Nucleon-Antinucleon Scattering and Vacuum Polarization

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The cross section for nucleon-antinucleon scattering is calculated by using the covariant form of the two-body equation in a simple approximation. The meson Green's function used is corrected for the first order radiative effects. The resulting S matrix is shown to be unitary. The cross section is nearly independent of the coupling constant. A graph is included.

IN the course of an examination of the vacuum polarization of the symmetrical pseudoscalar meson field, it was thought appropriate to calculate the cross section for the scattering of nucleons on antinucleons in an approximation where the polarization of the meson field plays an important role. Thus, we take the portion of the interaction operator which represents a single-quantum virtual annihilation of the nucleon-antinucleon pair.

In the notation of Karplus and Klein,[1] the two-body equation in the form appropriate to scattering problems, is

$$G^{-+}(12,34)=G^{-}(13)G^{+}(24)+G^{+}(24')G^{-}(13')I(3'4',1'2')G^{-+}(1'2',34). \quad (1)$$

The portion of the interaction operator which we retain is the point-interaction term which may be interpreted as a virtual, single-quantum annihilation:

$$I(3'4',1'2')G^{-+}(1'2',34)=-ig^2\gamma_5(\xi 3'3'')C(3''4)\mathcal{G}_+(\xi\xi')\times C^{-1}(2'4')\gamma_5(\xi'4'4'')G^{-}(4''3)G^{+}(2'4). \quad (2)$$

(We choose units such that $\hbar=c=1$ throughout.) We may note here that γ_5 contains implicitly the isotopic spin. We see that the integral equation (1) is solved when Eq. (2) is inserted. One may then simply calculate the S matrix from the two-body Green's function in the usual manner[2] which gives

$$(\lambda_1'p_1'\lambda_2'p_2'|S-1|\lambda_1p_1\lambda_2p_2)=ig^2(2\pi)^4\delta(p-p')\mathcal{G}_+(p^2)\times C_{12}C_{12}'(\bar{u}_{\lambda p1'}\gamma_5u_{\lambda p2'})(\bar{u}_{\lambda p2}\gamma_5u_{\lambda p1}) \quad (3)$$

where $p=p_1+p_2$, $p'=p_1'+p_2'$; 1, 2 symbolize the initial coordinates of the nucleon and antinucleon, $1'$, $2'$, their respective final coordinates. The $u_{\lambda p}$ are the spinors as defined by Schwinger;[3] hence λ_2 and λ_2' are negative and the $u_{\lambda p}$ also implicitly include the isotopic state of the particle. Further,

$$C_{12}=\left(\frac{(d\mathbf{p}_1)}{(2\pi)^3}\frac{(d\mathbf{p}_2)}{(2\pi)^3}\frac{M^2}{p_{01}p_{02}}\right)^{\frac{1}{2}},\quad p_0=(\mathbf{p}^2+M^2)^{\frac{1}{2}}, \quad (4)$$

is the normalization for the spinors. Finally $\mathcal{G}_+(k^2)$ is the meson Green's function given symbolically by:

$$\mathcal{G}_+(k^2)=\frac{1}{k^2+\mu^2+P(k^2)}, \quad (5)$$

where $P(k^2)$ is the vacuum polarization of the meson field. The approximate form of the S matrix thus obtained may be shown to be unitary if we can verify the identity

$$(i|(S-1)(S-1)^\dagger|i)=-2\,\mathrm{Re}(i|S-1|i), \quad (6)$$

where i symbolizes the initial state. The left side of this may be calculated by multiplying Eq. (3) by its conjugate and summing over the final states. This gives

$$(g^4/\pi)VT|\mathbf{p}|p_0|\mathcal{G}_+(-4p_0^2)|^2C_{12}^2|(\bar{u}_{\lambda p2}\gamma_5u_{\lambda p1})|^2 \quad (7)$$

in the center-of-mass system, where $p=p_1+p_2=2ip_0=2i(\mathbf{p}^2+m^2)^{\frac{1}{2}}$ is the common momentum of the particles of equal mass. This is to be compared with the right side of Eq. (6),

$$-2\,\mathrm{Re}[i\mathcal{G}_+(-4p_0^2)]g^2VT|(\bar{u}_{\lambda p2}\gamma_5u_{\lambda p1})|^2. \quad (8)$$

Equality of Eqs. (7) and (8) hinges upon

$$\mathrm{Im}\,P(-4p_0^2)=-1/(2\pi)g^2p_0|\mathbf{p}|. \quad (9)$$

One may establish Eq. (9) and complete the calculation of S by calculating the polarization P explicitly to the

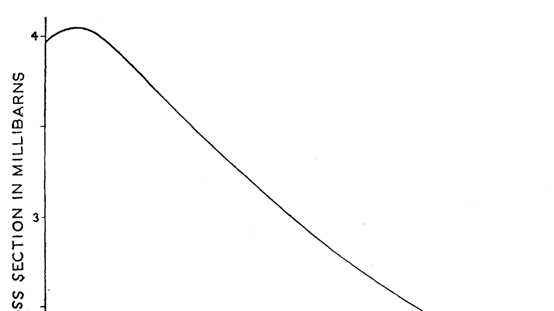

FIG. 1. Cross section as a function of the total kinetic energy of the incident particles in the center-of-mass system, $2p_0-2Mc^2=2(p^2c^2+M^2c^2)^{\frac{1}{2}}-2Mc^2$, for the $T_3=\pm1$ state. The abscissa gives the total kinetic energy in units of rest energy.

