different. It appears therefore that one needs a different Lagrangian function for each set of canonical variables, which means that this method is actually not a physical principle as in classical physics, but rather a set of rules prescribing how to find the equations of motion and the commutation rules for a given set of variables. The Lagrangian function is known when the matrix a_{kl} and the Hamiltonian operator are given; in other words, one has to know the commutation rules and the Hamiltonian beforehand. The variational method seems then to be completely superfluous.

Instead of taking a new Lagrangian function after a change of variables, it is also possible to keep the same Lagrangian expressed in the new variables. It is clear that the allowed variations for these new variables will now not be c numbers. Therefore, this procedure is just as unsatisfactory as the first.

[1] J. Schwinger, Phys. Rev. **91**, 713 (1953).

Electron Spectrum from μ-Meson Decay*

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In a recent Letter to the Editor,[1] Sagane *et al.* state that our measurements[2] of the μ-meson decay electron spectrum give a value $\rho\sim 0$, corresponding to a zero intercept at the upper-energy end. Unfortunately, the distortions due to end effects and bremsstrahlung are severe in the detector described, and we concluded that it was not possible for us to say anything about the value of ρ. The measured spectrum was shown in our article to illustrate a method of energy calibration using the end point, which we believe is not significantly affected by the above-mentioned effects.

* Work performed under the auspices of the U. S. Atomic Energy Commission.
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Nonuniform Charge Distributions and μ-Meson Capture

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EXTENSIVE calculations have been carried out to evaluate the transition probability for μ-meson capture in terms of several electric charge distributions, suggested by theoretical analyses of observations on μ-mesonic x-rays and high-energy electron-nucleus scattering.

By means of the density law $\rho(x)=\rho_0 f(x)\,(x=r/a)$ we have derived the maximum momentum $p(x)=P_0 f^{\frac{1}{3}}(x)$ of a proton at a distance x from the nucleus center: $P_0=[3Z\pi/4I(\infty)]^{\frac{1}{3}}(\hbar/a)$ is the maximum momentum at $x=0$ and $I(\infty)$ is the second order moment of the shape function $f(x)$.

According to a nuclear model based on this relationship between the density of particles $\rho(x)$ and the maximum momentum $p(x)$, the capture probability of the meson is:

$$\Lambda=g^2\frac{M^2c}{2\pi\hbar^4}\int\psi^*\psi\left\{\frac{\nu_{\max}{}^2-\nu_{\min}{}^2}{q}\right\}_{\text{Av}}dZ,$$

where $\psi(x)$ is the normalized meson wave function, $q=|\mathbf{N}+\boldsymbol{\nu}|$ ($\mathbf{N},\boldsymbol{\nu}$ neutron and neutrino momenta measured in units Mc). The quantity in brackets, averaged over the neutron-neutrino angle, is a sensitive function of the neutron excess: $\nu_{\max}$ and $\nu_{\min}$ are, respectively, the maximum and minimum neutrino momentum derived as functions of the proton momentum p in accordance with the exclusion principle and the conservation of energy and momentum.

It follows that the capture of the meson is forbidden for all protons having momentum $p<p_0(x)$; therefore, the summation over protons must be carried out in momentum space from p_0 to P_0, where

$$p_0(x)=[1+2N_0f^{\frac{1}{3}}(x)+P_0{}^2f^{\frac{2}{3}}(x)+2(\mu^2/3N_0)f^{-\frac{2}{3}}(x)-2(M_\mu/M)+2\epsilon_0]^{\frac{1}{2}}-1,$$

N_0 being the maximum neutron momentum at $x=0$, μ the meson momentum and ϵ_0 the meson binding energy on the K orbit. Consequently, the conservation of momentum requires that the integration in the ordinary space be extended from 0 to x_0, defined by the equation $p_0(x_0)=0$.

For simplicity's sake it is still possible to determine the *average* behavior of the capture probability according to the formula[1] $\Lambda=(1/\tau_0)(Z_{\text{eff}}/\bar{Z}_0)^4$ ($\tau_0=2.22$ μsec) provided the effective atomic number is defined through the relation $Z_{\text{eff}}=\{(\hbar^2/M_\mu e^2)^3\pi\rho_0\phi(Z)\}^{\frac{1}{4}}$, where

$$\phi(Z)=\int_0^{x_0}\{f(x)-[p_0(x)/P_0]^3\}(x\psi)^2dx\Big/\int_0^{\infty}(x\psi)^2dx.$$

The Schrödinger equation, with Coulomb potentials derived from the assumed charge distributions, has been solved by usual nonlinear variational methods.

Numerical inspection of the equation defining Z_{eff} shows that while the exclusion principle and the energy-momentum conservation tend to reduce the effective atomic number, an opposite effect arises from nonuniform charge distributions allowing larger central concentrations. For this reason the values of Z_{eff} for

TABLE I. Results for the exponential charge distribution: $\rho(x)=\rho_0 e^{-x}$. ($\bar{Z}_0=11.3$). $x=r/a$.

