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Tamm-Dancoff Methods and Nuclear Forces*†

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It is shown that by the use of 3-dimensional Feynman diagrams an equal-time two-particle wave equation can be written down. The procedure is the same one as Bethe and Salpeter used in deriving the 4-dimensional two-body equation. In this way all the terms of the new Tamm-Dancoff method of Dyson, in the form of the Lévy-Klein expansion of the interaction function, can easily be recorded.

The connection between the Bethe-Salpeter and Dyson equations is discussed. The spurious energy denominators of the fourth order interaction, in the adiabatic limit, can be eliminated by including all the contributions to the fourth order term arising from the 4- and 5-particle amplitudes that are coupled back to the two-particle amplitudes. The potential derived in the adiabatic limit to the fourth order term of the interaction operator is the same as the Lévy-Klein potential derived from the old Tamm-Dancoff method.

The Appendix contains a discussion of the Bethe-Salpeter equation and a simple method of derivation of equal-time equations from the 4-dimensional theories.

I. INTRODUCTION

THE field-theoretical investigations of the nature of nuclear forces received great impetus as a result of Lévy's[1] extension of the Tamm-Dancoff method to include nucleon pair effects in the intermediate states and higher order meson-exchange processes. The theory as used by Lévy was not fully relativistic. This had the consequence that renormalization was, in effect, not possible within his formulation of the theory. Furthermore, the use of the non-interacting vacuum state, in the definition of the Tamm-Dancoff amplitudes, introduced additional difficulties connected with the vacuum self-energy. It has been pointed out by Gell-Mann and Low,[2] and more recently by Dyson,[3] that most of the difficulties encountered in the old Tamm-Dancoff method can be avoided by defining the amplitudes with respect to the interacting vacuum state.

The observation that no vacuum self-energy appears when the amplitudes are defined with respect to the interacting vacuum state was, essentially, the starting point of Dyson[4] in his formulation of the new Tamm-Dancoff method.

In Dyson's formulation of the new Tamm-Dancoff method, just as in the 4-dimensional Bethe-Salpeter formalism, the wave function contains positive as well as negative energy parts. Actually, the statement of "negative energy" in the Bethe-Salpeter formalism has quite a different meaning: it is the energy of the "pair" which is positive. In Dyson's equal-time formalism pairs are mixed with the negative energies, which is a consequence of dealing with the equal-time amplitudes right at the starting point of the theory.

In the adiabatic limit the weak-coupling forms of the new and old Tamm-Dancoff methods do not differ substantially; the adiabatic limit of the new Tamm-Dancoff method is understood to include the statement that all the components of the two-particle wave function not referring to positive energy states of the particles vanish. It is hard to justify such an approximation, but the rejection of those components of the wave function referring to negative energies seems to be a necessity. It is known that there are some difficulties involved in the use of the interacting vacuum state as a boundary condition, but with the aforementioned approximation the vacuum difficulty does not arise. The vanishing of ψ_{+-}, ψ_{-+}, and ψ_{--} components of the wave function is not meant to be connected with the fact that the vacuum should be the state of lowest energy. The cancellation of the so-called spurious divergences in the second and fourth order interactions takes place only in the adiabatic limit. In the nonadiabatic form of the interactions they are not cancelled.

In this paper we have extended Dyson's formulation of the new Tamm-Dancoff method to include higher order amplitudes. In principle, it corresponds to Lévy's method of including the effects of the higher order amplitudes on the lower ones, as applied to the new Tamm-Dancoff method. Theoretically one considers an infinite number of linear integral equations for all the particle amplitudes. The use of the infinite set of equations in the elimination of all the Tamm-Dancoff amplitudes, except the amplitude for the two-particle state, if carried to completion, leads to a linear integral equation for the two-particle wave function.

We have carried out this procedure to the fourth order in the coupling constant G by assuming that the amplitudes of all states involving 5 or more particles should vanish. In Sec. II the most general form of the two-nucleon equation is first written down from the analogies using the methods of Bethe-Salpeter[5] and Lévy-Klein[6] in the derivation of the 4-dimensional and

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[1] M. M. Lévy, Phys. Rev. 88, 72 and 725 (1952).
[2] M. Gell-Mann and F. Low, Phys. Rev. 84, 350 (1951).
[3] F. J. Dyson, Phys. Rev. 90, 994 (1953).
[4] F. J. Dyson, Phys. Rev. 91, 1543 (1953).
[5] E. E. Salpeter and H. A. Bethe, Phys. Rev. 84, 1232 (1951).
[6] A. Klein, Phys. Rev. 90, 1101 (1953).

the old Tamm-Dancoff equations, respectively. The relation between the Dyson and Bethe-Salpeter wave functions is discussed briefly. Section III contains a derivation of the fourth order interaction operator together with its adiabatic limit leading to the derivation of the fourth order potential. The Tamm-Dancoff equations are given in Appendix A, and in Appendix B, we give a simple method of derivation of the old Tamm-Dancoff equation for two nucleons from the Bethe-Salpeter formalism.

II. THE SECOND ORDER INTERACTION

In this section we discuss the most general form of a 3-dimensional covariant equation for two nucleons. For this purpose it is convenient to represent the kernel of the equation in terms of 3-dimensional Feynman diagrams. In such a diagram a nucleon line will represent (initial and final states included) both positive and negative energy states. A second order graph, e.g., can be considered as the sum of 4 sub-graphs consisting of minus and plus particle states. There will be a single time-co-ordinate for any number of nucleon lines. If one of the nucleons emits a meson to be absorbed by the other nucleon or by itself, then, in the intermediate state, the energy of the emitted or absorbed meson is to be added to the energy of that nucleon with which it later interacts. The resulting expression can assume both positive and negative signs, according as they are plus or minus particles, respectively. By using this prescription we can write down all the terms of the interaction operator in the form of an infinite series in powers of the coupling constant G. We have verified the above rule by actual calculation with the new Tamm-Dancoff equations given in the Appendix A.

The two-nucleon equation used in this paper can be written as

$$[H_a(\mathbf{p})+H_b(-\mathbf{p})-E]\psi(\mathbf{p}) = -\int I(\mathbf{p},\mathbf{p}';E)\psi(\mathbf{p}')d^3p', \quad \text{(II.1)}$$

where $H(p)=\boldsymbol{\alpha}\cdot\mathbf{p}+\beta M$, E is the total energy in the center-of-mass system, and $\psi(\mathbf{p})$ is the two-nucleon component of the Tamm-Dancoff wave functions. The word wave function is not used in the strict sense of a probability amplitude.

The first term $I_2(\mathbf{p},\mathbf{p}',E)$ of the interaction operator $I(\mathbf{p},\mathbf{p}';E)$, which was already given by Dyson, consists of the four diagrams shown in Fig. 1. The first diagram

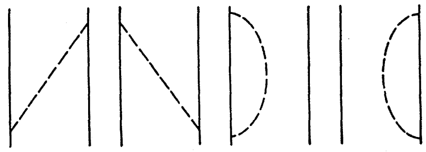


FIG. 1. Three-dimensional Feynman diagrams contributing to the second-order interaction of two nucleons.

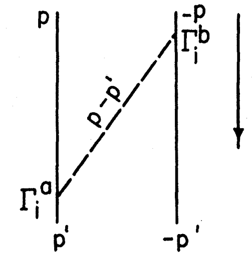


FIG. 2. Labels used in writing the matrix element for the first diagram of Fig. 1. The arrow indicates the time direction of the interaction.

(which is repeated with labels in Fig. 2) represents the matrix element:

$$\frac{G^2}{(2\pi)^3}\frac{1}{2\omega_{p-p'}}\Gamma_i^a[\eta_a(\mathbf{p})(E_p+\omega_{p-p'})+\eta_b(-\mathbf{p}')E_p'-E]^{-1}\Gamma_i^a,$$

where

$$\Gamma_i^a=(i\beta\gamma_5)_a\tau_i^a, \quad \eta_a(\mathbf{p})=H_a(p)/E_p.$$

This matrix element can easily be understood in terms of the prescription outlined at the beginning of this section.

We have not succeeded in deriving Eq. (II.1) from the Bethe-Salpeter equation. However, we shall now discuss its relation to the Bethe-Salpeter theory. In Dyson's theory, for a state Ψ of one proton and one neutron in interaction, we singled out the 3-dimensional two-particle wave function $\psi(\mathbf{p})$ satisfying Eq. (II.1). The function $\psi(\mathbf{p})$ is the lowest component of the Tamm-Dancoff wave functions. The 4-dimensional wave function $\chi(p_\mu)$ satisfies the Bethe-Salpeter equation. In Bethe-Salpeter theory one also defines an equal-time wave function $\varphi(\mathbf{p})$ by

$$\varphi(\mathbf{p})=\int\chi(\mathbf{p},p_0)dp_0. \quad \text{(II.2)}$$

If any relation exists between the Dyson and the Bethe-Salpeter theories it should be one that relates the functions $\psi(\mathbf{p})$ and $\varphi(\mathbf{p})$. Originally the function $\varphi(\mathbf{p})$ was not defined as a Tamm-Dancoff wave function, but it is reasonable to hope for the possibility of deriving the Bethe-Salpeter equation from a 4-dimensional formalism analogous to the Tamm-Dancoff method. In this case it is natural to expect a correlation between the functions $\psi(\mathbf{p})$ and $\varphi(\mathbf{p})$. We have been able to verify that $\psi(\mathbf{p})$ and $\varphi(\mathbf{p})$ satisfy the same equation if only one fermion's second order interaction is taken into account. In the fourth order we obtain different equations. For the two-particle case we do not know whether further transformations of the interaction and the equations would bring the two expressions into agreement. The similarity is, certainly, not apparent in the present form of Dyson's theory. The investigations were made for both one- and two-nucleon systems. Especially in the former case, the result may be considerably modified by renormalization which is not obvious for the 3-dimensional equation.

