

# Experimental Investigation of Conduction Band of GaSb†

A. SAGAR

Westinghouse Research Laboratories, Pittsburgh, Pennsylvania, and University of Pittsburgh, Pittsburgh, Pennsylvania

(Received July 24, 1959)

The conduction band of GaSb was investigated by making the following measurements on a number of *n*-type samples with different carrier concentrations: (1) Hall effect and conductivity between 1.5°K and 370°K; (2) the change of resistance and Hall effect of the samples under hydrostatic pressure (up to  $14 \times 10^9$  dynes/cm<sup>2</sup>) at room temperature; and (3) the change of resistance due to uniaxial stress between 77°K and 370°K. Our data can be explained on the basis of a double conduction band for this material with the lowest band-minimum in the center of the Brillouin Zone characterized by spherical constant-energy surfaces and the next minima along [111] directions in *k*-space characterized by ellipsoidal constant-energy surfaces. Our data can be further interpreted by assuming that the deformation potentials for the two bands in GaSb are similar to those of the corresponding bands in *n*-type germanium. The energy separation of the two bands at room temperature is estimated to be about 0.08 ev. The observed temperature dependence of the piezoresistance could be explained by assuming that the energy separation  $\Delta E$  changes with temperature at a rate  $(d\Delta E/dT) \approx -3 \times 10^{-4}$  ev/°C. Throughout the analysis, the relaxation times for the electrons in the two bands were assumed to be independent of energy.

## I. INTRODUCTION

THE band structure of GaSb has been investigated by optical methods by Roberts and Quarrington,<sup>1</sup> Ramdas and Fan,<sup>2</sup> and Zwerdling *et al.*<sup>3</sup> The magnetoabsorption measurements at liquid helium temperature by Zwerdling *et al.*<sup>3</sup> revealed an oscillatory spectrum, attributable to direct transitions. Assuming the valence band of GaSb to be similar to that of Ge and Si, they conclude that the conduction band minimum in GaSb is in the center of the Brillouin Zone. They find the energy gap  $E_g = (0.813 \pm 0.001)$  ev and the electron effective mass  $m^* = (0.047 \pm 0.003)m_e$ . Leifer and Dunlap<sup>4</sup> have measured the resistivity and Hall coefficient for *p*-type GaSb in the intrinsic range of temperatures. Assuming deformation-potential scattering and a spherical energy band, they find the electron effective mass  $m^* = 0.20m_e$  and  $E_g$  (at  $T=0$ ) = 0.8 ev. The effective mass values derived from these two experiments differ by a factor of about five. This inconsistency can be understood qualitatively if one assumes the existence of two conduction bands characterized by different effective masses and close to one another in energy. On the basis of such a model, the electrical experiments at high temperatures would measure the properties of electrons in both bands so that the effective-mass value derived from such an experiment would be some kind of an average of the effective masses in the two bands. The magnetoabsorption experiments by Zwerdling *et al.*<sup>3</sup> involve transitions to the lower band only and would measure the properties of carriers only in that

band. The absorption edge studies in GaSb by Ramdas and Fan<sup>2</sup> indicate a temperature-dependent tail below  $\alpha = 100$  cm<sup>-1</sup>. They attribute this tail to indirect transitions. Their results on magnetoresistance<sup>2</sup> in GaSb do not show any anisotropy or longitudinal effects, which indicates a spherical band. It is not possible however, to critically analyze their results for lack of data on their experiments.

We have made the following measurements on *n*-type GaSb in order to understand the band-structure of this material: (a) Hall effect and conductivity measurements between 1.5°K and 370°K on samples of different carrier concentrations; (b) the effect of hydrostatic pressure on resistivity and Hall coefficient at room temperature; and (c) piezoresistance effect between 77°K and 370°K.

## II. EXPERIMENTAL

### 1. Material Preparation

The compound was prepared from 99.999% pure Ga and Sb. The undoped GaSb was always *p*-type and zone refining did not prove useful for purifying the compound. The *p*-type GaSb had acceptor impurities  $\approx 10^{17}$ /cm<sup>3</sup>. The origin of these impurities is not known. The *n*-type GaSb was prepared by doping the compound with tellurium. Single crystals were grown by controlling the growth of a single seed from a melt of GaSb in a hydrogen atmosphere. Single crystals of about 1 in.  $\times$  1 in.  $\times$   $\frac{1}{2}$  in. size were obtained by this method.

### 2. Sample Preparation

The crystals were oriented to within  $\pm 2^\circ$ . The samples were lapped and etched after cutting. Electrical contacts were made with an ultrasonic soldering iron, using tin or indium as the soldering material. The dimensions of the sample were about 1 mm  $\times$  2 mm  $\times$  12 mm.

