to give a sufficiently large, positive B_2/B_0 if we simultaneously require that $\delta \geqslant +0.45$ and that the reaction goes mostly by the $s=2$ channel. The large value of δ demands the rather high value of 5 or greater for $[\gamma_3^2/\gamma_1^2]^{\frac{1}{2}}$. If this assignment is correct, we have an indication that $[\gamma_{l(p)+2}/\gamma_{l(p)}{}^2]^{\frac{1}{2}}$ may undergo large deviations from unity. It would be desirable to measure angular distributions of other reaction products at this resonance to corroborate or disprove this assignment.

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Proton Polarization in (d,p) Reactions*†

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The polarization of the proton produced in a (d,p) reaction is calculated under the assumption that "stripping" is the primary mechanism operative. The model adopted for the polarization production consists in a proton-nucleus interaction potential of the spin-orbit type which has been successful in describing polarization effects in neutron scattering. The model is applied to the specific reaction $C^{12}(d,p)C^{13}$ for $E_d=3.29$ Mev.

I. INTRODUCTION

THE apparent usefulness of (d,p) and (d,n) reactions as tools in nuclear spectroscopy stems from the now classic paper on these reactions by Butler.[1] The form of the differential cross section for such reactions predicted by the stripping theory is in surprisingly good agreement with the experimental results, especially if deuterons of not too low an energy are used as projectile particles. However, the results of experiments using deuterons of about 3 Mev exhibit features which the stripping theory has difficulty in explaining, even if the modifications[2] of the theory suggested subsequent to Butler's paper are taken into account. Particular reference is made here to the pronounced "resonances" observed in the excitation function and the form of the differential cross sections measured off and on resonance.[3] In brief summary, the resonance behavior of the excitation function is caused by a very pronounced change in magnitude of the forward-angle stripping peak whereas the back-angle yield is relatively constant over the range of energies including the resonances.

It has been suggested[4–6] that the nucleon produced as a product in the stripping reaction is polarized. The model sketched below for producing such polarization was chosen because of its apparent reasonableness in that such a model correctly describes the elastic scattering of polarized neutrons[7] in the medium-energy range. Since the stripping approximations are used throughout the polarization calculation, the extent of disagreement between the herein predicted polarization and the eventually forthcoming experimental results will indicate the extent to which the stripping assumptions are suspect at the deuteron energies considered below, namely, $E_d \sim 3$ Mev.

II. POLARIZATION CALCULATION

In the so-called "Born approximation" discussion[8] of stripping reactions, the final state nucleon is assumed to have no specifically nuclear interaction with the residual nucleus and no polarization of the proton results. In the calculation below, the assumption will be made that the final state nucleon is scattered in a spin-orbit potential superimposed upon the "clouded crystal ball" or complex central potential of Feshbach *et al.*[9] which adequately describes the elastic scattering of low-energy neutrons.[10] Strictly speaking, therefore, the model applies to (d,n) reactions; it will be applied to (d,p) reactions as well, with justification supplied by the

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† The results of this calculation were first reported at the Minneapolis meeting of the American Physical Society, June, 1954.

[1] S. T. Butler, Proc. Roy. Soc. (London) **A208**, 559 (1951).

[2] See, for example, W. Tobocman and M. H. Kalos, Phys. Rev. **95**, 605(A) (1954).

[3] Van Patter, Simmons, Stratton, and Zipoy, Bull. Am. Phys. Soc. **29**, No. 5, 14 (1954).

[4] H. C. Newns, Proc. Phys. Soc. (London) **A401**, 477 (1953).

[5] J. Horowitz and A. Messiah, Phys. Rev. **92**, 1326 (1953).

[6] N. Francis and K. Watson, Phys. Rev. **93**, 313 (1954). The model used in the present discussion was suggested in this article.

[7] Darden, Fields, and Adair, Phys. Rev. **93**, 931 (1954).

[8] See, for example, P. Daitch and J. French, Phys. Rev. **87**, 900 (1952).

[9] Feshbach, Porter, and Weisskopf, Phys. Rev. **90**, 166 (1953).

