PHYSICAL REVIEW VOLUME 117, NUMBER 1 JANUARY 1, 1960

Transient Temperature Variations During the Self-Heating of a Plasma by Thermonuclear Reactions*

HOWARD D. GREYBER†
Radiation Laboratory, University of California, Livermore, California
(Received April 3, 1959)

The ion and electron temperature variations in an idealized controlled thermonuclear reactor burning various ratios of deuterium and tritium are calculated. The variation in the fraction of energy deposited to the nuclear and electron gases with temperature is specifically included. One qualitative result is that it would be unsafe to assume nuclear temperature always equal to or greater than electron temperature for a nuclearly reacting plasma with tritium concentration greater than ten percent.

TO discuss the energy balance in a fully ionized gas with nuclei of very low Z and with a temperature in the region of 10–1000 kiloelectron volts (kev), it is necessary to consider the temperatures of the ion and electron gases separately. This is because the energy transfer mechanism between ions and electrons becomes, in this region, smaller than the characteristic energy sources and sinks for the two gases.

If, in addition, the radiation produced escapes more or less freely from our system,[1] the radiation present is not in equilibrium with the matter (and, in fact, the concept of radiation temperature is not applicable). If we state that the energy source for our fully ionized gas is the thermonuclear reactions among its constituents, then the combination of the aforesaid conditions is usually described as "nonequilibrium thermonuclear burning": (1) $T_n \neq T_e$, where T_n = nuclear or ion temperature, T_e = electron temperature; (2) $T_n, T_e \gg$ "radiation temperature."

Such a process is probably at least a rough approximation to the thermonuclear burning contemplated in some form of controlled thermonuclear reactor. Due to the extremely low density of the plasma, the system is essentially "open" to radiation. Also, if a 50% d, 50% t plasma is contemplated, for temperatures above 10 kiloelectron volts, T_n will be appreciably different than T_e.

The motivation for this work arose from an observation by Rosenbluth that in a different but related physical situation, T_e could exceed T_n during transient heating. This was in direct contradiction to the picture commonly invoked for a controlled thermonuclear reactor that the energy is produced by the ions, transmitted from ions to electrons, and lost by bremsstrahlung radiation from the electrons. The explanation is simply that the fast product charged particles from the thermonuclear reactions deposit energy to both the ion and electron gases. We have undertaken to trace the transient temperatures to be expected in an idealized physical situation that still bears some resemblance to what one envisions for the controlled thermonuclear reactor.

If we assume a system completely open to neutrons and radiation and closed as far as loss of charged particles or any other form of energy drain other than neutrons and radiation is concerned, we may write for a constant-volume system, i.e., magnetic field increasing in such a way as to keep constant volume,

$$(1.6\times10^{-9})\times\tfrac{3}{2}nT_n = aP_{dd}+bP_{dt}-P_{ie}, \tag{1a}$$

$$(1.6\times10^{-9})\times\tfrac{3}{2}nT_e = (1-a)P_{dd}+(1-b)P_{dt}+P_{ie}-P_B, \tag{1b}$$

where T is the kinetic temperature in kiloelectron volts, n is the number of particles per cubic centimeter, P_{dd} is the energy production rate from dd reaction, and is equal to $\frac{1}{2}n_d^2\langle(\sigma v)_{dd}\rangle_{\text{Av}}\times[2.4+\frac{1}{2}(3.5)]\times(1.6\times10^{-6})$ ergs/cm^3 sec. Only deposition of charged particle energy is counted. One half of a dt reaction is added for each dd reaction. We assume no depletion and instantaneous energy deposition. P_{dt} is the energy production rate from dt reaction, and equals $n_dn_t\langle(\sigma v)_{dt}\rangle_{\text{Av}}\times3.5(1.6\times10^{-6})$ ergs/cm^3 sec, n_d is the number of deuterons per cm^3, and n_t is the number of tritons per cm^3.

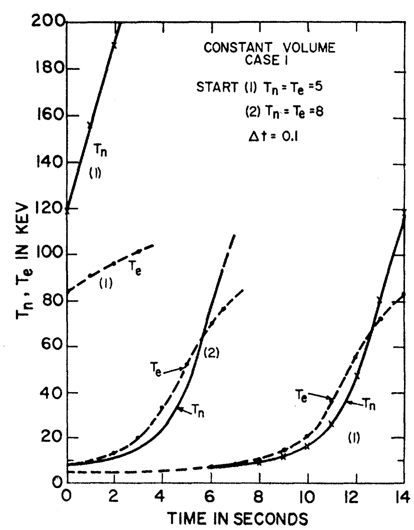

FIG. 1. Electron and nuclear temperature evolution for a 50% t, 50% d mixture.

* A preliminary report of this work appeared in U.S.A.E.C. report, Washington 289, June 1955.

† Now at the Geophysics Corporation of America, Boston, Massachusetts. Research supported by the U. S. Atomic Energy Commission.

[1] Dimensions "reasonable," i.e., 0.1 to 100 meters; density is low.