To show the connection between $\psi(\mathbf{p})$ and $\varphi(\mathbf{p})$, let us first consider the one-particle Bethe-Salpeter equation for a particle interacting with itself, which in momentum space is

$$\chi(p_\mu)=-\frac{G^2}{(2\pi)^4}S_F(p_\mu)\int\gamma_5\tau_i S_F(p_\mu-p_\mu')\times\gamma_5\tau_i\Delta_F(p_\mu')d^4p'\chi(p_\mu)$$

$$=\frac{G^2}{(2\pi)^4}\frac{1}{H(\mathbf{p})-p_0}\int\Gamma_i\frac{1}{H(\mathbf{p}-\mathbf{p}')-p_0+p_0'}\times\Gamma_i\frac{d^4p'\chi(p_\mu)}{(\omega_{p'}-p_0')(\omega_{p'}+p_0')}. \qquad \text{(II.3)}$$

Now, a one-particle, 4-dimensional wave function in momentum space can be defined by

$$\chi(p_\mu)=\chi(\mathbf{p})\delta(p_0-E), \qquad \text{(II.4)}$$

where E is the energy of a proton of momentum $\mathbf{p}$, including self-energy. On substituting (II.4) in (II.3) and integrating both sides of the resulting equation with respect to p_0, we get

$$[H(\mathbf{p})-E]\chi(\mathbf{p})=\frac{G^2}{8\pi^3}\int\frac{d^3p'}{2\omega_{p'}}\times\Gamma_i[\eta(\mathbf{p}-\mathbf{p}')(E_{p-p'}+\omega_{p'})-E]^{-1}\Gamma_i\chi(\mathbf{p}). \qquad \text{(II.5)}$$

This equation is identical with the one Dyson derived from his new Tamm-Dancoff method.

The two-particle Bethe-Salpeter wave function is not separable according to (II.4) and, therefore, the above method cannot be carried through. It is of interest, however, to record the result that can be obtained for the second order interaction. The Bethe-Salpeter equation for the one-meson interaction of two nucleons in the centimeter system is given by

$$\chi(p_\mu)=\frac{G^2}{(2\pi)^4}S_{Fa}(\tfrac{1}{2}K_\mu+p_\mu)S_{Fb}(\tfrac{1}{2}K_\mu-p_\mu)\beta_a\beta_b\Gamma_i^a\Gamma_i^b\times\int\Delta_F(p_\mu-p_\mu')\chi(p_\mu')d^4p'. \qquad \text{(II.6)}$$

The integration of both sides of (II.6) over the relative energy variable p_0 can be effected in accordance with the hole theory (see Appendix B), giving the result

$$[H_a(\mathbf{p})+H_b(-\mathbf{p})-E]\varphi(\mathbf{p})=\frac{G^2}{8\pi^3}\int\frac{d^3p'dp_0'}{2\omega_{p-p'}}\{\Gamma_i^b[\eta_a(\mathbf{p})(E_p+\omega_{p-p'})-\tfrac{1}{2}E-p_0']^{-1}\Gamma_i^a+\Gamma_i^a[\eta_b(-\mathbf{p})(E_p+\omega_{p-p'})-\tfrac{1}{2}E+p_0']^{-1}\Gamma_i^b\}\chi(\mathbf{p}'p_0'). \qquad \text{(II.7)}$$

It was not possible to find a suitable ansatz for the function $\chi(\mathbf{p}',p_0')$ on the right-hand side of the equation, so the integration over p_0' had to be left as it is. The comparison with Dyson's theory shows some similarities, but they disappear in the fourth order where some curious energy denominators of the form $(E_p+E_{p'}+\omega_{p-p'})^{-1}$ show up which are independent of p_0'. The energy denominators of that form also arose in the derivation[6] of the old Tamm-Dancoff two-nucleon equations[7] from the Bethe-Salpeter theory and were among the differences between the original Tamm-Dancoff method used by Lévy and the one derived from the Bethe-Salpeter equation. There is, however, one unexpected result that can be obtained from (II.7). If we use the ansatz (5) of Appendix A with the total energy E having a small negative imaginary part, in Eq. (II.7) we obtain, after the p_0' integration, Dyson's second order equation for two-nucleons. This is a mixture of hole and one-particle theories. It is not possible to derive a general conclusion from this result, but it certainly is not accidental.[8]

A complete correspondence, to the second order, between the positive energy parts of the wave functions $\psi(\mathbf{p})$ and $\varphi(\mathbf{p})$ can be established if we assume that (i) the p_0-dependence of the Bethe-Salpeter wave function has the form (see Appendix B)

$$\chi(\mathbf{p},p_0')=\frac{1}{2\pi i}[S_{Fa}(\tfrac{1}{2}K_\mu+p_\mu)\beta_a+S_{Fb}(\tfrac{1}{2}K_\mu-p_\mu)\beta_b]\Phi(\mathbf{p});$$

(ii) all components of the Dyson wave function vanish which do not refer to positive energy states.

Under these assumptions one easily obtains the relation

$$\psi_{++}(\mathbf{p})=\Phi_{++}(\mathbf{p}),$$

where the function $\psi_{++}(\mathbf{p})$ satisfies the old Tamm-Dancoff equation of second order. The approximation (ii) eliminates, to second order, the complications related to the vacuum being used as a boundary condition. For higher order terms of the interaction operator, the condition (ii) is not enough to eliminate the spurious energy denominators, but in the adiabatic limit to the interaction it is a sufficient condition. The last statement has been verified only for the fourth order interaction. A general method of elimination of the spurious denominators of the two-nucleon equation is not known; if this can be done consistently, then one expects to get the results of the old Tamm-Dancoff except for small deviations.

III. THE FOURTH ORDER INTERACTION

It has been observed by Bethe, on the basis of the perturbation theory, that the fourth order interaction

[7] A simpler derivation is given in Appendix A.

[8] We were not able to explain this curious result, but one thing is clear: it does not arise in the derivation of the old Tamm-Dancoff equation from the Bethe-Salpeter equation.

term needs to be included in the $ps-ps$ theory. There are, at present, no definite conclusions concerning the behavior of the whole series of interactions. The convergence of the adiabatic nuclear potential, without renormalization terms, has been discussed by Klein.[9] In the following we discuss the fourth order term in the interaction operator of Eq. (II.1). The aim of this investigation is to see in what way the qualitative features of the γ_5-interaction are altered in the fourth order, by a 3-dimensional covariant theory. The fourth order terms which are radiative corrections to the second order interactions will not be included. There exist about 30 of these corrections that may contribute significantly, even in the adiabatic limit. The 12 no-pair terms of the old Tamm-Dancoff method, with a different interpretation, constitute part of the interaction kernel. In the new Tamm-Dancoff method only 12 of the 24 one-pair terms of the old Tamm-Dancoff method appear and the 12 two-pair terms of the old method do not arise; this is a consequence of the use of the interacting vacuum state and the covariance of the theory. In this case the creation of three particles out of the vacuum, proceeding in the same time direction at a given vertex, is not allowed. This is the reason for the non-appearance of the aforementioned terms in

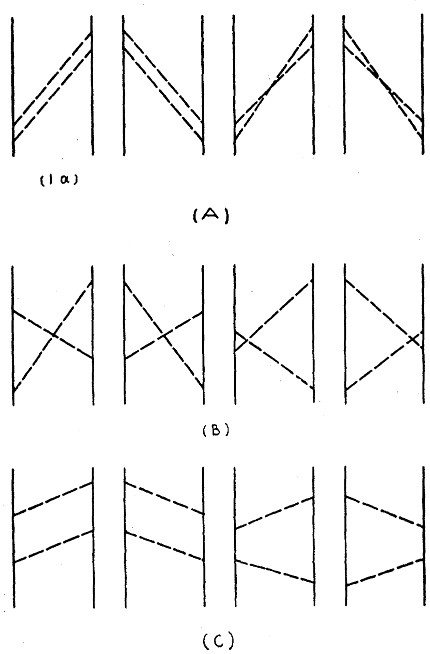
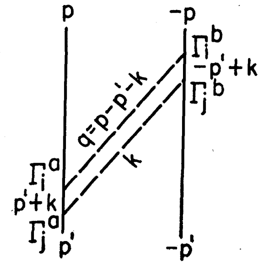


FIG. 3. The set (A) of fourth order diagrams contain the main spurious divergences in Dyson's new Tamm-Dancoff method. The sets (B) and (C) do not contain terms proportional to $(2M)^{-2}$. Because of the time ordering of the interaction, all the diagrams in the Tamm-Dancoff method can be divided into pairs. Of the above 12 interaction terms one need only calculate the 6 corresponding matrix elements. The remaining six can be obtained by a reflection of time. As an example, we give in Eq. (III.1) the matrix element that corresponds to the diagram labeled (1a).

[9] A. Klein, Phys. Rev. 92, 1017 (1953).