[10] Note, however, that the depth of the well suggested by Feshbach *et al.* to explain the scattering data is too small by a factor ~ 2 to harmonize with the data on neutron bound states interpreted on the basis of the shell model. [See A. Bohr and B. Mottelson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **27**, 159 (1953); R. K. Adair, Phys. Rev. **94**, 737 (1954).]

apparent charge independence of nuclear forces. The Coulomb distortion of the deuteron and proton waves is ignored; this should not change the order of magnitude of the results obtained. The additional restriction will be made to spin-zero target nuclei since it is for these targets that the Butler approximation should most nearly apply, i.e., that the nucleon absorbed by the target exists in a single, definite state of orbital angular momentum, at least for a time as long as the interaction period.

Since the final-state proton will scatter in a spin-orbit potential, it will be useful to express the wave function for the proton as a linear superposition of eigenfunctions of the $\mathbf{L}\cdot\mathbf{S}$ operator:

$$\Psi_p(\mu_p)=\sum_{L,M_L}\sum_{J,M_J} a(L,M_L) \times C_{L,\frac{1}{2}}(J,M_J;M_L,\mu_p)\Psi(J,L,M_J), \quad (1)$$

where the C-symbol is the Clebsch-Gordan coefficient in the notation of Blatt and Weisskopf.[11] The remaining undefined symbols in Eq. (1) have their usual meaning. If it is assumed that the residual nucleus may be represented as a neutron in a definite orbital angular momentum l about the target nucleus (spin zero) as a core, the state vector for the residual nucleus is:

$$\Psi_f(\mu_f)=\sum_{m,\mu_n} C_{l,\frac{1}{2}}(j_f,\mu_f;m,\mu_n)\Psi(l,m)\chi(\tfrac{1}{2},\mu_n)\Phi_0, \quad (2)$$

where $\Psi(l,m)$ = single-particle neutron orbital, $\chi(\frac{1}{2},\mu_n)$ = neutron spin function, and Φ_0 = target nucleus (final state core) wave function. It is convenient to make a partial wave expansion of the incident deuteron wave:

$$\Phi_d(\mu_d)=\sum_{L_d,M_d}\sum_{\mu_n',\mu_p'} b^*(L_d,M_d) \times\Phi(L_d,M_d)\chi(\tfrac{1}{2},\mu_n')\chi(\tfrac{1}{2},\mu_p') \times C_{\frac{1}{2},\frac{1}{2}}(1,\mu_d;\mu_n',\mu_p'). \quad (3)$$

If one adopts the symbol T for the transition operator representing the stripping reaction, the matrix element for the reaction may be written as a linear combination of terms of the type:

$$\langle\Psi(J,L,M_J)\Psi(l,m)\chi(\tfrac{1}{2},\mu_n) \times|T|\Phi(L_d,M_d)\chi(\tfrac{1}{2},\mu_n')\chi(\tfrac{1}{2},\mu_p')\rangle. \quad (4)$$

The axis of quantization will be defined by the vector product between the incident deuteron wave vector ($\mathbf{K}$) and the final state proton wave vector ($\mathbf{k}$). With this designation of the quantization axis, the matrix element, Eq. (4), of the transition operator reduces to:

$$\langle\Psi(J,L,M_J)\Psi(l,m)\chi(\tfrac{1}{2},\mu_n)|T|\Phi(L_d,M_d)\chi(\tfrac{1}{2},\mu_n')\chi(\tfrac{1}{2},\mu_p')\rangle =C_{L,\frac{1}{2}}(J,M_J;M_L,\mu_p)\langle\Psi(J,L,M_L)\Psi(l,m) \times|T|\Phi(L_d,M_d)\rangle\delta(\mu_n,\mu_n')\delta(\mu_p,\mu_p'). \quad (5)$$

[11] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952).

In establishing the relation of Eq. (5), use has been made of the fact that with the quantization axis chosen, a nucleon produced in a definite state of spin orientation in the original stripping act will maintain this orientation after scattering in the spin-orbit potential. The stripping matrix element, $\mathfrak{M}(\mu_d\rightarrow\mu_f,\mu_p)$, may now be written via Eq. (5) and the properties of the Clebsch-Gordan coefficients:

$$\mathfrak{M}(\mu_d\rightarrow\mu_f,\mu_p) =\sum_{L_d,L,J,M_d,M_L} a(L,M_L)b^*(L_d,M_d) \times C_{L,\frac{1}{2}}^2(J,M_L+\mu_p;M_L,\mu_p) \times C_{l,\frac{1}{2}}(j_f,\mu_f;\mu_f-\mu_d+\mu_p,\mu_d-\mu_p) \times C_{\frac{1}{2},\frac{1}{2}}(1,\mu_d;\mu_d-\mu_p,\mu_p) \times\langle\Psi(J,L,M_L)\Psi(l,m)|T|\Phi(L_d,\mu_d)\rangle. \quad (6)$$

