(7), one then obtains the delta-ray yield as a function of the thickness and atomic number of the target.

From an analysis similar to that already outlined, one can determine the energy distribution of the emergent secondaries, and it is found that almost all of them have energies between 10 and 100 kev.[10] Consequently, assumption (b) is valid, as is (a) for all but the most tightly-bound target electrons. Assumption (c) is not so well justified, since the path of an emerging secondary is in general longer than the distance from its point of origin to the surface, and the computed yields are consequently too high by about a factor of two. The ratio of the computed to the experimentally-determined yield is, however, substantially the same in all cases, indicating that the dependence of the delta ray yield on the thickness and atomic number of the target is correctly given by Eq. (7).

The computed yields normalized to fit the experimental value for 46 mg cm^{-2} Ni are plotted in Fig. 5. Theory and experiment are in agreement within the experimental uncertainties, except for the case of 5 mg cm^{-2} Au. This discrepancy may be caused by the fact that the scattering of the secondaries has not been taken into account in the theory. This effect would be more pronounced in a high-Z material such as gold.

[10] A lower limit to the delta-ray energies has been determined by an independent method of observing the energy distribution of the low-energy secondary electrons, and no current of delta rays with energies less than 2 kev has been detected [Shatas, Marshall, and Pomerantz, Phys. Rev. **94**, 757 (1954)].

ACKNOWLEDGMENT

The authors wish to acknowledge the assistance of Arthur E. Smith in the experimental program.

PHYSICAL REVIEW VOLUME 96, NUMBER 5 DECEMBER 1, 1954

Hall Effect in Bismuth at Low Temperatures*

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(Received July 30, 1954)

The dc Hall effect in bismuth has been investigated over a range of fields from 0 to 8000 gauss and at temperatures in the liquid helium region. It is observed that the Hall effect oscillates in a manner which is periodic in H^{-1} and that this period is independent of temperature. The dependence of the amplitude of the oscillations on field and temperature is of the general form given by the theoretical work of Grimsal and Levinger.

At low temperatures the magnetic susceptibility of bismuth and many other metals shows oscillations which are periodic in the reciprocal of the magnetic field intensity.[1–3] At high fields such oscillations occur to a smaller extent in the magnetoresistance.[4] The Peierls,[5] Blackman,[6] and Landau[7] theory relates the oscillations of the de Hass-van Alphen effect to the Larmor precessions of the conduction electrons and to the quantization of this motion in the plane perpendicular to the magnetic field. As this precession gives rise to the Hall effect, it is not surprising that the Hall effect in bismuth is also periodic in $1/H$.[8] Grimsal and Levinger[9] have recently obtained a formula for the periodic part of the Hall effect in bismuth, based on Blackman's formulation[6] of the theory of the de Haas-van Alphen effect. The data reported here are interpreted in terms of Grimsal and Levinger's formulation.

EXPERIMENTAL

The single crystals were grown in vacuum by the Bridgman method, from Johnson, Matthey bismuth (Johnson, Matthey & Co., Ltd., London). Crystal Bi-1, reported on earlier,[8] was in the form of a right parallelopiped of dimensions 25.5 by 7.5 by 0.88 mm. The orientation of its crystallographic axes were such that when mounted in the flask the current passed parallel to a binary axis and the magnetic field was at an angle of 25° with the trigonal axis.

In order to allow measurements to be made on one crystal, both with the field parallel and with it perpendicular to the trigonal axis, a crystal (Bi-4) was prepared which was more nearly cubical in shape ($6.7\times5.2\times5.2$ mm), with one pair of faces perpendicular to the trigonal axis, another pair perpendicular to a binary axis, and the third pair of faces parallel to the plane of

* Supported by the National Science Foundation.
[1] W. J. de Haas and P. M. van Alphen, Leiden Comm. No. **212a** (1930) and No. **220d** (1932).
[2] D. Schoenberg, Proc. Roy. Soc. (London) **A170**, 341 (1939).
[3] D. Schoenberg, Trans. Roy. Soc. (London) **A245**, 1 (1952).
[4] P. B. Alers and R. T. Webber, Phys. Rev. **91**, 1060 (1953).
[5] R. Peierls, Z. Physik **80**, 763 (1933).
[6] M. Blackman, Proc. Roy. Soc. (London) **A166**, 1 (1938).
[7] L. D. Landau, see Appendix to reference 2.
[8] Reynolds, Leinhardt, and Hemstreet, Phys. Rev. **93**, 247 (1954).
[9] J. S. Levinger and E. G. Grimsal, Phys. Rev. **94**, 772(A) (1954); and E. G. Grimsal, Thesis, Louisiana State University (1954) (Unpublished—copies available from L. S. U. Physics Department.)