FIG. 4. Labels used in writing the matrix element [Eq. (III.1)] for diagram (1a) of Fig. 3.



the new Tamm-Dancoff method and precisely this has been verified by actual calculation. Altogether one is left with 24 interaction terms. A detailed discussion of these points for the one-particle case is given by Klein[9], and therefore they will not be elaborated in this paper.

First, we shall consider the 8 irreducible interactions and those that follow from the iteration of the second order interaction [Figs. 3 (A), (B), and (C)]. Of the set shown in Fig. 3(C) we shall include only those contributions arising from the pair processes in the intermediate states. The argument for the contribution from positive-energy intermediate states is the same as given by Klein.[6]

The prescription given in Sec. II is not complete for writing down the matrix elements corresponding to the set shown in Fig. 3 (C). It has to be amended for those interactions which contain one or two mesons in the intermediate states and also for those in which one of the nucleon lines is bent. In this case we can state the rules for writing down the interaction terms as follows:

(i) If the emission (absorption) of a meson is the earliest event in the diagram, then the latest absorbed (emitted) meson is either in the state Ψ or in the state Ψ_0 and is absorbed (emitted) by the second (first) nucleon in a positive (negative) energy state, respectively.

(ii) The energy of the remaining meson is to be added to that nucleon energy with which it interacts later. The sum can take both positive and negative signs. In the above, for convenience, we assume that the mesons are always emitted and absorbed by the first and second nucleon, respectively.

As an example we give the matrix element corresponding to the diagram (1a) of Fig. 3(A) (see also Fig. 4):

$$\left[\frac{G^2}{8\pi^3}\right]^2 \int \frac{d^3k}{2\omega_k 2\omega_{p-p'-k}} \sum_{\pm} \Gamma_i{}^b [E-\eta_a(\mathbf{p})(E_p+\omega_q) -\eta_b(-\mathbf{p}'-\mathbf{k})E_{p'+k}]^{-1}\Gamma_j{}^b[E-\eta_a(\mathbf{p})(E_p+\omega_q) -\eta_b(-\mathbf{p}')E_{p'}\pm\omega_k]^{-1}\Gamma_i{}^a[E-\eta_b(-\mathbf{p}')E_{p'} \pm(E_{p'+k}+\omega_k)]^{-1}\Lambda_{\mp}{}^a(\mathbf{p}'+\mathbf{k})\Gamma_j{}^a. \quad \text{(III.1)}$$

In calculating the adiabatic limit we shall assume, as

mentioned in (II), that the components $\psi_{+-}(\mathbf{p})$, $\psi_{-+}(\mathbf{p})$, $\psi_{--}(\mathbf{p})$ of the wave function $\psi(\mathbf{p})$ vanish. Thus, the unit operators $\eta_a(\mathbf{p})$, $\eta_b(-\mathbf{p})$, $\eta_a(\mathbf{p}')$ and $\eta_b(-\mathbf{p}')$ in the matrix elements, with the appropriate projection operators, will be replaced by their $+1$ eigenvalues. This corresponds to the fact that the nucleons in the initial and final states are in positive energy states. The remaining η's can take both $+1$ and -1 eigenvalues. In this way each graph corresponds to 4 terms according as the two remaining η's in the interaction function take the values $+1$ or -1. In connection with the calculation of the matrix elements of the sets (A), (B), and (C) of Fig. 3 with respect to positive-energy free-particle Dirac wave functions, we use the relations

$$\eta(\mathbf{p})\Lambda_+(\mathbf{p})=\eta(\mathbf{p})[\tfrac{1}{2}+\tfrac{1}{2}\eta(\mathbf{p})]=\Lambda_+(\mathbf{p}),$$
$$\eta(\mathbf{p})\Lambda_-(\mathbf{p})=\eta(\mathbf{p})[\tfrac{1}{2}-\tfrac{1}{2}\eta(\mathbf{p})]=-\Lambda_-(\mathbf{p}),$$

and the identity

$$\begin{aligned}[\Lambda+(\mathbf{p})+\Lambda_-(\mathbf{p})]_a[\Lambda_+(-\mathbf{q})+\Lambda_-(-\mathbf{q})]_b &=\Lambda_+{}^a(\mathbf{p})\Lambda_+{}^b(-\mathbf{q})+\Lambda_+{}^a(\mathbf{p})\Lambda_-{}^b(-\mathbf{q})\\ &\quad+\Lambda_-{}^a(\mathbf{p})\Lambda_+{}^b(-\mathbf{q})+\Lambda_-{}^a(\mathbf{p})\Lambda_-{}^b(-\mathbf{q})=1.\end{aligned}$$

The matrix element of the integrand of (III.1) in the adiabatic limit is given by

$$\begin{aligned}&\langle p,-p|I_4{}^{(1)}(\mathbf{p},\mathbf{p}',E)|\mathbf{p}-\mathbf{q}-\mathbf{k},-\mathbf{p}+\mathbf{q}+\mathbf{k}\rangle\\ &=(3-2\boldsymbol{\tau}\cdot\boldsymbol{\tau}_b)\left\{\frac{1}{(2M)^2}\frac{1}{4\omega_k\omega_q(\omega_k-\omega_q)}+\frac{1}{(2M)^3}\left[\frac{(\boldsymbol{\sigma}\cdot\mathbf{q}\boldsymbol{\sigma}\cdot\mathbf{k})_b}{4\omega_k\omega_q{}^2(\omega_k-\omega_q)}-\frac{(\boldsymbol{\sigma}\cdot\mathbf{q}\boldsymbol{\sigma}\cdot\mathbf{k})_a}{4\omega_q\omega_k{}^2(\omega_k+\omega_q)}\right]\right.\\ &\quad\left.+\frac{1}{(2M)^4}\left[-\frac{(\boldsymbol{\sigma}\cdot\mathbf{q}\boldsymbol{\sigma}\cdot\mathbf{k})_a(\boldsymbol{\sigma}\cdot\mathbf{q}\boldsymbol{\sigma}\cdot\mathbf{k})_b}{4\omega_k{}^2\omega_q{}^2(\omega_k+\omega_q)}+\frac{(\boldsymbol{\sigma}\cdot\mathbf{q}\boldsymbol{\sigma}\cdot\mathbf{k})_a+(\boldsymbol{\sigma}\cdot\mathbf{q}\boldsymbol{\sigma}\cdot\mathbf{k})_b-[\boldsymbol{\sigma}\cdot\mathbf{p}\boldsymbol{\sigma}\cdot(\mathbf{q}+\mathbf{k})]_a-[\boldsymbol{\sigma}\cdot\mathbf{p}\boldsymbol{\sigma}\cdot(\mathbf{q}+\mathbf{k})]_b}{4\omega_k\omega_q(\omega_k-\omega_q)}\right]\right\},\end{aligned}\tag{III.2}$$

where

$$\mathbf{q}=\mathbf{p}-\mathbf{p}'-\mathbf{k}.$$

The terms associated with $(2M)^{-5}$ and higher are neglected. The adiabatic limit includes also the approximation of replacing the terms $[2M-\omega_q]^{-1}$ and $[2M+\omega_k]^{-1}$ by $[2M]^{-1}$. This is equivalent to cutting off the high-energy contributions of the virtual mesons, so that the resulting potential is not valid for all r. The sets (B) and (C) do not contain terms proportional to $[2M]^{-2}$. The largest contributions come from the set (A).

Apart from the sets (A), (B), and (C), we also have to consider the coupling of the 4-fermion amplitudes back to the 2-fermion wave function $\psi(\mathbf{p})$. If one neglects 5-particle amplitudes of the new Tamm-Dancoff method, then there are only two more terms contributing to the fourth order potential. They are shown as the two upper diagrams [set (D)] of Fig. 5. The first diagram (1.3) in Fig. 5 represents the matrix element:

$$\begin{aligned}&\left[\frac{G^2}{8\pi^3}\right]^2\int\frac{d^3k}{2\omega_k2\omega_q}\sum_{\mp}\\ &\pm\Gamma_i{}^b[E-E_p-\eta_b(-\mathbf{p}+\mathbf{k})E_{p-k}\pm\omega_k]^{-1}\Gamma_i{}^a\\ &\times\Lambda_-{}^a(\mathbf{p}'+\mathbf{k})[E-E_p-E_{p'}-E_{p'+k}\\ &-\eta_b(-\mathbf{p}+\mathbf{k})E_{p-k}]^{-1}\Gamma_j{}^a[E-E_{p'}-\eta_b(-\mathbf{p}+\mathbf{k})\\ &\times(E_{p-k}+\omega_q)]^{-1}\Gamma_j{}^b.\end{aligned}\tag{III.3}$$

In the adiabatic limit it contains a term that is infinite. This is similar to the situation that arises in the study of the terms (C), but in the present case such terms, when combined algebraically before the adiabatic limit is taken, cancel out. The remaining part contributes a term of the form

$$-\frac{(3-2\boldsymbol{\tau}_a\cdot\boldsymbol{\tau}_b)[(\boldsymbol{\sigma}\cdot\mathbf{k}\boldsymbol{\sigma}\cdot\mathbf{q})_b+(\boldsymbol{\sigma}\cdot\mathbf{q}\boldsymbol{\sigma}\cdot\mathbf{k})_a]}{(2M)^32\omega_k{}^2\omega_q{}^2},$$

where the second term corresponds to the contribution of the second diagram.