In the Born approximation treatment (neglect of proton-nucleus interaction) of the (d,p) stripping reaction, Gerjuoy[12] has shown that the reaction amplitude, f, for spinless nucleons takes the form:[13]

$$f=-\frac{2M}{\hbar^2}\frac{1}{4\pi}\int\exp(i\mathbf{k}\cdot\mathbf{r}_p)\Psi^*(\mathbf{r}_n) \times V^{np}(\mathbf{r}_n,\mathbf{r}_p)\Phi_d(\mathbf{r}_n,\mathbf{r}_p)d\mathbf{r}_n d\mathbf{r}_p, \quad (7)$$

where $\Psi(\mathbf{r}_n)$ = neutron wave function in the final (bound) state, and V^{np} = neutron-proton interaction potential. To take account of the nuclear scattering of the final (free) state proton, the usual replacement is made:

$$\exp(i\mathbf{k}\cdot\mathbf{r}_p)\chi(\tfrac{1}{2},\mu_p)\rightarrow\exp(i\mathbf{k}\cdot\mathbf{r}_p)\chi(\tfrac{1}{2},\mu_p) -\sum_{L,J} i^L[4\pi(2L+1)]^{\frac{1}{2}}C_{L,\frac{1}{2}}(J,\mu_p;0,\mu_p) \times h_L^{(1)}(kr_p)\beta(L,J)\mathfrak{Y}(L,J,M_J=\mu_p), \quad (8)$$

where $\mathfrak{Y}(L,J,M_J=\mu_p)$ = normalized eigenfunction of the total angular momentum, $h_L^{(1)}(kr_p)$ = spherical Hankel function of the first kind, and $\beta(L,J)\equiv\frac{1}{2}\eta(L,J)$, a complex number describing the distortion of the proton wave by the nuclear potential (see reference 11). To evaluate the reaction amplitude of Eq. (7) using the modification of Eq. (8), it is convenient to employ the "zero-range" $n-p$ interaction potential. The reaction amplitude conveniently separates into two parts, f^{Born} and f^{scat}, where f^{Born} is the reaction amplitude of Eq. (7) unmodified by the proton-nucleus interaction. Con-

[12] E. Gerjuoy, Phys. Rev. **91**, 645 (1953).

[13] This form of the scattering amplitude neglects the indistinguishability between the proton in the deuteron and a proton in the target nucleus during the interaction period. Antisymmetrization of the total wave function for the system with respect to the protons leads to an exchange amplitude in addition to the direct amplitude of Eq. (7). The neglect of the former with respect to the latter should be legitimate for the type of targets considered here. However, for odd-proton nuclei, the exchange amplitude may be comparable in magnitude to the direct amplitude (A. French and K. Case, private communication).

TABLE I. Proton polarization in $_6C^{12}(d,p)_6C^{13}$ for $E_d=3.29$ Mev.

θ (in degrees)	P (in percent)
0	0
10	−22
20	−14
30	−17
40	−26

stants common to both parts of the reaction amplitude will be omitted since the only characteristics of importance in the polarization calculation are their relative magnitudes and phases.

To calculate f^{Born}, the assumption of stripping theory that $\Psi(\mathbf{r}_n)$ represents a single particle orbital of prescribed orbital angular momentum will be made: i.e.,

$$\Psi^*(\mathbf{r}_n)=\text{const}\sum_m h_l^{(1)}(itr)Y_l^m(\theta_n,\phi_n),$$

where $\hbar^2t^2/2M=$ binding energy of absorbed neutron in residual nucleus, yielding

$$f_m^{\text{Born}}\sim 4\pi i^l Y_l^m(\theta_q)\int_{R_0}^{\infty} j_l(qr)h_l^{(1)}(itr)r^2dr, \quad (9)$$

where $-\mathbf{q}=\mathbf{K}-\mathbf{k}=$ wave vector of recoil (residual) nucleus, and θ_q is the angle between $\mathbf{q}$ and $\mathbf{K}\times\mathbf{k}$.