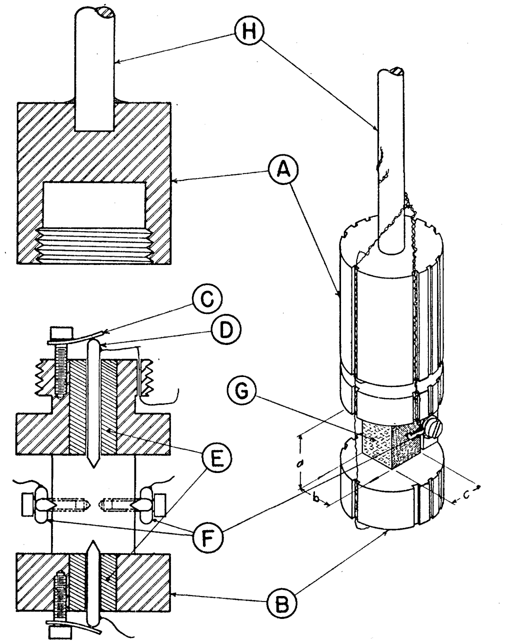

FIG. 1. Crystal holder. A, Lucite cap; B, Lucite holder; C, phosphor-bronze spring; D, current probes; E, brass bushings; F, Hall probes; G, Bi crystal; H, Lucite support.

the trigonal and the binary axis. The crystal was grown in the form of a cylinder and then cut to shape on a water-cooled carborundum saw.

The crystals were mounted in Lucite holders (Fig. 1) in direct contact with liquid helium. The Hall probes (F) and the current probes (D) were made of phosphor-bronze and were 0.84 mm in diameter. The current probes passed through brass bushings (F) for ease in alignment. To assure good contact, both the specimen and the probe tips were etched in nitric acid immediately before mounting.

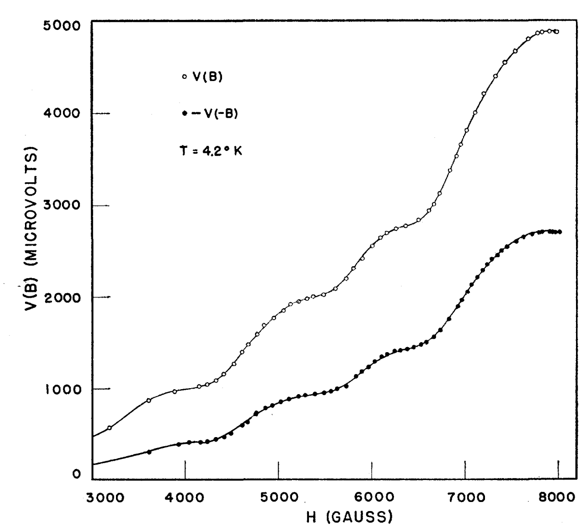

FIG. 2. The potential observed on the Hall probes at 4.2°K. Field perpendicular to the trigonal axis.

