P-wave phase shifts seem to be very small.[20] The fourth order phase shifts in Chew's theory give some indication that this sort of behavior might be predicted by a more complete calculation, the fourth order corrections reinforcing the second order phase shift for δ_{33} and detracting from it for δ_{11} and $\delta_{13}=\delta_{31}$. And, indeed, a TD calculation by Chew[19] has succeeded in reproducing the experimental phase shifts as far as they are known. The results of the calculations for the relativistic theory given in Eq. (52) are not nearly so encouraging. For the relativistic theory the fourth order correction to δ_{33} is very very small. If we invoke the damping of the pair effects again, the phase shifts in the relativistic theory will reduce essentially to those given by Chew's theory, and things will look more encouraging.

The chances both for the success of the TD method as applied to the pseudoscalar coupling theory and for the success of this theory in agreeing with experiment would be considerably improved if it could be unambiguously shown that the effects due to pair production are damped out by higher order radiative corrections. Unfortunately, this has not been done as yet.

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Renormalization of a Neutral Vector Meson Interaction

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A proof is given that the vector interaction of neutral vector mesons and fermions can be renormalized, to replace a fallacious proof by Matthews. A new interaction representation, formally different from the usual one, is used. The basic idea, not a new one, is that the objectionable part of the interaction can be identified with a certain scalar meson interaction, known to be illusory.

1. INTRODUCTION

IN general, vector and pseudovector meson theories cannot be renormalized by present-day methods, because derivatives of the Δ function in the free-field commutation rules give rise to an unlimited number of primitive divergent graphs in S-matrix calculations.[1] One exception to this rule is the vector interaction of neutral vector mesons with fermions. Matthews[2] has given a fallacious proof for this, using the formulation in the interaction representation of Miyamoto[3] and Stückelberg.[4]

We prove renormalizability by transforming to a new interaction representation, formally different from the usual one. The proof is based essentially upon identifying the objectionable part of the interaction with a certain scalar meson interaction, which is known to be illusory, and to this extent our work is simply a refinement of that of Matthews.

The fallacy in Matthews' work lay in the mathematical tools he used. He had previously proposed[5] a generalization of the Tomonaga-Schwinger theory to the Schrödinger representation,

$$i\hbar\delta\Psi_s[\sigma]/\delta\sigma(x)=H_s(\sigma,x)\Psi_s[\sigma] \qquad (1.1)$$

starting from a generalized form of the Heisenberg field equations,

$$i\hbar\delta\phi(x)/\delta\sigma(x')=[\phi(x), H(x)']. \qquad (1.2)$$

This generalization he assumed in his proof of renormalizability. However, it has been shown by the author (in an unpublished work) that Eq. (1.2) is not integrable in cases of physical interest, and that a direct transformation $U[\sigma]$ from the Heisenberg representation to this generalized Schrödinger representation cannot be found, since it would be defined by

$$i\hbar\delta U[\sigma]/\delta\sigma(x)=U[\sigma]H(x), \qquad (1.3)$$

(in the Heisenberg representation) which is not integrable either. Nor can an analogous transformation be found from the interaction representation. Thus, even if such a generalized formulation can be made in the Schrödinger representation in a self-consistent way, it cannot be equivalent to the usual theory.

2. STUCKELBERG'S FORMULATION

Stückelberg[4] has shown how to express the Proca field $\phi_\mu(x)$, with rest mass $\hbar\kappa$, in terms of a vector field $A_\mu(x)$ and a scalar field $C(x)$, using the Lagrangian,

$$L=-\tfrac{1}{2}(A_{\mu\nu}A_{\mu\nu}+\kappa^2A_\mu A_\mu+C_\nu C_\nu+\kappa^2C^2)+L', \qquad (2.1)$$

where L' describes any other fields, and their interactions with the A_μ and C fields. C and the four components A_μ are quantized in the usual way as inde-

[1] F. J. Dyson, Phys. Rev. **75**, 1736 (1949).
[2] P. T. Matthews, Phys. Rev. **76**, 1254 (1949).
[3] Y. Miyamoto, Progr. Theoret. Phys. **3**, 124 (1948).
[4] E. C. G. Stückelberg, Helv. Phys. Acta **11**, 299 (1938).
[5] P. T. Matthews, Phys. Rev. **75**, 1270 (1949).

pendent operators. ϕ_μ can now be expressed, in the Heisenberg representation, by

$$\phi_\mu(x)=A_\mu(x)+(1/\kappa)C_\mu(x), \tag{2.2}$$

if we also postulate two auxiliary conditions,

$$A_{\mu\mu}(x)+\kappa C(x)=0, \tag{2.3}$$

and its time-derivative,

$$[\int H(x')d^3x', A_{\mu\mu}(x)+\kappa C(x)]=0, \tag{2.4}$$

which are self-consistent. Of course, if the required Proca field interaction is such that $\phi_{\mu\mu}\neq 0$, Eqs. (2.3) and (2.4) are modified.

