TABLE I. Results for the exponential charge distribution: $\rho(x)=\rho_0 e^{-x}$. ($\bar{Z}_0=11.3$). $x=r/a$.

Nucleus	a (10^{-13} cm)	Z_{eff}	Λ (μ sec^{-1})
Be[a]	0.74	3.52	0.004
	0.90	3.49	0.004
C[b]	1.00	5.19	0.020
	1.6	27.9	16.6
Pb[a]	2.2	26.1	12.8
	2.9	23.9	9.0

[a] See reference 2.
[b] See reference 3.

lighter elements are not very different from those calculated by Wheeler on the basis of his simplified nuclear model.

Tables I and II give results for two limiting cases: exponential charge distribution and smoothed uniform charge distribution.

The size parameter of the exponential distribution giving the best fit in the atomic number interval including Hg, Bi, and Pb is $1.56 \leq a \leq 2.3$. Values of a, smaller than those determined by Schiff[2] using the Born approximation, are consistent with the phase shift analysis of data on fast electron scattering. By fitting the transition rates measured in Fe and Sb, the size parameter of the same distribution is found to be:

$$\text{Fe: } a=(1.29\pm0.20)\times10^{-13}\text{ cm};$$
$$\text{Sb: } a=(1.58\pm0.25)\times10^{-13}\text{ cm}.$$

The calculations with the distribution of Table II have been performed under the assumption that the thickness of the surface layer is independent of the mass number A. The calculated capture probabil'ty for C, Al, and S appear to be in good agreement with recent measurements of Alberigi-Quaranta and Pancini.[5]

According to Wheeler's theory and to shell model calculations,[6] the ratio $\Lambda_{\text{Pb}}/\Lambda_{\text{Ca}}$ is 14.3 and 6.1, respectively, whereas, on the basis of the assumed smoothed uniform distribution, one finds 6.91. This figure, however, cannot be considered as conclusive, either because increasing the rms radius by 5 to 10 percent would not be in conflict with the phase shift analysis of high-energy electron scattering data or because only the *average* behavior of the capture probability has been so far considered.

TABLE II. Results for the smoothed uniform charge distribution:[a] $\rho(x)=\rho_0[1+e^{(x-1)/b}]^{-1}$. ($\bar{Z}_0=11$). $b=(0.2869\times10^{-13}/r_0)A^{-\frac{1}{3}}$; $r_0=1.05\times10^{-13}$ cm. $x=r/a$.

Nucleus	Z	Z_{eff}	Λ (μ sec^{-1})
Be	4	3.59	0.005
C	6	5.28	0.024
O	8	7.29	0.087
Al	13	11.25	0.493
S	16	13.55	1.037
Ca	20	16.25	2.145
Fe	26	18.80	3.842
Zn	30	19.94	4.863
Ag	47	22.95	8.534
Sb	51	23.48	9.350
Ba	56	24.05	10.292
Hg	80	26.20	14.497
Pb	82	26.35	14.830
U	92	27.10	16.594

[a] See reference 4.

A detailed discussion on these topics will be published soon in the *Nuovo Cimento*.

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Evidence for Two-Body Spin-Orbit Forces in Nuclei

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IT is well known that the spins, level spectra, and other properties of many nuclei can be explained on the basis of a shell model in which one-body spin-orbit forces of the type $\xi(\mathbf{l}\cdot\mathbf{s})$ are assumed in addition to two-body central forces. A significant feature of this explanation is that different nuclei appear to be best fitted by different values of ξ. Apart from a possible general tendency for ξ to increase with mass number (as evidenced by the apparent improvement in the $j-j$ coupling approximation towards heavy nuclei), there is also a tendency for ξ to increase as particular shells are filled. Inglis[1] has shown that the value of ξ appears to double as the $1p$ shell is filled (the $1p_{\frac{3}{2}}-1p_{\frac{1}{2}}$ splitting is $\sim$3 Mev at He^5, $\sim$6.3 Mev at N^{15}). There is also some evidence of a trend towards $j-j$ coupling as the $2s$ and $1d$ shells are filled.[2] These facts suggest that a force of the simple type $\xi(\mathbf{l}\cdot\mathbf{s})$ may not be a real force in the nucleus, but rather that it is a "caricature" of a more complicated force. It has been suggested that this may be a tensor force[3] or a two-body spin-orbit force.[4]

We have investigated the second type of force:

$$T(12)\{(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot[(\mathbf{r}_1-\mathbf{r}_2)\times(\mathbf{p}_1-\mathbf{p}_2)]\}V(r_{12}), \quad (1)$$

where $T(12)$ may be 1 (neutral) or $(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)$ (symmetric), and where $V(r_{12})$ is some potential which we have taken to be:

$$V(r_{12})=V_0\frac{e^{-r_{12}/a}}{r_{12}/a}. \quad (2)$$

It has been shown before[5] that an interaction such as (1), acting between closed shells and a loose nucleon

outside, produces on the latter an effective force of the type $\xi(\mathbf{l}\cdot\mathbf{s})$. (The fact that ξ cannot be written as a Thomas-type constant because of exchange integrals[6] is of no consequence for our purposes). In a nucleus consisting of an unfilled shell outside closed shells, the loose nucleons experience their mutual spin-orbit forces in addition to this effective one-body force. It is the presence of the mutual forces that can cause the effective ξ to change as a shell is filled. In particular, the value of ξ for one hole in a shell may be considerably different from that for one particle.