Nucleus	a (10^{-13} cm)	Z_{eff}	Λ (μ sec^{-1})
Be[a]	0.74	3.52	0.004
	0.90	3.49	0.004
C[b]	1.00	5.19	0.020
	1.6	27.9	16.6
Pb[a]	2.2	26.1	12.8
	2.9	23.9	9.0

[a] See reference 2.
[b] See reference 3.

lighter elements are not very different from those calculated by Wheeler on the basis of his simplified nuclear model.

Tables I and II give results for two limiting cases: exponential charge distribution and smoothed uniform charge distribution.

The size parameter of the exponential distribution giving the best fit in the atomic number interval including Hg, Bi, and Pb is $1.56 \leq a \leq 2.3$. Values of a, smaller than those determined by Schiff[2] using the Born approximation, are consistent with the phase shift analysis of data on fast electron scattering. By fitting the transition rates measured in Fe and Sb, the size parameter of the same distribution is found to be:

$$\text{Fe: } a=(1.29\pm 0.20)\times 10^{-13}\text{ cm}; \qquad \text{Sb: } a=(1.58\pm 0.25)\times 10^{-13}\text{ cm}.$$

The calculations with the distribution of Table II have been performed under the assumption that the thickness of the surface layer is independent of the mass number A. The calculated capture probabil'ty for C, Al, and S appear to be in good agreement with recent measurements of Alberigi-Quaranta and Pancini.[5]

According to Wheeler's theory and to shell model calculations,[6] the ratio $\Lambda_{\text{Pb}}/\Lambda_{\text{Ca}}$ is 14.3 and 6.1, respectively, whereas, on the basis of the assumed smoothed uniform distribution, one finds 6.91. This

TABLE II. Results for the smoothed uniform charge distribution:[a] $\rho(x)=\rho_0[1+e^{(x-1)/b}]^{-1}$. ($\bar{Z}_0=11$). $b=(0.2869\times 10^{-13}/r_0)A^{-\frac{1}{3}}$; $r_0=1.05\times 10^{-13}$ cm. $x=r/a$.

Nucleus	Z	Z_{eff}	Λ (μ sec^{-1})
Be	4	3.59	0.005
C	6	5.28	0.024
O	8	7.29	0.087
Al	13	11.25	0.493
S	16	13.55	1.037
Ca	20	16.25	2.145
Fe	26	18.80	3.842
Zn	30	19.94	4.863
Ag	47	22.95	8.534
Sb	51	23.48	9.350
Ba	56	24.05	10.292
Hg	80	26.20	14.497
Pb	82	26.35	14.830
U	92	27.10	16.594

[a] See reference 4.

figure, however, cannot be considered as conclusive, either because increasing the rms radius by 5 to 10 percent would not be in conflict with the phase shift analysis of high-energy electron scattering data or because only the *average* behavior of the capture probability has been so far considered.

A detailed discussion on these topics will be published soon in the *Nuovo Cimento*.

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Evidence for Two-Body Spin-Orbit Forces in Nuclei

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IT is well known that the spins, level spectra, and other properties of many nuclei can be explained on the basis of a shell model in which one-body spin-orbit forces of the type $\xi(\mathbf{l}\cdot\mathbf{s})$ are assumed in addition to two-body central forces. A significant feature of this explanation is that different nuclei appear to be best fitted by different values of ξ. Apart from a possible general tendency for ξ to increase with mass number (as evidenced by the apparent improvement in the $j-j$ coupling approximation towards heavy nuclei), there is also a tendency for ξ to increase as particular shells are filled. Inglis[1] has shown that the value of ξ appears to double as the $1p$ shell is filled (the $1p_{\frac{3}{2}}-1p_{\frac{1}{2}}$ splitting is $\sim$3 Mev at He^5, $\sim$6.3 Mev at N^{15}). There is also some evidence of a trend towards $j-j$ coupling as the $2s$ and $1d$ shells are filled.[2] These facts suggest that a force of the simple type $\xi(\mathbf{l}\cdot\mathbf{s})$ may not be a real force in the nucleus, but rather that it is a "caricature" of a more complicated force. It has been suggested that this may be a tensor force[3] or a two-body spin-orbit force.[4]

We have investigated the second type of force:

$$T(12)\{(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot[(\mathbf{r}_1-\mathbf{r}_2)\times(\mathbf{p}_1-\mathbf{p}_2)]\}V(r_{12}), \quad (1)$$

where $T(12)$ may be 1 (neutral) or $(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)$ (symmetric), and where $V(r_{12})$ is some potential which we have taken to be:

$$V(r_{12})=V_0\frac{e^{-r_{12}/a}}{r_{12}/a}. \quad (2)$$

It has been shown before[5] that an interaction such as (1), acting between closed shells and a loose nucleon