If we were to confine ourselves only to the 2-, 3-, and 4-particle amplitudes, then the above 14 diagrams are all that would need to be included. There are actually other contributions to the fourth order potential arising from the coupling of the five-particle amplitudes back to the 2-fermion wave function.[10]

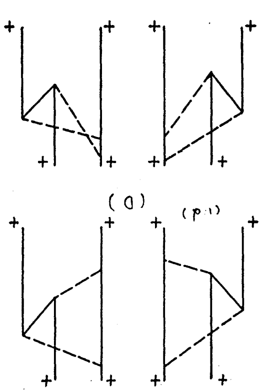


FIG. 5. These diagrams contain spurious interactions with signs opposite to those arising from the previous interactions. There are 8 more of these diagrams and they contribute in a manner similar to the ones shown here.

[10] This point was communicated to the author by A. Klein.

We also have to include interactions of the type that include 4 fermions and one meson in the intermediate states. The new Tamm-Dancoff equations in Appendix A do not include 5-particle amplitudes, but the method of writing down the matrix elements for any interaction term is now quite easy. The prescription given in Sec. II can be generalized in all cases.[11]

Two typical interactions involving 5 particles in the intermediate state are shown in the two lower diagrams of Fig. 5. There are altogether 10 diagrams of this type. We shall write down the matrix element corresponding to the first one as

$$\left[\frac{G^2}{8\pi^3}\right]^2 \int \frac{d^3k}{2\omega_k 2\omega_q} \sum_{\mp} \pm\Gamma_i^b[E-E_p-\eta_b(-\mathbf{p}+\mathbf{k})E_{p-k}\pm\omega_k]^{-1}\Gamma_j^b \times[E-E_p-E_{p'}-\omega_q\pm\omega_k]^{-1}\Gamma_i^a\Lambda_-^a(\mathbf{p}'+\mathbf{k}) \times[E-E_p-2E_{p'}-E_{p'+k}-\omega_q]^{-1}\Gamma_j^a. \quad \text{(III.4)}$$

Its contribution to the fourth order potential in the adiabatic limit is

$$-\frac{3-2\boldsymbol{\tau}_a\cdot\boldsymbol{\tau}_b}{(2M)^2}\left[\frac{1}{4\omega_k\omega_q(\omega_k+\omega_q)}+\frac{1}{4\omega_k\omega_q(\omega_k-\omega_q)}\right]-\frac{(\boldsymbol{\sigma}\cdot\mathbf{q}\boldsymbol{\sigma}\cdot\mathbf{k})_b(3-2\boldsymbol{\tau}_a\cdot\boldsymbol{\tau}_b)}{(2M)^3}\left[\frac{1}{4\omega_k^2\omega_q(\omega_k+\omega_q)}-\frac{1}{4\omega_k^2\omega_q(\omega_k-\omega_q)}\right]$$
$$+\frac{(3-2\boldsymbol{\tau}_a\cdot\boldsymbol{\tau}_b)}{(2M)^4}\left[\frac{[\boldsymbol{\sigma}\cdot\mathbf{p}\boldsymbol{\sigma}\cdot(\mathbf{q}+\mathbf{k})]_a+[\boldsymbol{\sigma}\cdot\mathbf{p}\boldsymbol{\sigma}\cdot(\mathbf{q}+\mathbf{k})]_b-(\boldsymbol{\sigma}\cdot\mathbf{q}\boldsymbol{\sigma}\cdot\mathbf{k})_a-(\boldsymbol{\sigma}\cdot\mathbf{q}\boldsymbol{\sigma}\cdot\mathbf{k})_b}{4\omega_k\omega_q(\omega_k+\omega_q)}\right.$$
$$\left.+\frac{[\boldsymbol{\sigma}\cdot\mathbf{p}\boldsymbol{\sigma}\cdot(\mathbf{q}+\mathbf{k})]_a+[\boldsymbol{\sigma}\cdot\mathbf{p}\boldsymbol{\sigma}\cdot(\mathbf{q}+\mathbf{k})]_b-(\boldsymbol{\sigma}\cdot\mathbf{q}\boldsymbol{\sigma}\cdot\mathbf{k})_a-(\boldsymbol{\sigma}\cdot\mathbf{q}\boldsymbol{\sigma}\cdot\mathbf{k})_b}{4\omega_k\omega_q(\omega_k-\omega_q)}\right]. \quad \text{(III.5)}$$

The number of interaction terms contributing to the fourth order potential are thus 24 in all. Each interaction consists of 4 terms so that one must include altogether 96 interactions. When summed, the spurious divergences of the type $(\omega_k-\omega_q)^{-1}$ cancel out and the result is the same as the one obtained by Klein from the old Tamm-Dancoff method. The one-pair terms also give a spin-orbit coupling term with the same coefficient as obtained by Klein from the old method. We note that the cancellation of the spurious divergence of the type $(\omega_k-\omega_q)^{-1}$ occurs only in the adiabatic limit adopted in this paper. A nonadiabatic cancellation of such interaction terms does not occur. In the sense of the adiabatic approximation the role of the interacting vacuum state in the definition of the new Tamm-Dancoff amplitudes is reduced to the one played by the non-interacting vacuum state. The last statement is, of course, verified only up to the fourth order interaction. However, it is quite reasonable to conclude that in the adiabatic limit the new Tamm-Dancoff method will not differ from the old one in higher orders.

IV. CONCLUSION

An attempt to study the entire series of interactions in the new Tamm-Dancoff method, even without renormalizations, would certainly be an ambitious enterprise. The inclusion of the renormalizations would make the whole problem next to impossible. In any case, the problem of renormalization in this theory is not well understood, and from this point of view the old Tamm-Dancoff method as derived from the Bethe-Salpeter equation is in a better situation.

It is very unlikely that in the near future we shall know much about the complete series. We are, therefore forced to base all our arguments on the fourth-order interaction which, of course, does not do justice to the theory. However, if we attribute a special place to the fourth order interaction, then our results show that the extreme nonrelativistic approximation to a relativistic theory does not lead to sensible results. If the Tamm-Dancoff method of approximation is to be maintained, then it is necessary to abandon the adiabatic approximation to the kernel of the equation. The same conclusion is implied by the work of Klein.[6]

An important problem is, now, the investigation of the nonadiabatic terms. It may well be that the nonadiabatic parts will contribute effectively near the core and change the sign of the interaction that was repulsive in the nonrelativistic region without the nonadiabatic terms. We have no reason for assuming that the nonadiabatic terms in a $ps-ps$ theory are small. This aspect of the problem is being investigated for the second order equation.

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[11] More general rules of writing down the matrix elements for the one-particle case has also been discussed by A. Klein, Phys. Rev. **95**, 1061 (1954).

APPENDIX A

In this Appendix we give the coupled set of integral equations mentioned in the introduction to this paper, and we shall use the notation of Dyson.[4] The Schrödinger equation for the new Tamm-Dancoff amplitudes was derived by Dyson as

$$E(\Psi_0{}^*C(N)A(N')\Psi)=[\Pi(N)\Pi(N')]^{-\frac{1}{2}}(\Psi_0{}^*[C(N)A(N'),H_0+H']\Psi).$$

The new Tamm-Dancoff equations for a state Ψ representing two nucleons in interaction with, at most, four fermions at a given time can be obtained by calculating the commutators,

$$[b_p{}^u d_q{}^v,H'],\quad [b_{p-k}{}^w d_q{}^z a_k,H'],\quad [b_{p-k}{}^w d_q{}^z a_{-k}{}^*,H'],\quad [b_{p-k-s}{}^{u'}d_q{}^{v'}a_k a_s,H'],\quad [b_{p-k-s}{}^{u'}d_q{}^{v'}a_{-k}{}^*a_s,H'],$$
$$[b_{p-k-s}{}^{u'}d_q{}^{v'}a_{-k}{}^*a_{-s}{}^*,H'],\quad [N(b_{p-k}{}^u d_q{}^v d_s{}^{*w}d_{k+s}{}^z),H'],\quad [N(b_{p-k}{}^u b_s{}^{*w}b_{k+s}{}^z d_q{}^v,H'].$$

In the last commutator, N stands for the normal product of the 4 operators. We shall use the following definitions in arranging the various commutators according to the number of particles that take part in the interaction:

$$\sum u^*\gamma_5 u=0,\quad \textstyle\sum_u \bar{u}u(\mathbf{p})\theta_u=\Lambda_+(\mathbf{p}),$$
$$\langle b_p{}^{*u}b_q{}^v\rangle_0=(1-\theta_u)\delta_{uv}\delta_{pq},\quad \langle b_p{}^u b_q{}^{*v}\rangle_0=\theta_u\delta_{uv}\delta_{pq},$$

where

$$\theta_u=\begin{cases}1 & \text{for proton states}\\ 0 & \text{for anti-proton states.}\end{cases}$$

