

# Magnetic Anisotropy at 0°K\*

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The results of the spin-wave method of Holstein and Primakoff are extended to obtain approximate values for the dependence of the energy of magnetically saturated cubic crystals upon the direction of magnetization. The anisotropy at 0°K is calculated for the cases of dipolar ferromagnetism and exchange ferromagnetism. In a nearest neighbor approximation, the results agree with those obtained by Van Vleck from a treatment of ferromagnetic anisotropy. However, in the case of dipolar interaction, the nearest neighbor approximation is not sufficient and more distant neighbors are included. For dipolar ferromagnetism, the method gives the following results: In a face-centered cubic (fcc) lattice of spins  $S$ , the anisotropy constant  $K_1$  is a maximum when there is no external field and is then given by  $K_1 = -0.11 M_0^2/S$ , where  $M_0$  is the saturation magnetization. In a body-centered cubic (bcc) lattice, with no external field,  $K_1 = -0.15 M_0^2/S$ . An external field must be applied to obtain saturation in a simple cubic (sc) lattice;  $K_1 = +0.43 M_0^2/S$  when the external field  $H = 0.6 M_0$ . More generally, in the presence of an external field  $H$ , then  $K_1 = \kappa(M_0^2/S)((4\pi/3) + H/M_0)^{-1}$ , where  $\kappa = -0.46$  for an fcc lattice,  $-0.63$  for a bcc lattice, and  $+2.06$  for an sc lattice. All values given are for specimens having zero demagnetizing field.

## 1. INTRODUCTION

CLASSICALLY, a saturated cubic lattice of dipoles has no anisotropy energy.<sup>1</sup> The dipole interaction energy of the lattice, when all the dipoles are aligned parallel, is independent of the direction of polarization. Nor does the quantum mechanical exchange interaction introduce any anisotropy. However, Kittel<sup>2</sup> has extended to magnetic dipole interactions an argument derived by Van Vleck<sup>3</sup> in treating ferromagnetic anisotropy, and finds that anisotropy arises quantum mechanically because the lowest energy state is not one of complete saturation. The dipoles cannot be aligned parallel as in a classical model, and it is therefore possible for the dipolar energy to depend upon the direction of polarization, or magnetization.

Holstein and Primakoff<sup>4</sup> have noted the applicability of spin-wave theory to the calculation of anisotropy. A spin wave may be thought of as a wave-like disturbance of the  $z$  components of spin of the atoms in a crystal, resulting in the deviation of the  $z$  components of spin from their maximum possible value  $S$ . The *spin deviation*,  $S - S^z$ , is not localized on any one atom but is distributed throughout the crystal in wavelike fashion. In the presence of dipole-dipole type interactions, the quantum mechanical energy of the spin waves in the lowest energy state is dependent upon the direction of magnetization. The treatment below extends the results of HP and obtains approximate values for the anisotropy energy of a cubic lattice of dipoles at 0°K, both in the absence and presence of an exchange interaction, that is, for the cases of dipolar ferromagnetism and exchange ferromagnetism.

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<sup>1</sup> R. Becker, Z. Physik **62**, 253 (1930).

<sup>2</sup> C. Kittel, Phys. Rev. **82**, 965 (1951).

<sup>3</sup> J. H. Van Vleck, Phys. Rev. **52**, 1178 (1937).

<sup>4</sup> T. Holstein and H. Primakoff, Phys. Rev. **58**, 1098 (1940). See their footnote 17c. This paper will be referred to as HP.

## 2. THE HAMILTONIAN

Consider a system of identical atoms, each carrying a magnetic moment, and located at the lattice points of a cubic lattice. The Hamiltonian of the system, in the presence of an external field and including exchange interactions and magnetic dipole interactions, is

$$\mathcal{H} = -\frac{1}{2} \sum_{\substack{l, m=1 \\ l \neq m}}^N 2J(R_{lm}) \mathbf{S}_l \cdot \mathbf{S}_m + \frac{1}{2} a^{-3} \sum_{\substack{l, m=1 \\ l \neq m}}^N (g^2 \mu_B^2 / R_{lm}^5) \\ \times [R_{lm}^2 (\mathbf{S}_l \cdot \mathbf{S}_m) - 3 (\mathbf{S}_l \cdot \mathbf{R}_{lm}) (\mathbf{S}_m \cdot \mathbf{R}_{lm})] \\ - \sum_{l=1}^N g \mu_B S_l^z H. \quad (1)$$

The notation follows essentially that of HP.  $N$  is the total number of atoms;  $\mathbf{R}_{lm} = \mathbf{R}_l - \mathbf{R}_m$ , the vector from the  $m$ th atom to the  $l$ th atom, in units of the lattice constant  $a$ ;  $J(R_{lm})$  is the exchange integral between the  $l$ th and  $m$ th atoms;  $g \mu_B \mathbf{S}_l$  is the magnetic moment operator for the atom at  $\mathbf{R}_l$ ;  $H$  is the external magnetic field and is in the  $z$  direction.