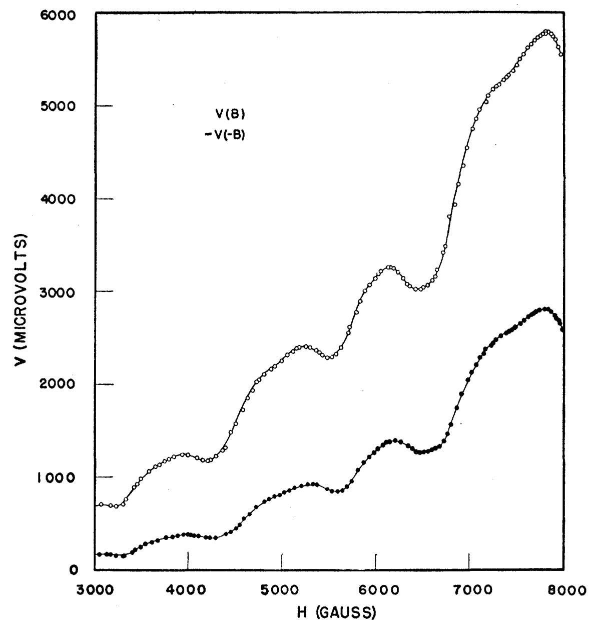

FIG. 3. The potentials observed on the Hall probes at 1.4°K. Field perpendicular to the trigonal axis.

The dc Hall voltage was measured by means of a White potentiometer, and an L and N Type K potentiometer was used to monitor the current through the sample. At each temperature the probe voltage was

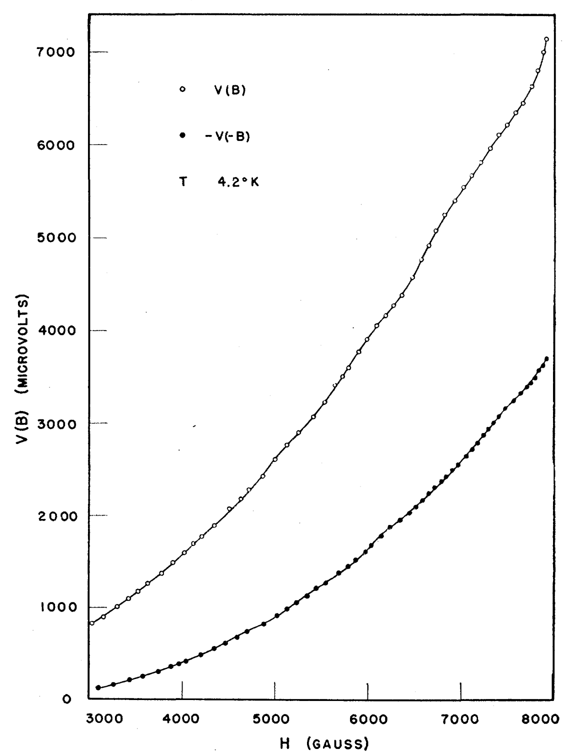

FIG. 4. The potentials observed on the Hall probes at 4.2°K. Field parallel to the trigonal axis.

measured at closely spaced intervals of the field value. After such a field sweep was complete, it was then repeated with the field reversed in direction. The Hall voltage is taken to be

$$V_H=\tfrac{1}{2}[V(B)-V(-B)]. \qquad (1)$$

This is done to eliminate the effect of the large magnetoresistance of bismuth.

RESULTS

(A) Field Dependence

In Figs. 2–5 $V(B)$ and $-V(-B)$, for sample Bi-4, are plotted against the magnetic field. The data in

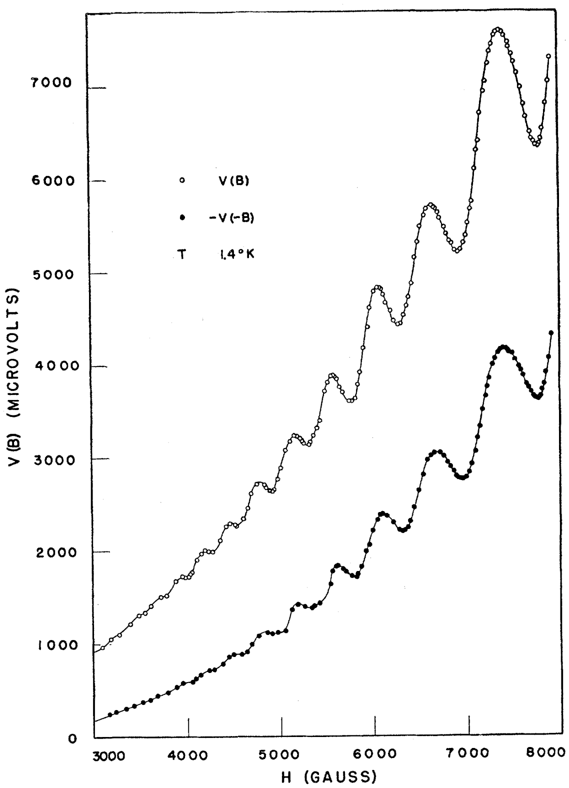

FIG. 5. The potentials observed on the Hall probes at 1.4°K. Field parallel to the trigonal axis.