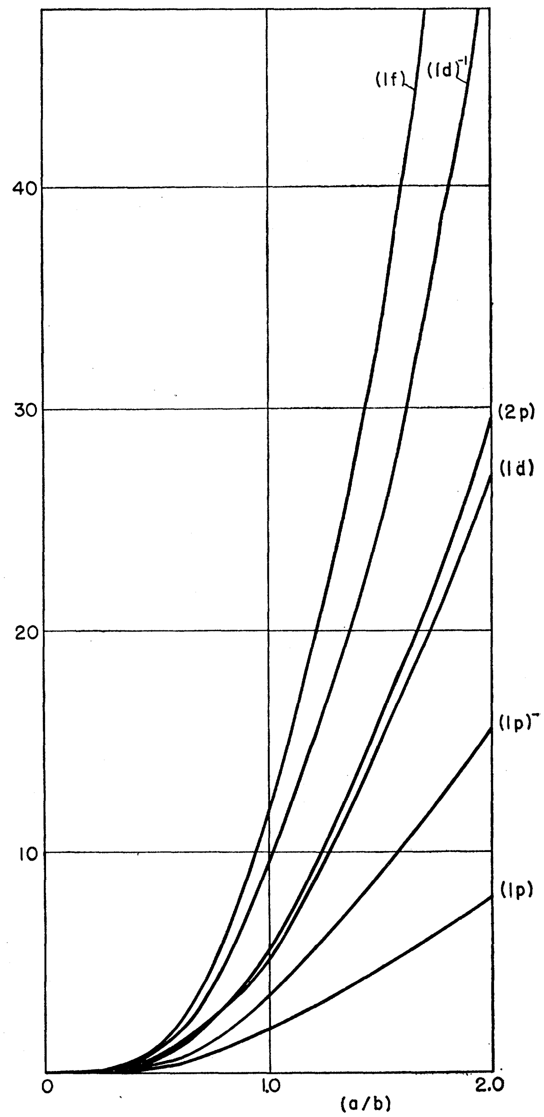

FIG. 1. Doublet splitting in units of V_0 (neutral interaction).

Using previously described methods,[7] we have evaluated the single-particle and single-hole splittings $\epsilon(nl)$ for various orbits (nl) (taking oscillator wave functions, e.g., $u_{1s}(r)=2\pi^{-\frac{1}{4}}\exp(-\frac{1}{2}\rho^2)$ where $\rho=r/b$). The results are given in Figs. 1 and 2 for neutral and symmetric forces. Rough values of the observed splittings are: $\epsilon(1p)\sim 3$ Mev,[6] $\epsilon(1p^{-1})\sim 6.3$ Mev, and $\epsilon(1d)\sim 5.1$ Mev[8] (from He^5, N^{15}, O^{17} respectively).[9]

It can be seen that, for all values of a/b, the neutral case gives qualitatively the correct ratios of the splittings, whereas the symmetric case is completely wrong.

When comparing the observed $(1p^{-1})$ and $(1d)$ splittings (ratio 6:5) with the curves (ratio 2:3), it must be remembered that we are taking the same value of a/b for both orbits, whereas consideration of Coulomb energies[10] suggests that b increases across the O^{16} closed-shell by ~ 15 percent. Allowing for this increase brings the two ratios close together.

An interesting feature of Fig. 1 is the tendency, at any given value of a/b, for $\epsilon(nl)$ to increase rapidly on going to higher orbits. In particular, the $2p$ and $4f$ orbits are split about three and six times more than the $1p$. This suggests that the $j-j$ coupling approximation improves for nuclei above Ca^{40}. The splitting of $1d^{-1}$ (K^{39}) is predicted to be $\sim 5/3$ of the $1d$ (O^{17}) splitting, i.e., about 9 Mev. (This increase arises partly from the presence of four extra $2s$ nucleons). It should be noted, however, that if b increases with A,[10] these effects will be diminished.

For nuclei with two or more particles in an unfilled shell, detailed studies show that the usual effect of mutual neutral forces acting between the loose particles is approximately to reinforce the effective one-body splitting arising from the interaction with closed shells. In the $1p$ shell, in particular, the two-body forces give rise to an effective force $\xi(\mathbf{l}\cdot\mathbf{s})$, where ξ increases

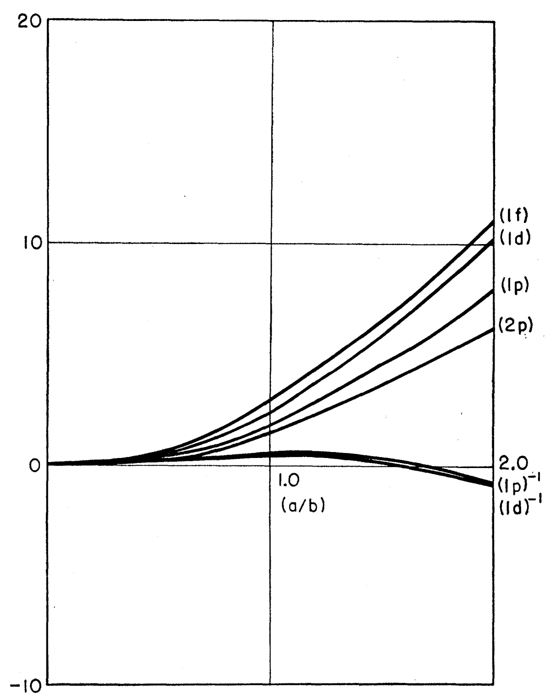

FIG. 2. Doublet splitting in units of V_0 (symmetric interaction).

steadily between He^5 and N^{15}. Consequently we conclude that a constant two-body neutral force can explain all data explicable with a one-body force of varying ξ.

Finally, we should mention that calculations similar to ours[4] made with wave functions $u_l(r)=r^l e^{-r/b}$ do not show nearly such a strong dependence of $\epsilon(nl)$ on (nl) as we have found. For $a/b=1$, their $1p$, $1d$, and $1f$ splittings are 0.75, 1.09, and 1.17 in units of V_0.

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