Holstein and Primakoff have given the approximate solution for the eigenvalues of this Hamiltonian when the magnetization  $M$  of the sample is very close to its maximum possible value  $M_0 = ng \mu_B S / a^3$ , that is  $M_0 - M \ll M_0$ . Here  $n$  is the number of atoms per unit cell.

The approximate solution is

$$E \equiv E_{N\mathbf{k}} = C + \sum_{\mathbf{k}} [\frac{1}{2} (A_{\mathbf{k}}^2 - |B_{\mathbf{k}}|^2)^{\frac{1}{2}} - \frac{1}{2} A_{\mathbf{k}}] \\ + \sum_{\mathbf{k}} (A_{\mathbf{k}}^2 - |B_{\mathbf{k}}|^2)^{\frac{1}{2}} N_{\mathbf{k}}. \quad (2)$$

The quantity  $C$  is the energy that would be obtained for the sample if it were considered as a classical assembly of magnetic dipoles and if all the dipoles were oriented in the same direction, that also being the direction of the magnetic field  $H$ . It includes, too, the exchange energy of such a model. In a cubic crystal, the energy  $C$  is independent of the direction of orientation

of the dipoles with respect to the crystal lattice<sup>1</sup> and therefore need not concern us further in calculating the anisotropy energy.

The  $N_k$ 's take on the values 0, 1, 2, ... and are the quantum numbers of the spin waves. The summation is over all the spin waves.  $A_k$  and  $B_k$  are given by:

$$\begin{aligned} A_k = & -\frac{3}{2}g^2\mu_B^2Sa^{-3}\sum_h R_h^{-3}(1-3z_h^2/R_h^2) \\ & +\frac{1}{2}g^2\mu_B^2Sa^{-3}\sum_h R_h^{-3}(1-3z_h^2/R_h^2) \\ & \times[1-\exp(i\mathbf{k}\cdot\mathbf{R}_h)]+2S\sum_h J(R_h) \\ & \times[1-\exp(i\mathbf{k}\cdot\mathbf{R}_h)]+g\mu_B H, \\ B_k = & -\frac{3}{2}g^2\mu_B^2Sa^{-3}\sum_h R_h^{-5}(x_h^2-y_h^2-2ix_hy_h) \\ & +\frac{3}{2}g^2\mu_B^2Sa^{-3}\sum_h R_h^{-5}(x_h^2-y_h^2-2ix_hy_h) \\ & \times[1-\exp(i\mathbf{k}\cdot\mathbf{R}_h)]. \quad (3) \end{aligned}$$

Here  $\mathbf{R}_h = (x_h, y_h, z_h)$  is used in place of  $\mathbf{R}_{lm}$ . It is the vector from one atom to any other atom in the lattice and the summation is over all such other atoms;  $\mathbf{k}$  is the wave vector of the spin waves in units of  $1/a$ . It takes on  $N$  discrete values, with its  $x$ ,  $y$ , and  $z$  components uniformly spaced over a range  $\sim -\pi$  to  $\sim +\pi$  determined by the imposition of periodic boundary conditions.

### 3. DIRECTIONAL DEPENDENCE

At  $T=0^\circ\text{K}$ , the sample will be in its lowest energy state, corresponding to all  $N_k=0$ . The  $A_k$  and  $B_k$  are functions of the  $x$ ,  $y$ , and  $z$  coordinates of the atoms situated at the lattice points and thus do depend upon the orientation of the lattice with respect to the  $z$  axis. The  $z$  direction is the direction of magnetization of the sample. We thus must evaluate

$$\sum_k [\frac{1}{2}(A_k^2 - |B_k|^2) - \frac{1}{2}A_k] \quad (4)$$

in order to obtain the anisotropy energy.