For vector interaction with fermions, the interaction Hamiltonian in a single-time interaction representation proves to be

$$H_1(x)=j_\mu(x)[A_\mu(x)+1/\kappa C_\mu(x)]+1/2\kappa^2[j_0(x)]^2, \tag{2.5}$$

where $j_\mu(x)=\frac{1}{2}ig[\bar{\psi}(x)\gamma_\mu\psi(x)-\bar{\psi}'(x)\gamma_\mu\psi'(x)]$ in the usual notation, g being a coupling constant. Miyamoto[3] considered a modification of the Stückelberg formalism, taking the auxiliary conditions (2.3) and (2.4) to be conditions on the Heisenberg state-vector Φ, and leaving C and the four components A_μ independent. He observed that if $j_0(x)$ in Eq. (2.5) is replaced by $j_\mu(x)n_\mu(x)$, where n_μ is the normal at x to a space-like surface σ, pointing to the future, the equation,

$$i\hbar\delta\Psi[\sigma]/\delta\sigma(x)=H_1(\sigma,x)\Psi[\sigma], \tag{2.6}$$

is integrable. Hence the theory can be stated in the Tomonaga-Schwinger formalism in the interaction representation.

It is not immediately obvious that the latter formulation is wholly equivalent to the canonical formulation in this case. For, when a transformation is made from the Heisenberg representation,

$$\Psi[\sigma]=V[\sigma]\Phi, \tag{2.7}$$

$$i\hbar\delta V[\sigma]/\delta\sigma(x)=V[\sigma]\mathcal{H}(x), \tag{2.8}$$

where $\mathcal{H}(x)$ contains field momenta, the transformed field variables are not simple point-functions in general, but depend on the orientation of the surface.[6] Miyamoto assumes no such surface-dependence in his interaction representation. However, since $A_\mu(x)$ and $C(x)$ prove to be surface-independent in this sense, and no use is made of the momentum conjugate to $C(x)$ or any other surface-dependent variables, the two formulations are in effect equivalent.

[6] F. J. Belinfante, Phys. Rev. **76**, 66 (1949).

3. QUASI-INTERACTION REPRESENTATION

We consider the transformation $W[\sigma]$ defined by the integrable equation,

$$i\hbar\delta W[\sigma]/\delta\sigma(x)=W[\sigma]A_\mu(x)j_\mu(x), \tag{3.1}$$

in the Heisenberg representation, and transform to a quasi-interaction representation (denoted by an asterisk),

$$\Psi^*[\sigma]=W[\sigma]\Phi, \tag{3.2}$$

$$i\hbar\delta\Psi^*[\sigma]/\delta\sigma(x)=A_\mu{}^*(x)j_\mu{}^*(x)\Psi^*[\sigma]. \tag{3.3}$$

It can easily be shown, by the methods of Schwinger[6,7] that this is effectively an interaction representation, for $A_\mu{}^*(x)$ obeys its free-field equations, and $\bar{\psi}^*(x)$, $\psi^*(x)$, $C^*(x)$ behave as if their free fields are coupled by a term $-(1/\kappa)j_\mu(x)C_\mu(x)$ in the Lagrangian. It is well known that the latter coupling is quite illusory,[8] and that a transformation $\Psi'=e^{iS}\Psi$ [where $S=(\hbar\kappa)^{-1}\int j_0(x)C(x)d^3x$], in the Schrödinger representation, removes the interaction altogether. Carrying out the corresponding transformation in the Heisenberg representation, we see that $\bar{\psi}^*(x)$, $\psi^*(x)$, $C^*(x)$ have free-field (anti-) commutation rules.

The S-matrix in this quasi-interaction representation closely resembles that for electrodynamics,[1] and can clearly be renormalized by the same methods. The primitive divergent graphs are the same in both cases, but the graphs corresponding to photon self-energy parts here lead to genuine divergences. Ward's[9] formal proof that photon-photon scattering graphs lead to no divergences can be applied to the corresponding graphs in this theory too. It is possible consistently to account for all divergences in terms of renormalization of the meson and fermion masses, and of the coupling constant, by means of Dyson's technique.[1] κ-renormalization must of course affect the C-field too, for consistency.

The auxiliary conditions on the state vector can be transformed to this new representation without causing further difficulties.

The methods of this section fail for charged mesons, and pseudovector mesons,[8] since the coupling terms which correspond to $-(1/\kappa)j_\mu(x)C_\mu(x)$ represent genuine interactions.

In conclusion, my thanks are due to Professor P. A. M. Dirac for stimulating discussions.

[7] J. Schwinger, Phys. Rev. **74**, 1439 (1948).
[8] F. J. Dyson, Phys. Rev. **73**, 929 (1948). Matthews has pointed out an error in this paper: there should be no residual $[j_0(x)]^2$ term after the transformation.
[9] J. C. Ward, Phys. Rev. **77**, 293 (1950).