The notation is the same as in Dyson's paper. We assume that all Tamm-Dancoff amplitudes which contain five or more particles vanish. In the actual case this assumption was not made in calculating the fourth order potential. Since our aim is to derive general rules for writing the matrix elements, there is no harm in the above assumption. In this case one obtains equations that involve the wave functions $\psi(\mathbf{p},\mathbf{q})$, $\psi^+(\mathbf{p}-\mathbf{k},\mathbf{q},\mathbf{k})$, $\psi^-(\mathbf{p}-\mathbf{k},\mathbf{q},\mathbf{k})$, $\psi^{++}(\mathbf{p}-\mathbf{k},\mathbf{q}-\mathbf{s},\mathbf{k},\mathbf{s})$, $\psi^{-+}(\mathbf{p}-\mathbf{k},\mathbf{q}-\mathbf{s},\mathbf{k},\mathbf{s})$, $\psi^{--}(\mathbf{p}-\mathbf{k},\mathbf{q}-\mathbf{s},\mathbf{k},\mathbf{s})$, $\phi^p(\mathbf{p}-\mathbf{k},-\mathbf{s},\mathbf{k}+\mathbf{s},\mathbf{q})$, $\phi^N(\mathbf{p}-\mathbf{k},\mathbf{q},-\mathbf{s},\mathbf{k}+\mathbf{s})$, for two and four fermions (in states Ψ or Ψ_0) and one and two mesons in the states Ψ and Ψ_0 as plus or minus particles. The four-fermion wave function is defined as

$$\phi^p(\mathbf{p}-\mathbf{k},-\mathbf{s},\mathbf{k}+\mathbf{s},\mathbf{q})=\sum_{uvzw^*}[\Psi_0{}^*N(b_{p-k}{}^u b_s{}^{*w}b_{k+s}{}^z d_q{}^v)\Psi]uvw^*z,$$
$$\phi^n(\mathbf{p}-\mathbf{k},\mathbf{q},-\mathbf{s},\mathbf{k}+\mathbf{s})=\sum_{uvzw^*}[\Psi_0{}^*N(b_{p-k}{}^u d_q{}^v d_s{}^{*w}d_{k+s}{}^z)\Psi]uvw^*z.$$

The integral equations satisfied by the first six wave functions are given by

$$[E-\eta_a(\mathbf{p})E_p-\eta_b(\mathbf{q})E_q]\psi(\mathbf{p},\mathbf{q})$$
$$=G\textstyle\sum_s(2v\omega_s)^{-\frac{1}{2}}\{\gamma^a[\psi^+(\mathbf{p}-\mathbf{s},\mathbf{q},\mathbf{s})+\psi^-(\mathbf{p}-\mathbf{s},\mathbf{q},\mathbf{s})]\}+\{\gamma^b[\psi^+(\mathbf{p},\mathbf{q}-\mathbf{s},\mathbf{s})+\psi^-(\mathbf{p},\mathbf{q}-\mathbf{s},\mathbf{s})]\}. \tag{A.1}$$

$$[E-\eta_a(\mathbf{p}-\mathbf{k})E_{p-k}-\eta_b(\mathbf{q})E_q-\omega_k]\psi^+(\mathbf{p}-\mathbf{k},\mathbf{q},\mathbf{k})$$
$$=G(2v\omega_k)^{\frac{1}{2}}[\Lambda_+{}^a(\mathbf{p}-\mathbf{k})\gamma^a\psi(\mathbf{p},\mathbf{q})+\Lambda_+{}^b(\mathbf{q})\gamma^b\psi(\mathbf{p}-\mathbf{k},\mathbf{q}+\mathbf{k})]+G\textstyle\sum_s(2v\omega_s)^{-\frac{1}{2}}[\gamma^a(\psi^{++}(\mathbf{p}-\mathbf{k}-\mathbf{s},\mathbf{q},\mathbf{s},\mathbf{k})$$
$$+\psi^{-+}(\mathbf{p}-\mathbf{k}-\mathbf{s},\mathbf{q},\mathbf{s},\mathbf{k}))+\gamma^b(\psi^{++}(\mathbf{p}-\mathbf{k},\mathbf{q}-\mathbf{s},\mathbf{s},\mathbf{k})+\psi^{-+}(\mathbf{p}-\mathbf{k},\mathbf{q}-\mathbf{s},\mathbf{s},\mathbf{k}))]$$
$$+G(2v\omega_k)^{-\frac{1}{2}}\textstyle\sum_s[\gamma^a\phi^p(\mathbf{p}-\mathbf{k},-\mathbf{s},\mathbf{k}+\mathbf{s},\mathbf{q})+\gamma^b\phi^n(\mathbf{p}-\mathbf{k},\mathbf{q},-\mathbf{s},\mathbf{k}+\mathbf{s})], \tag{A.2}$$

$$[E-\eta_a(\mathbf{p}-\mathbf{k})E_{p-k}-\eta_b(\mathbf{q})E_q+\omega_k]\psi^-(\mathbf{p}-\mathbf{k},\mathbf{q},\mathbf{k})$$
$$=G(2v\omega_k)^{-\frac{1}{2}}[\Lambda_-{}^a(\mathbf{p}-\mathbf{k})\gamma^a(\mathbf{p},\mathbf{q})+\Lambda_-{}^b(\mathbf{q})\gamma^b\psi(\mathbf{p}-\mathbf{k},\mathbf{q}+\mathbf{k})]+G\textstyle\sum_s(2v\omega_s)^{-\frac{1}{2}}[\gamma^a(\psi^{-+}(\mathbf{p}-\mathbf{k}-\mathbf{s},\mathbf{q},\mathbf{k},\mathbf{s})$$
$$+\psi^{--}(\mathbf{p}-\mathbf{k}-\mathbf{s},\mathbf{q},\mathbf{k},\mathbf{s}))+\gamma^b(\psi^{-+}(\mathbf{p}-\mathbf{k},\mathbf{q}-\mathbf{s},\mathbf{k},\mathbf{s})+\psi^{--}(\mathbf{p}-\mathbf{k},\mathbf{q}-\mathbf{s},\mathbf{k},\mathbf{s}))]$$
$$+G(2v\omega_k)^{-\frac{1}{2}}\textstyle\sum_s[\gamma^a\phi^p(-\mathbf{s},\mathbf{k}+\mathbf{s},\mathbf{p}-\mathbf{k},\mathbf{q})+\gamma^b\phi^n(\mathbf{p}-\mathbf{k},-\mathbf{s},\mathbf{k}+\mathbf{s},\mathbf{q})], \tag{A.3}$$

$$[E-\eta_a(\mathbf{p}-\mathbf{k}-\mathbf{r})E_{p-k-r}-\eta_b(\mathbf{q})E_q-\omega_k-\omega_r]\psi^{++}(\mathbf{p}-\mathbf{k}-\mathbf{r},\mathbf{q},\mathbf{k},\mathbf{r})$$
$$=G(2v\omega_k)^{-\frac{1}{2}}[\Lambda_+{}^a(\mathbf{p}-\mathbf{k}-\mathbf{r})\gamma^a\psi^+(\mathbf{p}-\mathbf{r},\mathbf{q},\mathbf{r})+\Lambda_+{}^b(\mathbf{q})\gamma^b\psi^+(\mathbf{p}-\mathbf{k}-\mathbf{r},\mathbf{q}+\mathbf{k},\mathbf{r})]$$
$$+G(2v\omega_r)^{-\frac{1}{2}}[\Lambda_+{}^a(\mathbf{p}-\mathbf{k}-\mathbf{r})\gamma^a\psi^+(\mathbf{p}-\mathbf{k},\mathbf{q},\mathbf{k})+\Lambda_+{}^b(\mathbf{q})\gamma^b\psi^+(\mathbf{p}-\mathbf{k}-\mathbf{r},\mathbf{q}+\mathbf{r},\mathbf{k})], \tag{A.4}$$

$$[E-\eta_a(\mathbf{p}-\mathbf{k}-\mathbf{r})E_{p-k-r}-\eta_b(\mathbf{q})E_q-\omega_k+\omega_r]\psi^{-+}(\mathbf{p}-\mathbf{k}-\mathbf{r},\mathbf{q},\mathbf{r},\mathbf{k})$$
$$=G(2v\omega_k)^{-\frac{1}{2}}[\Lambda_+{}^a(\mathbf{p}-\mathbf{k}-\mathbf{r})\gamma^a\psi^-(\mathbf{p}-\mathbf{r},\mathbf{q},\mathbf{r})+\Lambda_+{}^b(\mathbf{q})\gamma^b\psi^-(\mathbf{p}-\mathbf{k}-\mathbf{r},\mathbf{q}+\mathbf{k},\mathbf{r})]$$
$$+G(2v\omega_r)^{-\frac{1}{2}}[\Lambda_-{}^a(\mathbf{p}-\mathbf{k}-\mathbf{r})\gamma^a\psi^+(\mathbf{p}-\mathbf{k},\mathbf{q},\mathbf{k})+\Lambda_-{}^b(\mathbf{q})\gamma^b\psi^+(\mathbf{p}-\mathbf{k}-\mathbf{r},\mathbf{q}+\mathbf{r},\mathbf{k})]. \quad (A.5)$$

$$[E-\eta_a(\mathbf{p}-\mathbf{k}-\mathbf{r})E_{p-k-r}-\eta_b(\mathbf{q})E_q+\omega_k+\omega_r]\psi^{--}(\mathbf{p}-\mathbf{k}-\mathbf{r},\mathbf{q},\mathbf{k},\mathbf{r})$$
$$=G(2v\omega_k)^{-\frac{1}{2}}[\Lambda_-{}^a(\mathbf{p}-\mathbf{k}-\mathbf{r})\gamma^a\psi^-(\mathbf{p}-\mathbf{r},\mathbf{q},\mathbf{r})+\Lambda_-{}^b(\mathbf{q})\gamma^b\psi^-(\mathbf{p}-\mathbf{k}-\mathbf{r},\mathbf{q}+\mathbf{k},\mathbf{r})]$$
$$+G(2v\omega_r)^{-\frac{1}{2}}[\Lambda_-{}^a(\mathbf{p}-\mathbf{k}-\mathbf{r})\gamma^a\psi^-(\mathbf{p}-\mathbf{k},\mathbf{q},\mathbf{k})+\Lambda_-{}^b(\mathbf{q})\gamma^b\psi^-(\mathbf{p}-\mathbf{k}-\mathbf{r},\mathbf{q}+\mathbf{r},\mathbf{k})]. \quad (A.6)$$