The summations in the expressions (3) for  $A_k$  and  $B_k$  are evaluated by HP as follows: The first summation, in each case, is broken up into two parts. One part is the sum over all atoms contained within a sphere described about  $R_h=0$ . This part vanishes for a cubic crystal. The second part is the sum over the remaining atoms outside the sphere. This is evaluated by replacing the sum by an integral, in accordance with the prescription

$$g\mu_B Sa^{-3}\sum_h f(\mathbf{R}_h) \rightarrow M_0 \int f(\mathbf{R})d\mathbf{R}.$$

This prescription amounts to smearing out the atoms uniformly through the crystal instead of localizing them at the lattice points.

The second summation in Eqs. (3) for  $A_k$  and  $B_k$  is evaluated by HP by immediately replacing the sum by an integral throughout the entire crystal. As far as distant atoms are concerned, this has no significant effect. But with regard to near neighbors, this eliminates any effects due to the discrete localization of the atoms

at the lattice points and any consequent dependence of the sum upon the orientation of the crystallographic axes with respect to the direction of magnetization. Thus the sums must be evaluated in greater detail to show the anisotropy energy. This is pointed out in HP.

We introduce the following symbols:

$$\begin{aligned} F &= g^2\mu_B^2 S/a^3, \\ T_1 &= -F \sum_h^{R_0} R_h^{-3}(1-3z_h^2/R_h^2), \\ T_{2k} &= -\frac{1}{2}F \sum_h^{R_0} R_h^{-3}(1-3z_h^2/R_h^2) \exp(i\mathbf{k}\cdot\mathbf{R}_h), \\ T_{3k} &= -\frac{3}{2}F \sum_h^{R_0} R_h^{-5}(x_h^2-y_h^2-2ix_hy_h) \exp(i\mathbf{k}\cdot\mathbf{R}_h), \\ T_4 &= 2S \sum_h J(R_h), \\ T_{5k} &= -2S \sum_h J(R_h) \exp(i\mathbf{k}\cdot\mathbf{R}_h), \\ I_1 &= -g\mu_B M_0 \int_{R_0}^{o.b.} R^{-3}(1-3z^2/R^2)d\mathbf{R}, \\ I_{2k} &= -\frac{1}{2}g\mu_B M_0 \int_{R_0}^{o.b.} R^{-3}(1-3z^2/R^2) \exp(i\mathbf{k}\cdot\mathbf{R})d\mathbf{R}, \\ I_{3k} &= -\frac{3}{2}g\mu_B M_0 \int_{R_0}^{o.b.} R^{-5}(x^2-y^2-2ixy) \exp(i\mathbf{k}\cdot\mathbf{R})d\mathbf{R}. \quad (5) \end{aligned}$$

The sums  $A_k$  and  $B_k$  can then be approximated as follows:

$$\begin{aligned} A_k &= T_0 + I_1 + T_{2k} + I_{2k} + T_4 + T_{5k} + g\mu_B H, \\ B_k &= T_{3k} + I_{3k}. \quad (6) \end{aligned}$$

In this approximation, a sphere of radius  $R_0$  is described about the atom at the origin and the summations in the expressions (3) for  $A_k$  and  $B_k$  are taken over all the atoms contained within the sphere. Beyond the sphere, the atoms are considered as uniformly smeared out and the summations are replaced by integrals. The symbol  $\sum_h^{R_0}$  means summation over all atoms for which  $R_h < R_0$  but excluding  $R_h=0$ ;  $\int_{R_0}^{o.b.}$  is an integration from the surface of the sphere to the outer boundary of the specimen. The specimen is taken as a needle-shaped ellipsoid with the major axis parallel to the direction of magnetization.

For a cubic crystal,  $T_1$  vanishes. Also, for every atom at  $\mathbf{R}_h$ , there is another atom at  $-\mathbf{R}_h$ . Thus, in the various sums above,  $\exp(i\mathbf{k}\cdot\mathbf{R}_h)$  can be replaced by  $2 \cos(\mathbf{k}\cdot\mathbf{R}_h)$  and the summation is then only over half the lattice space. Such a summation will be represented by the symbol  $\sum_h^{R_0}$ .

The integrals  $I_1$ ,  $I_{2k}$ , and  $I_{3k}$  can be evaluated by integration by parts and by the evaluation procedure described in HP, Appendix II. For  $kR_0 \ll 1$ , the surface integrals over the surface of the sphere of radius  $R_0$  can be evaluated by expanding  $\exp(i\mathbf{k}\cdot\mathbf{R})$ . For  $kR_0 \gtrsim 1$ , the integrals  $I_{2k}$  and  $I_{3k}$  approach zero because of the rapid variation of the integrand.