Figs. 2 and 3 were taken with the field perpendicular to the trigonal axis, while those in Figs. 4 and 5 were taken with the field parallel to the trigonal axis. In all these measurements the current was parallel to a binary axis. The oscillations are appreciable only at the lower temperature, and simplest for the field parallel to the trigonal axis.

According to Eq. (1) the Hall voltage is given in each figure by the average of the two curves. These averages were obtained graphically and from them values of the Hall coefficient were obtained. The Hall coefficients were then plotted against H^{-1}, and these curves are seen in Fig. 6 for three orientations at 1.4°K.

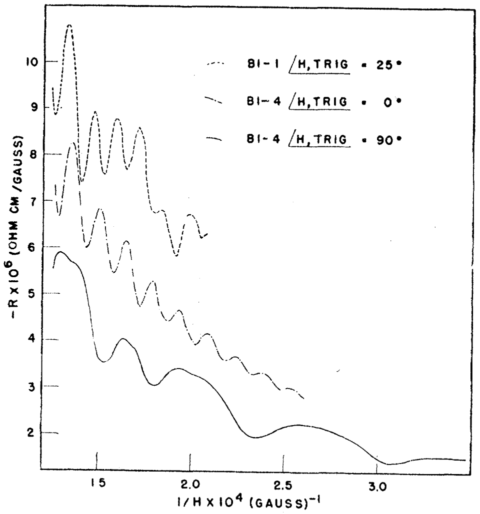

FIG. 6. The Hall coefficients. $T=1.4$°K.

The upper curve, for which the field is 25° from the trigonal axis, is taken from the data previously reported on sample Bi-1, while the lower and middle curves are taken from the data in Figs. 3 and 5, respectively. One can see that as the field approaches parallelism with the trigonal axis the oscillations simplify to a single period in H^{-1}. To illustrate the periodicity of the oscillations, the successive values of H^{-1} for which the Hall coefficient is a maximum or minimum are plotted against their corresponding integers (Fig. 7).

(B) Temperature Dependence

Some two months after the data shown in Figs. 4 and 5 were taken, sample Bi-4 was remounted in a flask and the Hall measurements were taken at seven different temperatures, ranging from 4.21°K down to 1.38°K. The field was chosen to be along the trigonal axis, because the oscillations have a simple period for this orientation. These data are shown in Fig. 8. One sees immediately from Fig. 8 that the period and phase of the oscillations are independent of temperature, while the amplitude decreases rapidly with increasing temperature.

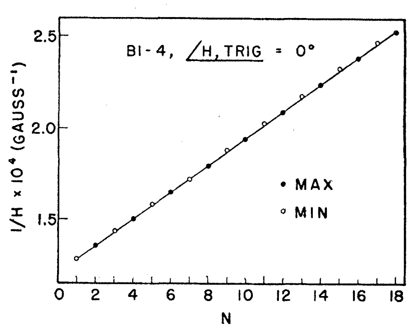

FIG. 7. Values of H^{-1} for which maxima and minima in the Hall coefficients occur, plotted against their corresponding integers.