The equation for the wave function $\phi^p(\mathbf{p}-\mathbf{k},-\mathbf{s},\mathbf{k}+\mathbf{s},\mathbf{q})$ follows from a careful study of the commutator,

$$[N(b_{p-k}{}^ub_s{}^{*w}b_{k+s}{}^zd_q{}^v),H']$$
$$=+G(2v\omega_k)^{-\frac{1}{2}}(z^*\gamma^aw)(1-\theta_w)(a_k+a_{-k}{}^*)b_{p-k}{}^ud_q{}^v-G(2v\omega_k)^{-\frac{1}{2}}(z^*\gamma^aw)(1-\theta_z)(a_k+a_{-k}{}^*)b_{p-k}{}^ud_q{}^v$$
$$+G(2v\omega_{p-k-s})^{-\frac{1}{2}}(u^*\gamma^aw)\theta_w(a_{p-k-s}+a_{-p+k+s})b_{k+s}{}^zd_q{}^v-G(2v\omega_{p-k-s})^{-\frac{1}{2}}(u^*\gamma^aw)\theta_u(a_{p-k-s}+a_{-p+k+s}{}^*)b_{k+s}{}^zd_q{}^v.$$

There are only 8 possibilities for the signs of the energies along the proton line. It is easy to see that if u, w, and z represent positive or negative energy spinors simultaneously, the right-hand side of the above equation vanishes. Therefore the $\phi_{+++}{}^p$ and $\phi_{---}{}^p$ components of the wave function must vanish. The sum of the remaining six components of the wave function satisfies the equation

$$\phi^p(\mathbf{p}-\mathbf{k},-\mathbf{s},\mathbf{k}+\mathbf{s},\mathbf{q})=-G(2v\omega_k)^{-\frac{1}{2}}\Lambda_-{}^a(\mathbf{k}+\mathbf{s})\Lambda_-{}^a(\mathbf{s})[E+\eta_a(\mathbf{p}-\mathbf{k})E_{p-k}-E_s+E_{k+s}-\eta_b(\mathbf{q})E_q]^{-1}$$
$$\times\gamma^a(\psi^+(\mathbf{p}-\mathbf{k},\mathbf{q},\mathbf{k})+\psi^-(\mathbf{p}-\mathbf{k},\mathbf{q},\mathbf{k}))+G(2v\omega_k)^{-\frac{1}{2}}\Lambda_+{}^a(\mathbf{k}+\mathbf{s})\Lambda_+{}^a(\mathbf{s})[E+\eta_a(\mathbf{p}-\mathbf{k})E_{p-k}+E_s-E_{k+s}-\eta_b(\mathbf{q})E_q]$$
$$\times\gamma^a(\psi^+(\mathbf{p}-\mathbf{k},\mathbf{q},\mathbf{k})+\psi^-(\mathbf{p}-\mathbf{k},\mathbf{q},\mathbf{k}))-G(2v\omega_{p-k-s})^{-\frac{1}{2}}\Lambda_+{}^a(\mathbf{p}-\mathbf{k})\Lambda_+{}^a(\mathbf{s})[E+\eta_a(\mathbf{k}+\mathbf{s})E_{k+s}-E_{p-k}+E_s$$
$$-\eta_b(\mathbf{q})E_q]^{-1}\gamma^a(\psi^+(\mathbf{k}+\mathbf{s},\mathbf{q},\mathbf{p}-\mathbf{k}-\mathbf{s})+\psi^-(\mathbf{k}+\mathbf{s},\mathbf{q},\mathbf{p}-\mathbf{k}-\mathbf{s}))+G(2v\omega_{p-k-s})^{-\frac{1}{2}}\Lambda_-{}^a(\mathbf{p}-\mathbf{k})\Lambda_-{}^a(\mathbf{s})$$
$$\times[E+\eta_a(\mathbf{k}+\mathbf{s})E_{k+s}+E_{p-k}-E_s-\eta_b(\mathbf{q})E_q]^{-1}\gamma^a(\psi^+(\mathbf{k}+\mathbf{s},\mathbf{q},\mathbf{p}-\mathbf{k}-\mathbf{s})+\psi^-(\mathbf{k}+\mathbf{s},\mathbf{q},\mathbf{p}-\mathbf{k}-\mathbf{s})). \quad (A.7)$$

In a similar way the function $\phi^N(\mathbf{p}-\mathbf{k},\mathbf{q},-\mathbf{s},\mathbf{k}+\mathbf{s})$ satisfies the equation

$$\phi^n(\mathbf{p}-\mathbf{k},\mathbf{q},-\mathbf{s},\mathbf{k}+\mathbf{s})=G(2v\omega_k)^{-\frac{1}{2}}\Lambda_+{}^b(\mathbf{k}+\mathbf{s})\Lambda_+{}^b(\mathbf{s})[E-\eta_a(\mathbf{p}-\mathbf{k})E_{p-k}+E_s-E_{k+s}+\eta_b(\mathbf{q})E_q]^{-1}$$
$$\times\gamma^b(\psi^+(\mathbf{p}-\mathbf{k},\mathbf{q},\mathbf{k})+\psi^-)-G(2v\omega_k)^{-\frac{1}{2}}\Lambda_-{}^b(\mathbf{k}+\mathbf{s})\Lambda_-{}^b(\mathbf{s})[E-\eta_a(\mathbf{p}-\mathbf{k})E_{p-k}-E_s+E_{k+s}+\eta_b(\mathbf{q})E_q]^{-1}$$
$$\times\gamma^b(\psi^+(\mathbf{p}-\mathbf{k},\mathbf{q},\mathbf{k})+\psi^-(\mathbf{p}-\mathbf{k},\mathbf{q},\mathbf{k}))+G(2v\omega_{q-s})^{-\frac{1}{2}}\Lambda_-{}^b(\mathbf{q})\Lambda_-{}^b(\mathbf{s})[E-\eta_a(\mathbf{p}-\mathbf{k})E_{p-k}-E_s+E_q+\eta_b(\mathbf{k}+\mathbf{s})E_{k+s}]^{-1}$$
$$\times\gamma^b(\psi^+(\mathbf{p}-\mathbf{k},\mathbf{k}+\mathbf{s},\mathbf{q}-\mathbf{s})+\psi^-(\mathbf{p}-\mathbf{k},\mathbf{k}+\mathbf{s},\mathbf{q}-\mathbf{s}))-G(2v\omega_{q-s})^{-\frac{1}{2}}\Lambda_+{}^b(\mathbf{q})\Lambda_+{}^b(\mathbf{s})[E-\eta_a(\mathbf{p}-\mathbf{k})E_{p-k}+E_s$$
$$-E_q+\eta_b(\mathbf{k}+\mathbf{s})E_{k+s}]^{-1}(\psi^+(\mathbf{p}-\mathbf{k},\mathbf{k}+\mathbf{s},\mathbf{q}-\mathbf{s})+\psi^-). \quad (A.8)$$

We have, thus, completed the derivation of the new Tamm-Dancoff equations. In carrying out the first Born approximation to these equations the radiative correction terms can easily be recognized and dropped from the interaction. This leads to the sets (A), (B), and (D).

APPENDIX B

In connection with the elimination of spurious plane wave solutions of the Bethe-Salpeter (integro-differential) equation we can introduce a transformation of the two-body wave function $\chi(12)$:

$$\chi(12)=\frac{1}{2\pi i}\left[\int S_{Fa}(11')\beta_a\phi(1'2)d1'+\int S_{Fb}(22')\beta_b\phi(12')d2'\right] \quad (B.1)$$

where the physical meaning of the spinor function $\phi(12)$ is not directly obvious. A simple interpretation for the function $\phi(12)$ can, however, be found in some special cases.

Now, the momentum space transform of (B.1) in the center-of-mass system can be written as

$$\chi(p_\mu)=\frac{1}{2\pi i}\left[\frac{1}{H_a(\mathbf{p})-\frac{1}{2}E-p_0}+\frac{1}{H_b(-\mathbf{p})-\frac{1}{2}E+p_0}\right]\phi(p_\mu). \quad (B.2)$$

In general there are two cases of interest: (*a*) we substitute the expression (B.2) in the Bethe-Salpeter equation and obtain an equation for $\phi(p_\mu)$,

$$[H_a(\mathbf{p})+H_b(-\mathbf{p})-E]\phi(p_\mu)=-\frac{G^2}{(2\pi)^4}\Gamma_i{}^a\Gamma_i{}^b\int\Delta_F(p_\mu-p_\mu{}')\left[\frac{1}{H_a(\mathbf{p}')-\frac{1}{2}E-p_0{}'}+\frac{1}{H_b(-\mathbf{p}')-\frac{1}{2}E+p_0{}'}\right]\phi(p_\mu{}')d^4p'. \quad \text{(B.3)}$$

When the particles interact instantaneously ($p_0=p_0{}'$ in Δ_F), the kernel of (B.3) is independent of p_0 so that the function $\phi(p_\mu)$ does not depend on the relative energy variable p_0. Equation (B.3), after the p_0-integration on the right-hand side, reduces to

$$[H_a(\mathbf{p})+H_b(-\mathbf{p})-E]a(\mathbf{p})=\frac{G^2}{8\pi^3}\Gamma_i{}^a\Gamma_i{}^b\int\frac{d^3p'}{\omega_{p-p'}{}^2}\Lambda_-{}^{ab}(\mathbf{p}')a(\mathbf{p}'), \quad \text{(B.4)}$$

where $a(p)$ is the wave function of two instantaneously interacting nucleons and

$$\Lambda_-{}^{ab}(\mathbf{p})=\Lambda_+{}^a(\mathbf{p})\Lambda_+{}^b(-\mathbf{p})-\Lambda_-{}^a(\mathbf{p})\Lambda_-{}^b(-\mathbf{p}).$$

In the nonrelativistic limit, for the $a_{++}(p)$-components, the wave equation (B.4) reduces to the usual Yukawa form of the $ps-ps$ theory. Thus, in the above special case the physical meaning of $\phi(p_\mu)$ is clear.

