μ -meson capture in terms of several electric charge distributions,

suggested by theoretical analyses of observations on μ -mesonic x-rays and high-energy electron-nucleus scattering.

By means of the density law $\rho(x) = \rho_0 f(x)$ ($x = r/a$) we have derived the maximum momentum $p(x) = P_0 f^{1/2}(x)$ of a proton at a distance x from the nucleus center: $P_0 = [3Z\pi/4I(\infty)]^{1/2}(\hbar/a)$ is the maximum momentum at $x=0$ and $I(\infty)$ is the second order moment of the shape function $f(x)$.

According to a nuclear model based on this relationship between the density of particles $\rho(x)$ and the maximum momentum $p(x)$, the capture probability of the meson is:

$$\Lambda = g^2 \frac{M^2 c}{2\pi \hbar^4} \int \psi^* \psi \left\{ \frac{\nu_{\max}^2 - \nu_{\min}^2}{q} \right\}_{Av} dZ,$$

where $\psi(x)$ is the normalized meson wave function, $q = |\mathbf{N} + \mathbf{v}|$ (\mathbf{N}, \mathbf{v} neutron and neutrino momenta measured in units Mc). The quantity in brackets, averaged over the neutron-neutrino angle, is a sensitive function of the neutron excess: ν_{\max} and ν_{\min} are, respectively, the maximum and minimum neutrino momentum derived as functions of the proton momentum p in accordance with the exclusion principle and the conservation of energy and momentum.

It follows that the capture of the meson is forbidden for all protons having momentum $p < p_0(x)$; therefore, the summation over protons must be carried out in momentum space from p_0 to P_0 , where

$$p_0(x) = [1 + 2N_0 f^{1/2}(x) + P_0^2 f(x) + 2(\mu^2/3N_0) f^{-3/2}(x) - 2(M_\mu/M) + 2\epsilon_0]^{1/2} - 1,$$

N_0 being the maximum neutron momentum at $x=0$, μ the meson momentum and ϵ_0 the meson binding energy on the K orbit. Consequently, the conservation of momentum requires that the integration in the ordinary space be extended from 0 to x_0 , defined by the equation $p_0(x_0) = 0$.

For simplicity's sake it is still possible to determine the average behavior of the capture probability according to the formula¹ $\Lambda = (1/\tau_0)(Z_{\text{eff}}/\bar{Z}_0)^4$ ($\tau_0 = 2.22 \mu\text{sec}$) provided the effective atomic number is defined through the relation $Z_{\text{eff}} = \{(\hbar^2/M_\mu e^2)^3 \pi \rho_0 \phi(Z)\}^{1/3}$, where

$$\phi(Z) = \int_0^{x_0} \{f(x) - [p_0(x)/P_0]^3\} (x\psi)^2 dx / \int_0^\infty (x\psi)^2 dx.$$

The Schrödinger equation, with Coulomb potentials derived from the assumed charge distributions, has been solved by usual nonlinear variational methods.

Numerical inspection of the equation defining Z_{eff} shows that while the exclusion principle and the energy-momentum conservation tend to reduce the effective atomic number, an opposite effect arises from nonuniform charge distributions allowing larger central concentrations. For this reason the values of Z_{eff} for