We obtain:

$$\begin{aligned} T_1 &= 0, \quad T_{2k} = -F \sum_{\frac{1}{2}h}^{R_0} R_h^{-3} (1 - 3z_h^2/R_h^2), \\ T_{3k} &= -3F \sum_{\frac{1}{2}h}^{R_0} R_h^{-5} (x_h - iy_h)^2 \cos \mathbf{k} \cdot \mathbf{R}_h, \\ T_4 &= 2S \sum_h J(R_h), \quad T_{5k} = -4S \sum_{\frac{1}{2}h} J(R_h) \cos \mathbf{k} \cdot \mathbf{R}_h, \\ I_1 &= (4\pi/3) g\mu_B M_0, \\ I_{2k} &= \frac{2}{3} \pi g\mu_B M_0 + 2\pi g\mu_B M_0 (-k_z^2/k^2 + k^2 R_0^2/30 \\ &\quad + k_z^2 R_0^2/30), \quad \text{for } kR_0 \ll 1, \end{aligned} \quad (7)$$

and

$$I_{2k} \rightarrow 0 \quad \text{for } kR_0 \gtrsim 1,$$

$$I_{3k} = 2\pi g\mu_B M_0 (k_x - ik_y)^2 (1 - k^2 R_0^2/10) k^{-2}, \quad \text{for } kR_0 \ll 1,$$

and

$$I_{3k} \rightarrow 0 \quad \text{for } kR_0 \gtrsim 1.$$

Let us now gather terms in  $A_k$  and write it as

$$A_k = Q + T_{2k} + T_{5k}, \quad (8)$$

where

$$Q = T_1 + I_1 + I_{2k} + T_4 + g\mu_B H. \quad (9)$$

Then,

$$\frac{1}{2} (A_k^2 - |B_k|^2)^{\frac{1}{2}} - \frac{1}{2} A_k \approx -|B_k|^2/4Q, \quad (10)$$

where the square root has been expanded and terms of higher order by a factor of  $T_{2k}/Q$  and  $T_{5k}/Q$  have been dropped as have higher order terms in  $B_k/Q$ . This expansion will be more accurate, then, if the terms  $T_4$  or  $g\mu_B H$ , in  $Q$ , are large; that is, in the presence of a large exchange interaction or large external field.

If, in accordance with (7), we consider values of  $\mathbf{k}$  such that  $I_{3k} = 0$ , then

$$|B_k|^2 = |T_{3k}|^2 = 9F^2 \sum_{\frac{1}{2}h}^{R_0} \sum_{\frac{1}{2}h'}^{R_0} \tau_h \tau_{h'}^* \times \cos(\mathbf{k} \cdot \mathbf{R}_h) \cos(\mathbf{k} \cdot \mathbf{R}_{h'}),$$

where

$$\tau_h = R_h^{-5} (x_h - iy_h)^2.$$

This expression for  $|B_k|^2$  can be broken up and written

$$|B_k|^2 = 9F^2 \sum_{\frac{1}{2}h}^{R_0} |\tau_h|^2 \cos^2(\mathbf{k} \cdot \mathbf{R}_h) + 9F^2 \sum_{\frac{1}{2}h, h \neq h'}^{R_0} \sum_{\frac{1}{2}h'}^{R_0} \tau_h \tau_{h'}^* \cos(\mathbf{k} \cdot \mathbf{R}_h) \cos(\mathbf{k} \cdot \mathbf{R}_{h'}).$$

We are now ready to evaluate

$$\sum_{\mathbf{k}} \left[ \frac{1}{2} (A_k^2 - |B_k|^2)^{\frac{1}{2}} - \frac{1}{2} A_k \right].$$

This summation is equal to  $N \langle \frac{1}{2} (A_k^2 - |B_k|^2)^{\frac{1}{2}} - \frac{1}{2} A_k \rangle_{Av}$ , where the average is over the values of  $\mathbf{k}$ .

If we pick  $R_0$  sufficiently large, then the number of values of  $\mathbf{k}$ , for which  $kR_0 < 1$ , can be made as small as desired as compared with the total number of values of  $\mathbf{k}$ . Thus, the contribution of those spin waves, for which  $kR_0 < 1$ , to the calculation of the above average, can be made negligibly small and we can take  $kR_0 \gtrsim 1$  for all the values of  $\mathbf{k}$ .