(*b*) Let us now assume that the system first propagates as two free particles and then the interaction takes place in a time-ordered way. We use an equal-time wave function $\varphi(\mathbf{p})=\int\chi(\mathbf{p},p_0)dp_0$ and the ansatz of replacing $\phi(p_\mu)$ in (B.2) by a function $\phi(\mathbf{p})$ independent of p_0, *viz.*,

$$\chi(p_\mu)=\frac{1}{2\pi i}\left[\frac{1}{H_a(\mathbf{p})-\frac{1}{2}E-p_0}+\frac{1}{H_b(-\mathbf{p})-\frac{1}{2}E+p_0}\right]\phi(\mathbf{p}). \quad \text{(B.5)}$$

The function $\phi(\mathbf{p})$ will not describe the system in a fully relativistic way, since its ϕ_{+-} and ϕ_{-+} components will not appear in the resulting 3-dimensional equation. This can be seen by integrating both sides of (B.5) with respect to p_0,

$$\varphi(\mathbf{p})=\int\chi(p_\mu)dp_0=\phi_{++}(\mathbf{p})-\phi_{--}(\mathbf{p}) \quad \text{(B.6)}$$

so that

$$\varphi_{+-}(\mathbf{p})=\varphi_{-+}(\mathbf{p})=0.$$

This is one of the reasons that the definition (B.5) cannot be used as an ansatz in the Bethe-Salpeter equation to derive Dyson's equation.

In deriving an equal-time formalism from the Bethe-Salpeter equation the operation of integration over the relative energy variables must precede the use of the ansatz (B.5); namely, we must first integrate both sides of the Bethe-Salpeter equation with respect to p_0 to include the contributions from the free particle states. In this case the function $\phi_{++}(\mathbf{p})$ can be identified as a Tamm-Dancoff wave function.

We now proceed to the discussion of the 3-dimensional wave function $\phi_{++}(\mathbf{p})$. The Bethe-Salpeter equation for one-and-two meson interactions of two nucleons is

$$\chi(p_\mu)=-S_{Fa}(\tfrac{1}{2}K_\mu+p_\mu)S_{Fb}(\tfrac{1}{2}K_\mu-p_\mu)\beta_a\beta_b\int\beta_a\beta_b[I_2(p_\mu,p_\mu{}';K_\mu)+I_4(p_\mu,p_\mu{}';K_\mu)]\chi(p_\mu{}')d^4p', \quad \text{(B.7)}$$

where $I_2(p_\mu,p_\mu{}';K_\mu)$ and $I_4(p_\mu,p_\mu{}';K_\mu)$ represent the ladder and crossed diagrams given by

$$\beta_a\beta_bI_2(p_\mu,p_\mu{}';K_\mu)=\frac{G^2}{(2\pi^4)}\Gamma_i{}^a\Gamma_i{}^b\Delta_F(p_\mu-p_\mu{}'), \quad \text{(B.8)}$$

$$\beta_a\beta_bI_4(p_\mu,p_\mu{}';K_\mu)=\left[\frac{G^2}{(2\pi)^4}\right]^2\int\Delta_F(p_\mu-p_\mu{}'-k_\mu)\Delta_F(k_\mu)$$
$$\times\Gamma_i{}^a[H_a(\mathbf{p}'+\mathbf{k})-\tfrac{1}{2}E-p_0{}'-k_0]^{-1}\Gamma_j{}^a\Gamma_j{}^b[H_b(-\mathbf{p}+\mathbf{k})-\tfrac{1}{2}E+p_0-k_0]^{-1}\Gamma_i{}^bd^4k. \quad \text{(B.9)}$$

In order to derive an equation for $\phi_{++}(\mathbf{p})$ we replace $\chi(p_\mu)$ in (B.7) by the expressions

$$\chi_{++}(p_\mu)=\frac{1}{2\pi i}\frac{2A_p}{(A_p-p_0)(A_p+p_0)}\phi_{++}(\mathbf{p}),\quad A_p=E_p-\tfrac{1}{2}E, \tag{B.10}$$

leading to

$$\frac{1}{2\pi i}\frac{2A_p}{(A_p-p_0)(A_p+p_0)}\phi_{++}(\mathbf{p})=-\frac{\Lambda_+^a(\mathbf{p})\Lambda_+^b(-\mathbf{p})}{(A_p-p_0)(A_p+p_0)}\int\beta_a\beta_b[I_2(p_\mu,p_\mu';K_\mu)+I_4(p_\mu,p_\mu';K_\mu)]$$
$$\times\frac{2A_{p'}}{2\pi i(A_{p'}-p_0')(A_{p'}+p_0')}\phi_{++}(\mathbf{p}')d^3p'idP_0'. \tag{B.11}$$

Because the p_0-integration has to precede the use of the ansatz, the factors $(A_p-p_0)^{-1}$, $(A_p+p_0)^{-1}$ on both sides of the equation (B.11) will not be cancelled out. The 3-dimensional equation can, now, be obtained by integrating both sides of (B.11) with respect to p_0,

$$\phi_{++}(\mathbf{p})=-\Lambda_+^a(\mathbf{p})\Lambda_+^b(-\mathbf{p})\int\frac{2A_{p'}d^3p'}{2\pi i(A_p-p_0)(A_p+p_0)(A_{p'}-p_0')(A_{p'}+p_0')}$$
$$\times\beta_a\beta_b[I_2(p_\mu,p_\mu';K_\mu)+I_4(p_\mu,p_\mu';K_\mu)]\phi_{++}(\mathbf{p}')dp_0idp_0'. \tag{B.12}$$

In this equation the first term to be integrated with respect to p_0 and p_0' is

$$M_2=\frac{G^2}{(2\pi)^4}\frac{1}{2\pi}\Gamma_i^a\Gamma_i^b\int_{-\infty}^{\infty}\frac{2A_{p'}dp_0dp_0'}{(A_p-p_0)(A_p+p_0)(A_{p'}-p_0')(A_{p'}+p_0')(\omega_{p-p'}-p_0+p_0')(\omega_{p-p'}+p_0-p_0')}. \tag{B.13}$$

Usually one carries out the integration over p_0 in the complex plane of p_0 and the resulting expression can then be integrated over p_0' in the complex plane of p_0'. This is a long and tedious business. Actually, we can accomplish both integrations simultaneously if we note that, because of the small negative imaginary parts in A_p's and $\omega_{p-p'}$, the expression (B.13) can be written as

$$M_2=\frac{G^2}{(2\pi)^5}(i)^6\Gamma_i^a\Gamma_i^b\int_{-\infty}^{\infty}dp_0dp_0'\int_0^{\infty}d\alpha d\beta d\gamma d\delta d\mu d\nu 2A_{p'}\exp[-i\alpha(A_p-p_0)-i\beta(A_p+p_0)-i\gamma(A_{p'}-p_0')$$
$$-i\delta(A_{p'}+p_0')-i\mu(\omega_{p-p'}-p_0+p_0')-i\nu(\omega_{p-p'}+p_0-p_0')],$$
$$=\frac{G^2}{8\pi^3}(i)^6\Gamma_i^a\Gamma_i^b\int_0^{\infty}2A_{p'}d\alpha d\beta d\gamma d\delta d\mu d\nu\exp[-i(\alpha+\beta)A_p-i(\gamma+\delta)A_{p'}-i(\mu+\nu)\omega_{p-p'}]$$
$$\times\delta(\beta-\alpha+\mu-\nu)\delta(\gamma-\delta+\mu-\nu), \tag{B.14}$$

where α, β, γ, δ, μ, and ν are positive parameters. From the two δ-functions we have two linear algebraic equations,

$$\beta-\alpha+\mu-\nu=0,\quad \gamma-\delta+\mu-\nu=0. \tag{B.15}$$

The positive character of the parameters α, β, γ, δ, μ, and ν does not allow an arbitrary elimination of two of them from the integral in (B.14). Let

$$\beta-\alpha=\pm a,\quad \gamma-\delta=\pm b,\quad \mu-\nu=\pm c,$$

where a, b, and c are also positive. The signs for the set a, b, and c can be fixed by considering the 8 possible signs for the set a, b, and c. There are only two possible combinations that are consistent with the positive character of the 6 parameters, namely the combinations $(++-)$ and $(--+)$. The two solutions of (B.15) are, therefore,

$$\text{(i)}:\ \beta=\alpha+a,\quad \gamma=\delta+a,\quad \nu=\mu+a,\qquad \text{(ii)}:\ \alpha=\beta+a,\quad \delta=\gamma+a,\quad \mu=\nu+a. \tag{B.16}$$