† This work is part of a thesis submitted to the Department of Physics, University of Pittsburgh, Pittsburgh, Pennsylvania, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

<sup>1</sup> V. Roberts and J. E. Quarrington, J. Electronics **1**, 152 (1955-56).

<sup>2</sup> A. K. Ramdas and H. Y. Fan, Bull. Am. Phys. Soc. Ser. II, **3**, 121 (1958).

<sup>3</sup> Zwerdling, Lax, Button, and Roth, J. Phys. Chem. Solids **9**, 320 (1959).

<sup>4</sup> H. N. Leifer and W. C. Dunlap, Jr., Phys. Rev. **95**, 51 (1954).

### 3. Measurements

#### a. Hall and Conductivity Measurements

Hall measurements were made at two different sections of each sample, to check the homogeneity of the carrier concentration. Samples with carrier concentration variations of more than 10% along their lengths were discarded. The magnetic fields used were between 500 and 6000 gauss. The apparatus used for measurements between 77°K and 370°K was similar to that used by Ure.<sup>5</sup> The temperature was maintained constant to within  $\pm 0.5^\circ\text{K}$ . Measurements in the liquid helium temperature range were made using conventional equipment. The temperature was maintained constant to within  $\pm 0.05^\circ\text{K}$ . The temperatures below 4.2°K were determined from helium vapor pressure measurements.<sup>6</sup> The accuracy of all Hall measurements was within 5%.

#### b. Piezoresistance Measurements

The piezoresistance phenomenon deals with the effect of stress (or strain) on the resistivity of the material and is described mathematically by a 4th rank tensor. The properties of this tensor for various crystal symmetries have been discussed in detail by Smith.<sup>7,8</sup> For a cubic crystal the piezoresistance tensor  $\pi$  is defined by

$$\delta\rho_i/\rho_0 = \sum_j \pi_{ij} X_j,$$

where  $\rho_0$  is the resistivity of the material without stress and  $X$  is the stress tensor. The indices  $i$  and  $j$  run from 1 to 6, as in the elasticity theory. For a cubic crystal there are three independent coefficients  $\pi_{11}$ ,  $\pi_{12}$ , and  $\pi_{44}$ .

We have made longitudinal piezoresistance measurements (with current and stress in the same direction) along various crystallographic directions. No transverse piezoresistance measurements (with stress direction perpendicular to current direction) were made because of the difficulty of making large-area low-electrical-resistance contacts.

The fractional change of resistance per unit stress (with current and stress in the same direction) in terms of the  $\pi$ 's is given by the expression<sup>9</sup>

$$\frac{1}{R} \frac{dR}{dP} = K' + \pi_{11} - 2(l^2 m^2 + m^2 n^2 + n^2 l^2)(\pi_{11} - \pi_{12} - \pi_{44}), \quad (1)$$

where  $l$ ,  $m$ ,  $n$  are the direction cosines of the specimen axis along which the stress is applied and  $K'$  is the correction term due to the dimensional changes of the sample.

The method used for piezoresistance measurements was similar to that used by Pollak.<sup>10</sup> The samples were subjected to an alternating stress (27 cps) of the order

<sup>5</sup> R. W. Ure, Jr., Rev. Sci. Instr. **28**, 836 (1957).

<sup>6</sup> C. T. Linder, Westinghouse Research Report R-94433-2-A, 1950 (unpublished).

<sup>7</sup> C. S. Smith, Phys. Rev. **94**, 42 (1954).

<sup>8</sup> C. S. Smith, *Solid State Physics*, edited by F. Seitz and D. Turnbull (Academic Press, Inc., New York, 1959), Vol. 6, p. 175.

<sup>9</sup> R. F. Potter, Phys. Rev. **108**, 652 (1957).

<sup>10</sup> M. Pollak, Rev. Sci. Instr. **29**, 639 (1958).

of  $5 \times 10^7$  dynes/cm<sup>2</sup>. The accuracy of all measurements was within 5%. The measurements by this method give adiabatic constants. A correction has to be made for converting these to isothermal values. This correction term is estimated to be  $\approx 1 \times 10^{-12}$  cm<sup>2</sup>/dyne. We have neglected this correction term in the calculation of our results, as it is small and does not affect our final conclusions.

Measurements between 77°K and 300°K were made in cooled methylbutane and the temperature was measured with a copper-constantan thermocouple.

#### c. Measurements under Hydrostatic Pressure

It is evident from Eq. (1) that all the three piezoresistance coefficients cannot be determined from the longitudinal measurements only. We have measured the effect of hydrostatic pressure on the resistance of these samples at room temperature to complete the information on the three coefficients.