We have, for  $kR_0 \gtrsim 1$ ,

$$\begin{aligned} \frac{1}{2} (A_k^2 - |B_k|^2)^{\frac{1}{2}} - \frac{1}{2} A_k &\approx -|B_k|^2/4Q \\ &= -(9F^2/4Q) \{ \sum_{\frac{1}{2}h}^{R_0} |\tau_h|^2 \cos^2(\mathbf{k} \cdot \mathbf{R}_h) \\ &\quad + \sum_{\frac{1}{2}h, h \neq h'}^{R_0} \sum_{\frac{1}{2}h'}^{R_0} \tau_h \tau_{h'}^* \\ &\quad \times \cos(\mathbf{k} \cdot \mathbf{R}_h) \cos(\mathbf{k} \cdot \mathbf{R}_{h'}) \}. \end{aligned} \quad (11)$$

When this expression is averaged over the range of  $\mathbf{k}$  values, the second term in the curly brackets vanishes because of the orthogonality of the cosine factors. For the first term,  $\langle \cos^2(\mathbf{k} \cdot \mathbf{R}_h) \rangle_{Av} = \frac{1}{2}$ . Thus,

$$\begin{aligned} \sum_{\mathbf{k}} \left[ \frac{1}{2} (A_k^2 - |B_k|^2)^{\frac{1}{2}} - \frac{1}{2} A_k \right] \\ = -N(9F^2/8Q) \sum_{\frac{1}{2}h}^{R_0} R_h^{-10} (x_h^2 + y_h^2)^2. \end{aligned} \quad (12)$$

This expression represents that portion of the lowest energy value which depends upon the direction of magnetization. In the lowest energy state, the energy, *per unit volume*, which depends upon the direction of magnetization, is then

$$-\frac{9}{8} \frac{n}{a^3} \frac{F^2}{Q} \sum_{\frac{1}{2}h}^{R_0} \frac{1}{R_h^6} \left( 1 - \frac{z_h^2}{R_h^2} \right)^2. \quad (13)$$

#### 4. ANISOTROPY CONSTANT, $K_1$

If the  $z$  axis, the direction of magnetization of the crystal, has direction cosines  $\gamma_1, \gamma_2, \gamma_3$  with respect to the crystallographic cubic axes,  $x', y'$ , and  $z'$ , then  $z_h = \gamma_1 x_h' + \gamma_2 y_h' + \gamma_3 z_h'$ . Now,  $\gamma_1^2 + \gamma_2^2 + \gamma_3^2 = 1$ , and  $\gamma_1^4 + \gamma_2^4 + \gamma_3^4 = 1 - 2(\gamma_1^2 \gamma_2^2 + \gamma_2^2 \gamma_3^2 + \gamma_3^2 \gamma_1^2)$ . The summation,  $\sum_{\frac{1}{2}h}^{R_0} R_h^{-6} (1 - z_h^2/R_h^2)^2$ , can be evaluated quite readily for each of the three cubic crystal types, the face-centered cubic (fcc), the body-centered cubic (bcc), and the simple cubic (sc). The summation is equal to an expression of the form  $b_1 + b_2 \Gamma$ , where  $b_1$  and  $b_2$  are constants and  $\Gamma = \gamma_1^2 \gamma_2^2 + \gamma_2^2 \gamma_3^2 + \gamma_3^2 \gamma_1^2$ .

The term  $b_2 \Gamma$  gives the directional dependence. Its values for the three crystal types are

$$6.50\Gamma \text{ for fcc, } 2.20\Gamma \text{ for bcc, } -1.84\Gamma \text{ for sc,} \quad (14)$$

where the first 7 zones of neighbors were used for the fcc calculation, the first 6 zones for the bcc, and the first 4 zones for the sc. In each case,  $R_0 = 2$ , that is,  $R_h = 2$  for the last zone used, and the contribution of the last zone to the total is  $-0.03\Gamma$ .