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[1] R. Karplus and A. Klein, Phys. Rev. **87**, 848 (1952).
[2] R. Karplus *et al.*, Phys. Rev. **90**, 1072 (1953).
[3] J. Schwinger, Phys. Rev. **92**, 1283 (1953).

lowest order. P is given to this order by[4]

$$(\xi|P|\xi')=-ig^2\,\mathrm{tr}(\gamma_5(\xi)G\gamma_5(\xi')G). \tag{10}$$

This expression can be easily cast into the symbolic form,

$$P(k^2)=4i\frac{g^2}{(2\pi)^4}\times\int(dp)\frac{p^2-\frac{1}{4}k^2+M^2}{[(p-\frac{1}{2}k)^2+M^2][(p+\frac{1}{2}k)^2+M^2]}, \tag{11}$$

where k is the meson momentum. The main effect of the polarization P is to alter the meson mass μ, and the strength of the coupling constant g^2. After these renormalizations have been carried out we are left with the finite portion of P which describes the nonlocal properties of the meson field:

$$P^{(1)}(k^2)=\frac{g^2}{4\pi^2}\left(\frac{2M}{k\mu}\right)^4 k^2(k^2+\mu^2)^2\times\int_0^1 dv\frac{v^2}{[v^2-(2M/k)^2-1][v^2+(2M/\mu)^2-1]^2}. \tag{12}$$

In deriving Eq. (12) from Eq. (11), use was made of the identity

$$\frac{1}{ab}=\int_{-1}^{+1}\tfrac{1}{2}dv\int_0^\infty -sds\times\exp\{-is[\tfrac{1}{2}(1-v)a+\tfrac{1}{2}(1+v)b]\}, \tag{13}$$

and the renormalization scheme as outlined by Karplus and Klein.[1] Explicit calculation of Eq. (12) yields[5]

$$\begin{aligned}P^{(1)}(k^2)=\frac{g^2}{4\pi}\frac{1}{2\pi}\Bigg\{&k^2\left[1+\left(\frac{2M}{k}\right)^2\right]^{\frac{1}{2}}\\&\times\left(\log\frac{1-[1+(2M/k)^2]^{\frac{1}{2}}}{1+[1+(2M/k)^2]^{\frac{1}{2}}}+\pi i\right)\\&+(k^2-\mu^2)[(2M/\mu)^2-1]^{\frac{1}{2}}\\&\times\tan^{-1}\{[(2M/\mu)^2-1]^{-\frac{1}{2}}\}\\&+(k^2+\mu^2)[1-[(2M/\mu)^2-1]^{-\frac{1}{2}}\\&\times\tan^{-1}\{[(2M/\mu)^2-1]^{-\frac{1}{2}}\}]\Bigg\}.\end{aligned} \tag{14}$$

[4] J. Schwinger, Proc. Natl. Acad. Sci. U. S. **37**, 452, 455 (1951).

[5] K. M. Watson and J. V. Lepore, Phys. Rev. **76**, 1157 (1949). Their results do not agree with our Eq. (12). The error lies in the renormalization of Eq. (8) of Watson and Lepore's paper, which is our Eq. (11).

One may now quickly establish the truth of Eq. (9), and hence the unitarity of the S matrix.

We can now calculate the cross section from Eq. (7) by dividing by the reaction volume VT, by the relative velocity of the particles in the center-of-mass system $2p/p_0$, and by the reciprocal of the density of particles in the initial state C_{12}^2. Thus

$$\sigma=\left(\frac{g^2}{4\pi}\right)^2\frac{\pi/2M^2}{|1-(\mu/2p_0)^2+[P^{(1)}(-4p_0^2)/-4p_0^2]|^2}. \tag{15}$$

Reduction of the spinor factors then yields

$$\sigma=\left(\frac{g^2}{4\pi}\right)^2\frac{\pi/2M^2}{|1-(\mu/2p_0)^2+[P^{(1)}(-4p_0^2)/(-4p_0^2)]|^2}\times\delta_{0i}\begin{Bmatrix}1\\2\end{Bmatrix}, \tag{16}$$

where $i=0$ for the singlet state, $i=1$ for the triplet state. The fact that $\sigma^2=0$ for the triplet state is a simple consequence of the null spin of the meson field, and the fact that the only interaction considered is the virtual annihilation of the pair of spin $\frac{1}{2}$ particles. The factor $\begin{Bmatrix}1\\2\end{Bmatrix}$ results from an evaluation of the isotopic spin operators, the upper term for the $T_3=\pm1$ state, the lower term for the $T_3=0$ state, where T is the isotopic spin of the pair. For calculational purposes we may neglect $(\mu/2M)^2$ compared to 1, and

$$\sigma\sim\left\{\frac{\pi}{2M^2}\Big/\left\{\left[\frac{4\pi}{g^2}+\frac{1}{2\pi}\left(\frac{|\mathbf{p}|}{p_0}\log\frac{p_0-|\mathbf{p}|}{p_0+|\mathbf{p}|}+2\right)\right]^2+\left(\frac{|\mathbf{p}|}{2p_0}\right)^2\right\}\right\}\delta_{0i}\begin{Bmatrix}1\\2\end{Bmatrix}. \tag{17}$$

We see that σ is nearly independent of $g^2/4\pi$ in the usual range of values. We have chosen $g^2/4\pi=10$ for the graph.

We have shown that, as in the work of Karplus, Kivelson, and Martin,[2] a unitary approximation to the S matrix may be achieved if one uses a Green's function, corrected for the radiative effects to the same order to which the interaction operator is taken. The damping gives a result nearly independent of the coupling constant.

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