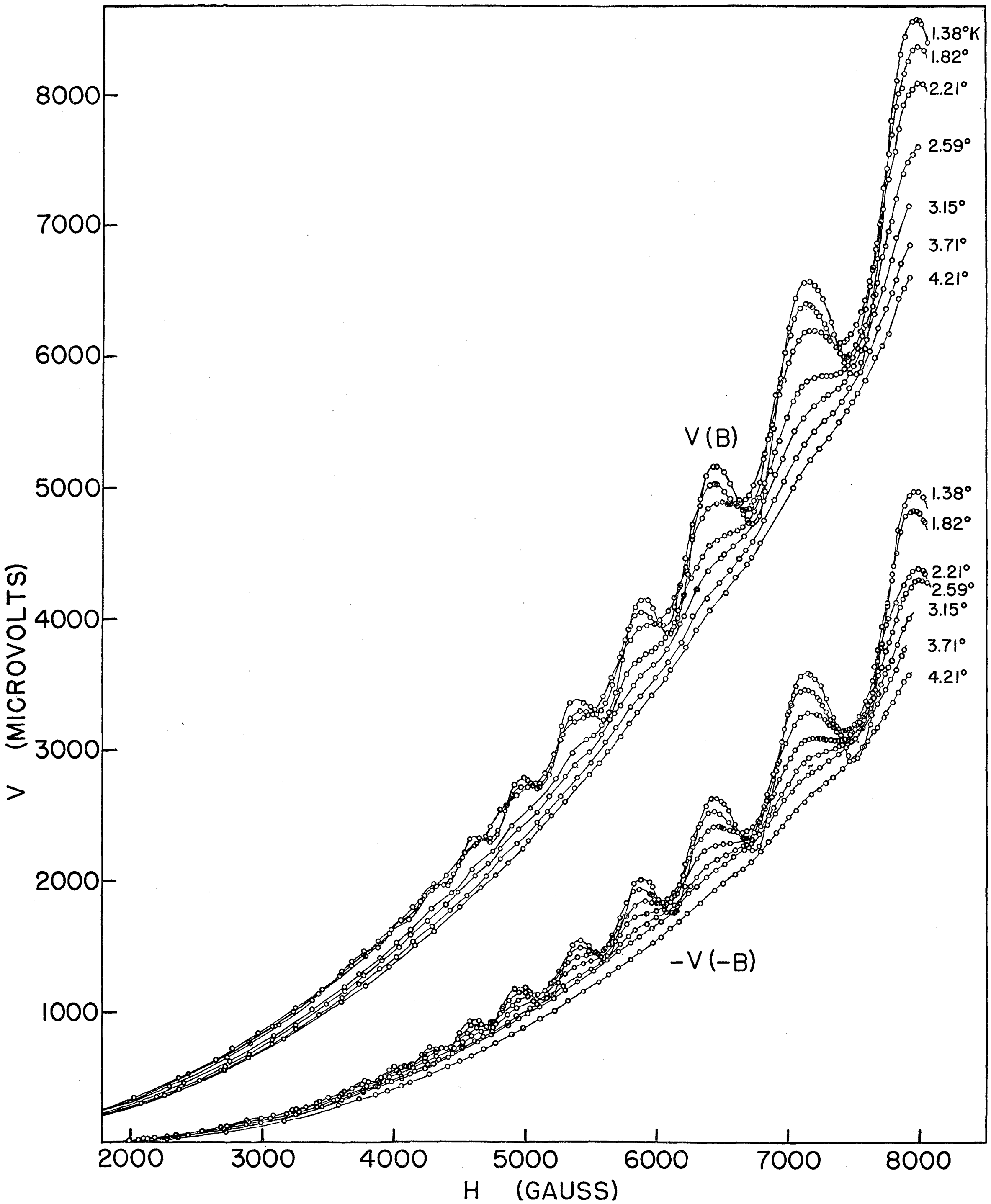

FIG. 8. The potentials observed on the Hall probes for various temperatures. Field parallel to the trigonal axis.

