Validity of Semiclassical Treatment of Coulomb Excitation*

P. B. Daitch, J. P. Lazarus, M. H. Hull, Jr., F. D. Benedict, and G. Breit
Yale University, New Haven, Connecticut
(Received October 4, 1954)

CALCULATIONS have been made by means of Eq. (11) of the preceding note in the form

$$32\eta^2\mathcal{C}_L=\int_L^{L+1} b(L)dL, \tag{1}$$

with

$$b(L)=L\eta^{-2}\sum |S_\mu^{(2)}|^2. \tag{2}$$

The choice of the range of integration in Eq. (1) is such as to obtain good agreement for the two sides. Here and below the notation of the preceding note is used.

The value $\eta=8.156$ was used so as to correspond to the experiment of Huus and Zupančič[1] for 2-Mev protons bombarding tantalum. Values of F_5 were kindly furnished by Dr. M. Abramowitz of the National Bureau of Standards. These were checked by the series of Eqs. (13) and (14) of Biedenharn *et al.*[2] at ρ_L to about 0.1 percent, in the range $10<\rho<18$ by numerical integration (NI) for nearly the same η with a correction for the difference of ~ 0.0003 in η, in the range $18<\rho<35$ by computing F'' by differences from F and hence $Q^2=-F''/F$. At $\rho=50$, the NI of the continuation was checked against asymptotic series, and a NI was used as a consistency check in $6<\rho<10$. The values of F_5 were used for the calculation of F_L for $L\leqslant 10$ in the range $0<\rho<35$. For $L=20$, 30, 50 the values at the classical turning point, ρ_L, have been computed by Eqs. (13) and (14) of reference 2 and continued by NI in steps of 0.50 in ρ to larger and smaller distances. The integrals in Eq. (2′) will be referred to as the (L,L) and $(L+1, L-1)$ integrals. They were calculated by evaluating the contributions from two ranges of ρ, the small ρ range and the tail range, the former furnishing the main contribution. For $0\leqslant L\leqslant 10$ the tail ranges began at the highest zero of F_L for $\rho<35$; for $L=20$ at $\rho\cong 47.3$, the third node of F above $\rho=0$; for $L=30$ at $\rho\cong 86.5$; for $L=50$ at $\rho\cong 101$. Different procedures were employed in the evaluation of contributions from the small ρ range and the tail. The work for the small range will be described first. For $L\leqslant 10$ Simpson's rule at intervals of 0.50 in ρ with checks by the Newton-Cotes fourth degree fit rule was used. The calculation of both types of integrals took place by direct evaluation of the integrand. For $L=20$, 30, 50 the (L,L) integral for the small ρ range was handled as for $L\leqslant 10$. The $(L+1, L-1)$ integrals were first transformed by means of recurrence relations and partial integrations to a form involving integrands of the type

$$(A/\rho^3+B/\rho^4+C/\rho^5)F_L^2. \tag{3}$$

The calculation then proceeded as for the (L,L) integrals.

The evaluation of the tail contributions for $(L+1, L-1)$ integrals was carried out by reducing them to the type of Eq. (3). These as well as the (L,L) forms are conveniently calculated by means of the phase amplitude quantities which yield

$$F_L^2=(A_L^2/2)-(A_L^2/2)\cos(2\varphi_L). \tag{4}$$

The second part oscillates with a high frequency so that its contribution is much smaller than that of the first, low frequency part. The first part has been obtained by means of a series[3] for A_L in descending powers of ρ. The last term of the series for A_L was 3 percent or less of the contribution of the tail, this relatively high fractional error occurring however for a tail contribution of about 10 percent of the desired integral. The error caused by breaking off the series for A_L is thus of the order of 0.3 percent or less in the value of the integral and this error was not exceeded in other cases. The high frequency part is of the order of the error caused by omission of the last term in the series for A_L. It has been evaluated by successive partial integrations giving terms in descending powers of ρ. The second term is of the order 0.001 of the first. The procedure has been checked by the consideration of the curvature of the factor multiplying $\cos 2\varphi$. The accuracy of the results for the integrals is thus believed to be generally 0.3 percent or better, corresponding to 0.6 percent in the value of contributions to $b(L)$. The results of the comparisons of contributions to $10^6\mathcal{C}_L$ and the corresponding SCT quantity are shown in Table I. The integral

$$(10^6/32\eta^2)\int_0^\infty b(L)dL=532.8$$

and the contribution to it from the region $L>50$ is ~ 12 percent of the whole. Assuming the last column to give about the same fractional difference for the remaining relevant region as for the last three entries the

Table I. Comparison of quantum and classical contributions to $(25/64\pi^2)[v\hbar^3/(Z_1^2e^2mkB(2))]\sigma\times 10^6$ with $\sigma=$ total cross section.