#### Dipolar Ferromagnetism

The anisotropy constant  $K_1$  is given by the coefficient of  $\Gamma$  in the expression (13). For the case of a cubic lattice of magnetic dipoles such that the exchange interaction is negligible, then

$$J = 0, \quad \text{and} \quad Q = (4\pi/3) g\mu_B M_0 + g\mu_B H.$$

If there is no external field, then  $Q = (4\pi/3) g\mu_B M_0$ . Remembering that  $M_0 = n g\mu_B S/a^3$  and that  $n = 4$  for an fcc lattice and  $n = 2$  for a bcc lattice, then  $K_1$  can be

obtained from (13) and (14). We get

$$\begin{aligned} K_1(\text{fcc}) &= -0.11M_0^2/S, \\ K_1(\text{bcc}) &= -0.15M_0^2/S, \\ &\text{for pure magnetic dipoles with } H=0. \end{aligned} \quad (15)$$

In the sc case, it is necessary to apply an external field in order to obtain a parallel arrangement. Otherwise, an antiparallel arrangement has lower energy.<sup>5</sup> The smallest value which the external field may have and still maintain a parallel arrangement of the dipoles is  $H=0.6M_0$ . This gives

$$\begin{aligned} K_1(\text{sc}) &= +0.43M_0^2/S, \\ &\text{for pure magnetic dipoles with } H=0.6M_0. \end{aligned} \quad (16)$$

In the presence of an external field  $H$ , the expressions for  $K_1$ , in the three cases, become

$$\begin{aligned} K_1(\text{fcc}) &= -0.46(M_0^2/S)(4\pi/3+H/M_0)^{-1}, \\ K_1(\text{bcc}) &= -0.63(M_0^2/S)(4\pi/3+H/M_0)^{-1}, \\ K_1(\text{sc}) &= +2.06(M_0^2/S)(4\pi/3+H/M_0)^{-1}, \\ &\text{for pure magnetic dipoles.} \end{aligned} \quad (17)$$

#### Exchange Ferromagnetism

In the presence of a large short-range exchange interaction, such that  $g\mu_B M_0$  and  $g\mu_B H$  are negligible compared to  $J$ , then the anisotropy constant  $K_1$ , from (13), is

$$K_1 = -\frac{9}{8} \frac{n}{a^3} \frac{F^2}{2zJS} b_2, \quad (18)$$

where  $z$  is the number of nearest neighbors, and  $b_2$  is gotten from (14). We obtain

$$\begin{aligned} K_1(\text{fcc}) &= -0.08(M_0^2/S)(g^2\mu_B^2 a^{-3}/J), \\ K_1(\text{bcc}) &= -0.08(M_0^2/S)(g^2\mu_B^2 a^{-3}/J), \\ K_1(\text{sc}) &= +0.17(M_0^2/S)(g^2\mu_B^2 a^{-3}/J). \end{aligned} \quad (19)$$

<sup>5</sup> J. A. Sauer, Phys. Rev. 57, 142 (1940).

This gives the anisotropy constant of a ferromagnet when the anisotropy energy is due to magnetic dipole interactions.

Spin-orbit effects are partially represented, according to Van Vleck,<sup>3</sup> by an equivalent dipole-dipole interaction which may be much larger than the true magnetic dipole-dipole interaction. Van Vleck estimates the equivalent dipole-dipole interaction,  $C'$ , for nearest neighbors to be  $\sim J/100$ , whereas  $g^2\mu_B^2 a^{-3} \sim J/10^4$ . The equivalent dipole-dipole interaction can be substituted for the true magnetic dipole-dipole interaction in the above calculation by substituting  $C'(r_0^3/a^3)$  for  $g^2\mu_B^2/a^3$  in (13);  $r_0$  is the nearest neighbor distance. Inasmuch as  $C'$  is of short-range character, therefore the summation in (13) should only be carried out over nearest neighbors. In place of (14), the dependence of the summation on  $\Gamma$  is then, for nearest neighbors only,

$$8.00\Gamma \text{ for fcc, } 4.21\Gamma \text{ for bcc, } -2.00\Gamma \text{ for sc.} \quad (20)$$

We then obtain

$$\begin{aligned} K_1(\text{fcc}) &= -1.13(n/a^3)(S^2 C'^2/2zJS), \\ K_1(\text{bcc}) &= -2.00(n/a^3)(S^2 C'^2/2zJS), \\ K_1(\text{sc}) &= +2.25(n/a^3)(S^2 C'^2/2zJS). \end{aligned} \quad (21)$$

This gives the anisotropy constant of a ferromagnet at  $T=0^\circ\text{K}$  when the anisotropy energy is due to the spin-orbit interaction considered as an equivalent short-range dipole-dipole interaction. The anisotropy values (21) are in exact agreement with the values of  $K_1$  which may be obtained from Eq. (32) of Van Vleck's paper<sup>3</sup> on the anisotropy of cubic ferromagnetic crystals.

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