The parameter R_0 appearing in Eq. (9) is the "stripping radius" corresponding to the smallest distance of separation of the target and deuteron for the stripping mechanism to be operative. The integral of Eq. (9) may be performed analytically:

$$\int_{R_0}^{\infty} j_l(qr)h_l^{(1)}(itr)r^2dr\equiv B(R_0)$$
$$=(q^2+t^2)^{-1}\left\{\frac{d}{dR_0}[j_l(qR_0)R_0]h_l^{(1)}(itR_0)R_0 - \frac{d}{dR_0}[h_l^{(1)}(itR_0)R_0]j_l(qR_0)R_0\right\}. \quad (10)$$

In passing from the stationary to time-dependent perturbation description of the reaction process, the Born approximation to the matrix element of Eq. (6) becomes (with arbitrary normalization):

$$\mathfrak{M}^{\text{Born}}(\mu_d\rightarrow\mu_f,\,\mu_p)=C_{l,\frac{1}{2}}(j_f,\mu_f;\,m,\mu_n)C_{\frac{1}{2},\frac{1}{2}}(1,\mu_d;\,\mu_n,\mu_p)\times 4\pi i^l Y_l^m(\theta_q)B(R_0). \quad (11)$$

If it is assumed further that the incoming deuteron wave is distorted by a nonspin dependent potential, the elements of the T operator become (normalized with respect to $\mathfrak{M}^{\text{Born}}$ of Eq. (11)):

$$\langle\Psi(J,L,M_J)\Psi(l,m)|T|\Phi(L_d,M_d)\rangle$$
$$=(-1)^{M_L}\left[\frac{(2L+1)(2L_d+1)}{4\pi(2l+1)}\right]^{\frac{1}{2}}$$
$$\times C_{L,L_d}(l,0;0,0)C_{L,L_d}(l,-m;m_L,-m_L-m)$$
$$\times\int_{R_0}^{\infty}[j_L(kr)-\beta(L,J)h_L^{(1)}(kr)]$$
$$\times[j_{L_d}(Kr)-\beta(L_d)h_{L_d}^{(1)}(Kr)]h_l^{(1)}(itr)r^2dr. \quad (12)$$

Evaluation of the integral appearing in the above expression is straightforward by numerical methods.

Finally, the degree of polarization, P, of the protons produced in the reaction is defined as:

$$P\equiv\frac{\sum_{\mu_d,\mu_f}\{|\mathfrak{M}(\mu_d\rightarrow\mu_f,\,\mu_p=+\frac{1}{2})|^2-|\mathfrak{M}(\mu_d\rightarrow\mu_f,\,\mu_p=-\frac{1}{2})|^2\}}{\sum_{\mu_d,\mu_f}\{|\mathfrak{M}(\mu_d\rightarrow\mu_f,\,\mu_p=+\frac{1}{2})|^2+|\mathfrak{M}(\mu_d\rightarrow\mu_f,\,\mu_p=-\frac{1}{2})|^2\}}. \quad (13)$$

III. APPLICATION TO THE REACTION $_6C^{12}(d,p)_6C^{13}$

The above formalism has been applied to the reaction $_6C^{12}(d,p)_6C^{13}$ for a deuteron bombarding energy of 3.29 Mev. The differential cross section for this reaction has been measured by Holmgren[14] who has found a pronounced stripping maximum in the angular distribution of the protons at approximately 20°. The angular distribution obtained is consistent with the assignment of $(\frac{1}{2},-)$ to the ground state of C^{13}.

The scattering potential experienced by the final state proton was taken to be:

$$V^p(\text{in Mev})=-19(1+0.05i)-2\mathbf{L}\cdot\mathbf{S}, \quad r\leqslant r_0$$
$$=0, \quad r>r_0,$$

where $r_0=1.45\times10^{-13}A^{\frac{1}{3}}$ cm. In addition, the $L_d=0$, 1 waves of the incoming deuteron were assumed distorted by a hard sphere potential whose radius was arbitrarily chosen to be R_0, the stripping radius (in this case $R_0=6.5\times10^{-13}$ cm). The results of such a calculation are contained in Table I.

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[14] H. Holmgren, Ph.D. thesis, University of Minnesota, 1954 (unpublished).