The effect of hydrostatic pressure on the resistance of the material in terms of the  $\pi$ 's is given by

$$d \ln R / dP = K'' + (\pi_{11} + 2\pi_{12}),$$

where  $K''$  is the dimensional correction term. The dimensional correction terms have been calculated by Smith<sup>7</sup> for different situations in terms of the elastic constants of the material. We have neglected these correction terms in our calculations for both the longitudinal and hydrostatic measurements, as they are small ( $\approx 2 \times 10^{-12}$  cm<sup>2</sup>/dyne) and would not affect our final conclusions.

The maximum pressure used was  $14 \times 10^9$  dynes/cm<sup>2</sup>. Pressure was measured to within  $\pm 2 \times 10^7$  dynes/cm<sup>2</sup>. The value of  $(\pi_{11} + 2\pi_{12})$  at  $P=0$  was derived by differentiating the  $R$  vs  $P$  curves and extrapolating the value of  $(d \ln R / dP)$  to  $P=0$ .

Hall effect measurements under hydrostatic pressure were made in a beryllium-copper chamber. The maximum pressure used for these measurements was  $8 \times 10^9$  dynes/cm<sup>2</sup>.

## III. EXPERIMENTAL RESULTS

### 1. Hall Coefficient vs Temperature

Results of Hall coefficient measurements between 1.5°K to 370°K are shown in Fig. 1. The Hall coefficient

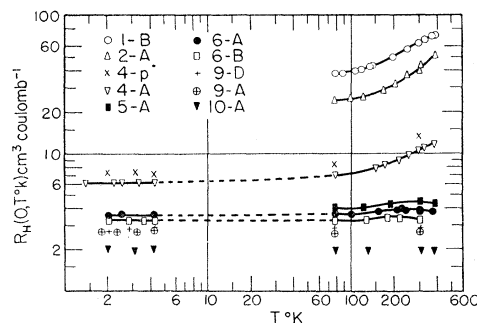


FIG. 1. Hall coefficient as a function of temperature for  $n$ -type GaSb samples of different carrier concentrations.

was found to be independent of the magnetic field strength, in the range of the fields used. The Hall coefficient increases steadily with increasing temperature for samples of highest purity. For samples with intermediate carrier concentrations, the Hall coefficient passes through a maximum. This maximum moves towards lower temperature and becomes less pronounced for samples with higher carrier concentrations. Finally for samples with very high carrier concentrations, the maximum disappears and the Hall effect remains constant with temperature.

## 2. Conductivity vs Temperature

Results of conductivity measurements between 1.5°K and 370°K are shown in Fig. 2. The values of  $\sigma$  and  $R_H$  at 4.2°K, 77°K, and 300°K for various samples are given in Table I. The conductivity increases uniformly with increasing carrier concentrations, except in samples 9-D and 10-A, where the conductivity values

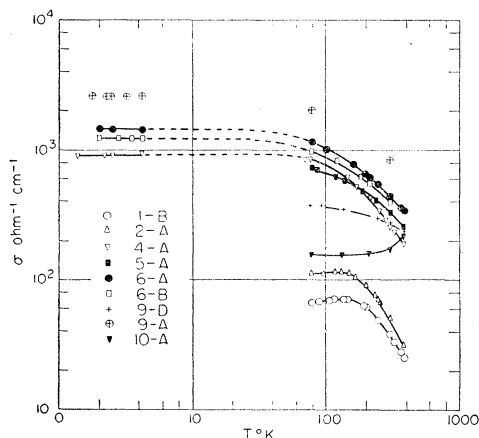


FIG. 2. Conductivity as a function of temperature for *n*-type GaSb samples with different carrier concentrations.

are anomalously low. These samples exhibit anomalous behavior (e.g., nonohmic resistivity and negative magnetoresistance) in the liquid helium temperature range. We have not attempted to interpret the data on these two samples.

## 3. Resistance vs Pressure Measurements

The effect of hydrostatic pressure on the resistance of various samples is shown in Fig. 3. The ordinate of the curves is the resistance at pressure  $P$ , normalized by division by the resistance at atmospheric pressure. Figure 4 gives the curves  $(d \ln R/dP)$  vs  $P$ , derived by differentiating the smooth curves  $R$  vs  $P$  drawn from the data of Fig. 3 for various samples. The value of the coefficient  $(d \ln R/dP)$  at  $P=0$  is higher for purer samples. The coefficient  $(d \ln R/dP)$  decreases with increasing pressure and tends to approach a small constant value ( $\sim 10 \times 10^{-12}$  cm<sup>2</sup>/dyne) for all samples

TABLE I. Conductivity and hall coefficient values at 300°K, 77°K, and 4.2°K for different samples.  $R_H$  is in cm<sup>3</sup> coulomb<sup>-1</sup>;  $\sigma$  is in ohm<sup>-1</sup> cm<sup>-1</sup>.