Analysis of Data

Dingle,[10] Grimsal, and Levinger[9] derive for the number of "de Haas-van Alphen electrons"

$$N=N_0+\sum_{P=1}^{\infty}\frac{(-1)^P}{P^{\frac{3}{2}}}\frac{m^{\frac{3}{2}}}{2\pi^3\hbar^3}\frac{P\chi}{\sinh P\chi}(\beta^*H)^{\frac{3}{2}}\times\sin\left(2\pi P\frac{E_0}{\beta^*H}-\frac{\pi}{4}\right), \quad (2)$$

where $\chi=2\pi^2kT/\beta^*H$. E_0 is the Fermi energy and β^* is the "effective" double Bohr magneton given by

$$\beta^*=e\hbar/m^*c,$$

where m^* is a suitable "effective" electron mass. Replacing hyperbolic sine by the exponential in Eq. (2) and putting this into the two-band "isotropic" Hall effect formula, they[9] get an expression for the Hall coefficient of the form

$$R=R_0+rH^{\frac{1}{2}}T\exp\left(-\frac{2\pi^2kT}{\beta^*H}\right)\sin\left(\frac{2\pi E_0}{\beta^*H}-\frac{\pi}{4}\right), \quad (3)$$

where R_0 and r are constants. (Using $P=1$ only.)

The amplitude of the oscillations in the Hall coefficient is then given by

$$A=rH^{\frac{1}{2}}T\exp(-2\pi^2kT/\beta^*H) \quad (4)$$

and a plot of $\ln(A/TH^{\frac{1}{2}})$ against T/H should yield a straight line whose slope is $-2\pi^2k/\beta^*$, from which β^* may be determined. (See Fig. 9.)

In the plot of $\ln(A/TH^{\frac{1}{2}})$ *versus* T/H, the scatter of the points is rather large. However, according to Dingle,[3] collision broadening of the energy levels, due to impurities, would reduce the amplitude of de Haas-van Alphen oscillations, as though the temperature in Eq. (2) had been raised by an amount Z, where

$$Z=h/2\pi^2k\tau$$

and τ is the collision time. Adding this term to the temperature improved the scatter of the points greatly. In plots of $\ln[A/(T+Z)H^{\frac{1}{2}}]$ *versus* $(T+Z)/H$ the points fell most nearly on a straight line for $Z=0.5$. This particular plot is seen in Fig. 9. The slope of the line drawn in Fig. 9 yields the value $\beta^*=3.0\times10^{-19}$ erg/gauss.

[10] R. B. Dingle, Proc. Roy. Soc. (London) **A211**, 500 (1952).

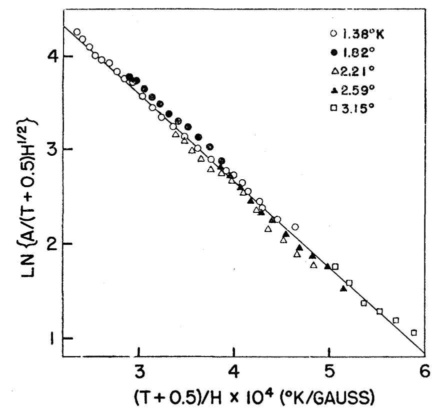

FIG. 9. Dependence of amplitude on temperature and field. Field parallel to the trigonal axis.

According to Eq. (2) the slope of the line in Fig. 6 should be $\frac{1}{2}(\beta^*/E_0)$. This gives $\beta^*/E_0=1.5\times10^{-5}$ gauss^{-1}. The above value of β^* then gives

$$E_0=2.0\times10^{-14}\text{ erg.}$$

From his susceptibility data, Shoenberg[3] gets a value

$$E_0=2.9\times10^{-14}\text{ erg,}$$

using Landau's[7] formulation of the theory. We have found no satisfactory explanation for the discrepancy. However, the theory employed in the analysis of the Hall effect data is not as yet refined to the point that one could expect very exact agreement in calculations of such parameters. Another objection to the theory as it now stands is that it does not account for the field dependence of the nonoscillatory part of the Hall effect, i.e., R_0 in Eq. (3) is a constant. Equation (3) does, however, describe the essential features of the oscillatory part of the Hall effect. It is periodic in H^{-1} and this period is independent of temperature. The dependence of the amplitude of the oscillations on field and temperature seem to be of the right form from the fit of the points in Fig. 9.

The authors wish to express their appreciation to Dr. J. S. Levinger, Dr. E. G. Grimsal, and Dr. D. C. Ralph for their valuable discussions.

Note added in proof.—The authors have learned through private communication from T. G. Berlincourt that the periodic Hall effect has been observed in graphite at the Naval Research Laboratory.