L	Quantum quantity $10^6\mathcal{C}_L$	Classical quantity $10^6 b(L)/(32\eta^2)$	Excess of quantum over classical in percent
0	0	6.21	
1	18.98	17.63	7.1
2	26.91	26.57	1.2
3	32.45	32.32	0.4
4	35.10	35.00	0.3
5	35.42	35.26	0.5
6	33.95	33.89	0.2
7	31.63	31.58	0.2
8	28.84	28.84	0.0
9	26.04	26.00	0.1
10	23.27	23.27	0.0
20	7.54	7.62	−1.1
30	3.10	3.14	−1.1
50	0.896	0.909	−1.5

excess of the quantum cross section over the SCT cross section is 0.1 percent of the whole. The numbers recorded in the table are not as accurate as the number of significant figures would suggest. The intended accuracy was about one percent but internal consistency indicates that it may be better. The second column minus the third when summed over different L contributes to the excess of $10^6 \sum \mathcal{C}_L$ over the SCT approximation the amounts -6.2, $+2.2$, -1.3, and -0.6 from the L ranges 0, 1–10, 11–50, 51—∞ respectively. The total difference is -5.9, about -1 percent of the total. At the end of the work a slight error was found in the quantum values for $L=20$, 30, 50. A crude correction replaces the -1.1, -1.1, -1.5 percent entries by roughly $+3$ percent to 1.7 percent, replacing the total quantum-classical difference by ~ 1.0, *i.e.*, ~ 0.2 percent of total. With either interpretation the difference is one percent or less of the total cross section.

* This research was supported by the Office of Ordnance Research, U. S. Army.

[1] T. Huus and Č. Zupančič, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **28**, No. 1 (1953).

[2] Biedenharn, Gluckstern, Hull, and Breit, Phys. Rev. (to be published).

[3] Bloch, Hull, Broyles, Bouricius, Freeman, and Breit, Phys. Rev. **80**, 553 (1950); B. E. Freeman, Dissertation, Yale University, 1950 (unpublished).

Spin, Magnetic Moment, and Hyperfine Structure of Rb^{81}*

J. P. Hobson,† J. C. Hubbs,‡ W. A. Nierenberg, and H. B. Silsbee

University of California, Berkeley, California

(Received October 11, 1954)

THE spin, hyperfine splitting, and nuclear moment of Rb^{81} (4.7 hr) have been measured. They are 3/2, 5000±125 Mc/sec, and 2.00±0.06 nuclear magnetons respectively. The zero-moment method of atomic beams was used. Figure 1 shows the zero-moment curve

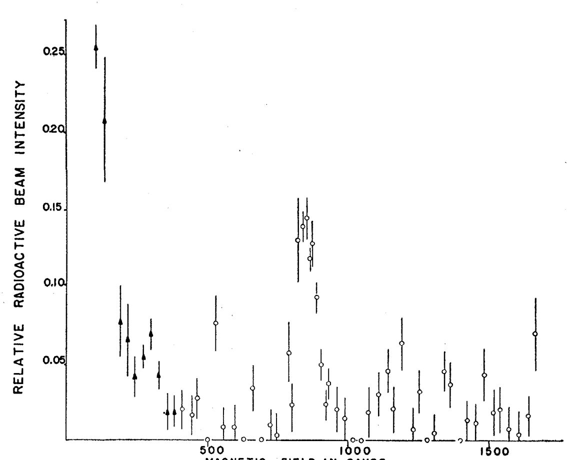

FIG. 1. Rb^{81} zero-moment curve.

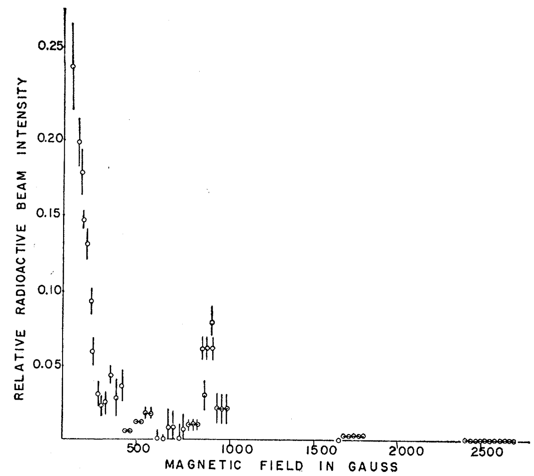

FIG. 2. Rb^{81} zero-moment pattern.

(beam intensity *vs* magnet current) for Rb^{81}. A peak appears at approximately 850 gauss. The run did not remove the possibility of spin 5/2 or 7/2. Figure 2 is another run which shows the 850-gauss peak but shows no peaks at 1700 or 2550 gauss, thus establishing the spin as 3/2. Calibration of the fields with natural Rb established the hyperfine splitting, and the ratio of radio-rubidium to natural rubidium hyperfine splittings determined the magnetic moment.

The detection was accomplished by collection of the neutral beam on sulfur buttons and counting the K x-rays. The sulfur surface collection was one of many methods tried and it apparently gave nearly full efficiency. The K x-rays were absorbed in a 1 mm$\times\frac{1}{2}$ in. $\times\frac{1}{2}$ in. NaI(TlI) crystal which gave nearly 50 percent efficiency with a minimum counting background.

The isotope was produced by α bombardment and therefore had carrier added to optimize the beam. It was identified by the chemistry, its decay curve, and the behavior of the sample as a function of α energy.

There is a suggestion of a peak at 290 gauss in Fig. 1, possibly due to Rb^{82}. Further runs are being made to improve the statistics in this region and at multiples of this field.

* This research has been supported by the U. S. Office of Naval Research.

† Now at the National Research Council of Canada, Ottawa, Canada.

‡ Now at Columbia University, New York, New York.

$E3$ Isomer in Ir^{191}*

J. W. Mihelich,† M. McKeown, and M. Goldhaber

Brookhaven National Laboratory, Upton, New York

(Received October 15, 1954)

IT has been established[1] that Os^{191} (16 day) decays via a β transition to an excited level in Ir^{191}, followed by two γ-ray transitions in cascade (42 and 129 kev).