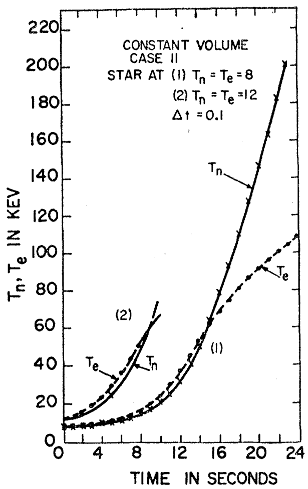

FIG. 2. Electron and nuclear temperature evolution for a 10% t, 90% d mixture.

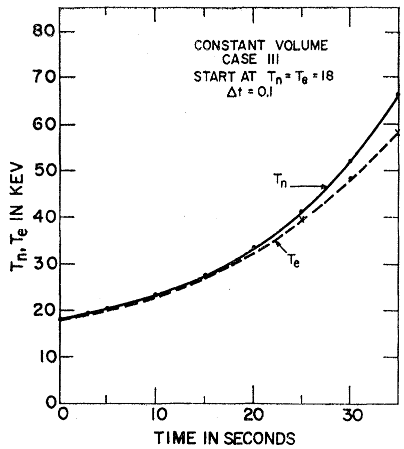

FIG. 3. Electron and nuclear temperature evolution for a 1% t, 99% d mixture.

The coefficients a and b indicate the fraction of nuclear energy deposited to the nuclear and electron gases and are approximations to graphs showing the slowing down cross section *versus* energy of the charged product particle. a is taken to be $0.3+0.005\ T_e$ and b is $0.01\ T_e$. The expression for a happens to be identical with that used by Fermi and Ulam in work at Los Alamos. P_{ie} is the energy transfer rate from nuclear gas to electron gas and is equal to $1.244\times10^{-21}n^2 \times[(T_1-T_2)/T_2^{3/2}]$ ergs/cm³ sec, while P_B is the energy production rate in bremsstrahlung radiation and equals $5.38\times10^{-24}n^2T_e^{1/2}$ ergs/cm³ sec.

We have done the calculations for $n=10^{14}$ but it is clear that one may scale the results simply for number density. The time scale will vary inversely with density; hence for $n=10^{15}$ cm³, divide times on accompanying graphs by 10, etc.

The integrations were carried out on an IBM card-programmed calculator, and the results are shown in Figs. 1, 2, and 3. Three cases were run: (I) 50% t and 50% d; (II) 10% t, 90% d; and (III) 1% t, 99% d. Some cases were run for two different starting temperatures.

A check calculation was run with $\Delta t=0.01$, 0.1, and 1.0 second and the comparison showed that the interval of $\Delta t=0.1$ sec used gives graphs quite close to those for $\Delta t=0.01$ sec and hence, fairly accurate.

However, a much larger source of error is our assumption of instantaneous deposition of energy by the fast reaction-product charged particles. A rough estimate of the time for a 3.5-Mev He^4 nucleus (or a 1-Mev t) to slow down to zero energy in a deuterium-tritium mixture at $n=10^{14}$ is given by

$$\tau=0.4(T_e^{1/2}),$$

where T_e is in kev and τ is in seconds.

Thus, for $T_e=25$ kev, $\tau=2.0$ seconds. One can infer how the curves would look were this factor correctly introduced into the calculation. First we note that qualitatively, since a fast charged particle spends most of its time losing energy to the electron gas, and, only near the end of its range, speedily loses the remainder to the nuclear gas, (1) the T_e curve will exceed the T_n curve by a larger amount in the range below 70 kev than calculated; (2) the crossover points where $T_e=T_n$ will be at higher values than calculated. Also, the time scales for the rise of temperature with time will be increased, i.e., the time to reach $T_n=100$ kev may be greater than calculated by perhaps as much as a factor of 2.

The results of the calculation, although rough, give us an idea of the time scales involved in the self-heating process. Of course, other losses than considered here, such as energy lost in particles diffusing to the walls, energy lost in particles leaking through magnetic mirrors, or useful output such as work against moving mirrors, or particles exiting in a divertor, etc., would all tend to lengthen the time for a given temperature rise. It is clear that our Eqs. (1a) and (1b) could easily be generalized to include extra terms such as those mentioned above, or energy input terms such as compression or magnetic pumping while burning, if simple expressions are available. The time-dependent energy deposition feature, although messy, could also be added.

We derived the equations for the case of a plasma at constant pressure while burning (constant magnetic field—experimentally obtained by "crowbarring" the magnet coils) and integrated them also on the card-programmed computer. Unfortunately, there appears to have been a numerical error in these calculations and no graphs are given for these cases.

One qualitative result of this work is that it would be unsafe to assume nuclear temperature always equal to or greater than electron temperature in a discussion of plasma instabilities for a plasma with tritium concentration equal to or greater than 10%.

ACKNOWLEDGMENTS

It is a pleasure to thank Dr. Marshall Rosenbluth and Dr. George Bing for several stimulating discussions.