Other solutions are thus excluded. By using the solutions (i) and (ii) in (B.14), we obtain the matrix elements

$$M_2=-2\frac{G^2}{8\pi^3}\Gamma_i^a\Gamma_i^b(i)^4\int_0^{\infty}2A_{p'}d\alpha'd\beta'd\gamma'd\delta'\exp[-i\alpha'(2A_p)-i\beta'(2A_{p'})-i\gamma'(2\omega_{p-p'})-i\delta'(A_p+A_p'+\omega_{p-p'})].$$

Hence

$$M_2=\frac{G^2}{8\pi^3}\Gamma_i{}^a\Gamma_i{}^b\frac{2}{(2E_p-E)(2\omega_{p-p'})(E-E_p-E_{p'}-\omega_{p-p'})}. \tag{B.17}$$

The second order equation is, therefore, given by

$$(E-2E_p)\phi_{++}(\mathbf{p})=\frac{G^2}{8\pi^3}\Lambda_+{}^a(\mathbf{p})\Lambda_+{}^b(-\mathbf{p})\Gamma_i{}^a\Gamma_i{}^b\int\frac{\phi_{++}(\mathbf{p}')d^3p'}{\omega_{p-p'}(E-E_p-E_{p'}-\omega_{p-p'})}. \tag{B.18}$$

For the fourth order case we use the Casimir projection operators in the second term of (B.11). We have altogether 4 terms to be integrated in the integrand of the second term, the first one of which is

$$M_4{}^{(1)}=\left[\frac{G^2}{(2\pi)^4}\right]^2\frac{1}{2\pi}\int 2A_{p'}O(\mathbf{p},\mathbf{p}';\mathbf{k})d^3kdp_0dp_0'dk_0$$
$$\times\frac{1}{(A_p-p_0)(A_p+p_0)(A_{p'}-p_0')(A_{p'}+p_0')(A_{p'+k}-p_0'-k_0)(A_{p-k}+p_0-k_0)}$$
$$\times\frac{1}{(\omega_k-k_0)(\omega_k+k_0)(\omega_{p-p'-k}-p_0+p_0'+k_0)(\omega_{p-p'-k}+p_0-p_0'-k_0)}, \tag{B.19}$$

where

$$O(\mathbf{p},\mathbf{p}';\mathbf{k})=\Lambda_+{}^a(\mathbf{p})\Lambda_+{}^b(-\mathbf{p})(\Gamma_i\Lambda_+(\mathbf{p}'+\mathbf{k})\Gamma_j)_a(\Gamma_j\Lambda_+(-\mathbf{p}+\mathbf{k})\Gamma_i)_b.$$

As before, the integrations over p_0, p_0', k_0 can be effected in accordance with the hole theory, so that (B.19) can be written as

$$M_4{}^{(1)}=\left[\frac{G^2}{(2\pi)^4}\right]^2\frac{(i)^{10}}{2\pi}\int 2A_{p'}O(\mathbf{p},\mathbf{p}';\mathbf{k})d^3kdp_0dp_0'dk_0\int_0^\infty d\alpha d\beta d\gamma d\delta d\epsilon d\lambda d\mu d\nu d\rho d\sigma \exp[-i\alpha(A_p-p_0)$$
$$-i\beta(A_p+p_0)-i\gamma(A_{p'}-p_0')-i\delta(A_{p'}+p_0')-i\epsilon(A_{p'+k}-p_0'-k_0)-i\lambda(A_{p-k}+p_0-k_0)-i\rho(\omega_k-k_0)$$
$$-i\sigma(\omega_k+k_0)-i\mu(\omega_{p-p'-k}+p_0-p_0'-k_0)-i\nu(\omega_{p-p'-k}+p_0'+k_0-p_0)]$$

or

$$M_4{}^{(1)}=\left[\frac{G^2}{8\pi^3}\right]^2(i)^{10}\int\int_0^\infty 2A_{p'}O(\mathbf{p},\mathbf{p}';\mathbf{k})d^3kd\alpha d\beta\cdots d\sigma\exp[-i(\alpha+\beta)A_p-i(\gamma+\delta)A_{p'}-i(\rho+\sigma)\omega_k$$
$$-i(\mu+\nu)\omega_{p-p'-k}-i\epsilon A_{p'+k}-i\lambda A_{p-k}]\delta(-\alpha+\beta+\lambda+\mu-\nu)\delta(-\gamma+\delta-\epsilon-\mu+\nu)$$
$$\times\delta(-\epsilon-\lambda-\mu+\nu-\rho+\sigma). \tag{B.20}$$

We have to eliminate 3 of the 10 parameters in the expression (B.20) by using the 3 linear algebraic equations for 10 unknowns,

$$\epsilon=(\delta-\gamma)+(\nu-\mu),\quad \lambda=(\alpha-\beta)+(\nu-\mu),\quad (\alpha-\beta)+(\delta-\gamma)+(\rho-\sigma)+(\nu-\mu)=0. \tag{B.21}$$

We put

$$\alpha-\beta=\pm a,\quad \delta-\gamma=\pm b,\quad \rho-\sigma=\pm c,\quad \nu-\mu=\pm d,$$

and then consider the 16 possible signs for the quantities a, b, c, and d. The only sets of signs consistent with the positive character of α, β, γ, δ, μ, ν, λ, ϵ, ρ, and σ are 6 in number and are given by the combinations $(++-+)$, $(++--)$, $(-+-+)$, $(--++)$, $(---+)$, and $(+--+)$, leading to the solutions:

(i)	(ii)	(iii)	(iv)	(v)	(vi)
$\alpha=\beta+a$,	$\alpha=\beta+\lambda+a$,	$\alpha=\beta+a$,	$\beta=\alpha+a$,	$\beta=\alpha+\epsilon+a$,	$\beta=\alpha+a$,
$\delta=\gamma+b$,	$\delta=\gamma+\epsilon+a$,	$\gamma=\delta+b$,	$\delta=\gamma+b$,	$\gamma=\delta+\lambda+a$,	$\gamma=\delta+b$,
$\sigma=\rho+a+b+c$,	$\sigma=\rho+\epsilon+\lambda+a$,	$\sigma=\rho+\epsilon+a$,	$\sigma=\rho+\lambda+b$,	$\rho=\sigma+a$,	$\sigma=\rho+c$,
$\nu=\mu+c$,	$\mu=\nu+a$;	$\nu=\mu+\epsilon+b$,	$\nu=\mu+\lambda+a$,	$\nu=\mu+\epsilon+\lambda+a$;	$\nu=\mu+a+b+c$,
$\epsilon=b+c$,		$\lambda=\epsilon+a+b$;	$\epsilon=\lambda+a+b$;		$\lambda=b+c$,
$\lambda=a+c$;					$\epsilon=a+c$.

On using these in (20), we obtain $M_4^{(1)}$ as a sum of 6 matrix elements:

$$M_4^{(1)}=\left[\frac{G^2}{8\pi^3}\right]^2\frac{1}{2E_p-E}\int\frac{O(\mathbf{p},\mathbf{p}',\mathbf{k})d^3k}{2\omega_k 2\omega_{p-p'-k}}$$
$$\times\left[\frac{1}{(E_p+E_{p-k}+\omega_k-E)(E_{p'}+E_{p'+k}+\omega_k-E)(E_{p'+k}+E_{p-k}+\omega_k+\omega_{p-p'-k}-E)}\right.$$
$$\times\frac{1}{(E_p+E_{p'}+\omega_{p-p'-k}+\omega_k-E)(E_p+E_{p-k}+\omega_k-E)(E_{p'}+E_{p'+k}+\omega_k-E)}$$
$$\times\frac{1}{(E_p+E_{p-k}+\omega_k-E)(E_{p'}+E_{p-k}+\omega_{p-p'-k}-E)(E_{p-k}+E_{p'+k}+\omega_k+\omega_{p-p'-k}-E)}$$
$$\times\frac{1}{(E_p+E_{p'+k}+\omega_{p-p'-k}-E)(E_{p'}+E_{p'+k}+\omega_k-E)(E_{p-k}+E_{p'+k}+\omega_{p-p'-k}+\omega_k-E)}$$
$$+\frac{1}{(E_p+E_{p'}+\omega_{p-p'-k}+\omega_k-E)(E_p+E_{p-k}+\omega_{p-p'-k}-E)(E_{p'}+E_{p-k}+\omega_{p-p'-k}-E)}$$
$$\left.+\frac{1}{(E_p+E_{p'+k}+\omega_{p-p'+k}-E)(E_{p'}+E_{p-k}+\omega_{p-p'-k}-E)(E_{p-k}+E_{p'+k}+\omega_k+\omega_{p-p'-k}-E)}\right]. \quad \text{(B.22)}$$

These are just the no-pair terms of the old Tamm-Dancoff method, so that each of the above solutions corresponds to a Feynman diagram.

The remaining 3 terms of the second term of (B.10) can be treated in the same manner, each giving 6 matrix elements. Altogether the crossed diagram corresponds to 24 Tamm-Dancoff diagrams. The method can be applied to the iterated ladder and the results will come out in the form of reducible and irreducible terms. Finally, we note that the above method of integration can be generalized to more complicated cases.