Sample	$R_H$ (300°K)	$\sigma$ (300°K)	$R_H$ (77°K)	$\sigma$ (77°K)	$R_H$ (4.2°K)	$\sigma$ (4.2°K)
1-B (single crystal)	64	38	37.6	67	X	X
2-A (polycrystalline)	40	51	24	111	X	X
4-P (single crystal)	13	X	8.3	X	7.1	X
4-A (single crystal)	10.5	260	7	825	6.25	924
5-A (polycrystalline)	4.5	331	4	715	X	X
6-A (polycrystalline)	3.75	436	3.6	1130	3.6	1420
6-B (polycrystalline)	3.3	410	3.25	975	3.3	1200
9-D (polycrystalline)	2.9	275	2.8	370	2.8	nonohmic
9-A (single crystal)	2.7	830	2.85	2030	2.8	2600
10-A (polycrystalline)	1.96	174	1.96	158	1.9	nonohmic

at high pressures; it seems to approach this constant value at lower pressures for purer samples than for impure samples. Furthermore, resistance vs pressure data for sample 9-D (which has anomalously low conductivity and shows anomalous behavior at low temperatures) seems to fall into a consistent pattern with the data on other samples.

## 4. Hall Effect vs Pressure

The Hall coefficients of samples 1-B, 4-A, and 9-A were measured as a function of pressure. The results are shown in Fig. 5. The Hall coefficient for sample 9-A decreases steadily with pressure. For samples 1-B and 4-A, it passes through a maximum and then decreases steadily with pressure. The maximum is more pronounced and occurs at higher pressure in sample 1-B than in sample 4-A.

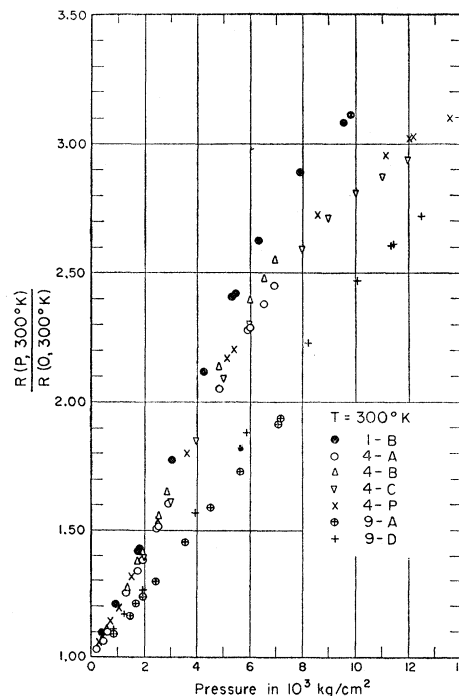


FIG. 3. Resistance vs pressure for *n*-type GaSb samples with different carrier concentrations at room temperatures.

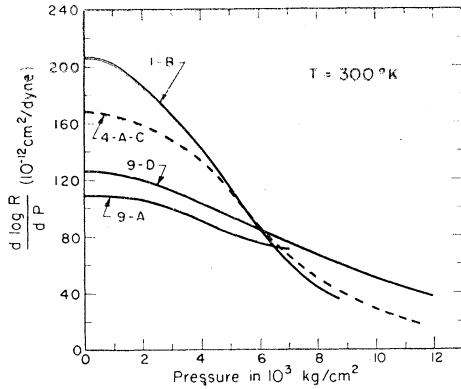


FIG. 4.  $(d \ln R / dP)$  vs pressure. These curves were drawn by differentiating the smooth curves  $R$  vs  $P$  taken from the data of Fig. 3.

### 5. Piezoresistance Measurements

The results of longitudinal measurements along with the results of pressure experiments at 300°K are shown in Table II. The piezoresistance effect was found to be sensitive to the carrier concentration of the samples. We need to make piezoresistance measurements on at least two samples with the same carrier concentrations, to determine the three piezoresistance coefficients  $\pi_{11}$ ,  $\pi_{12}$ , and  $\pi_{44}$ . The Hall coefficient values for the samples 4-A-B-C were the same within 5%. These samples were considered identical for the analysis of the piezoresistance data. The small differences in the carrier concentrations of the samples 4-A-B-C, would introduce an error in the values of  $\pi_{11}$ ,  $\pi_{12}$ , and  $\pi_{44}$  derived from the measurements on these three samples. This error will be small in the temperature range where the values of  $\pi$ 's are big, and will not affect our final conclusions. In the low-temperature range, however, where the values of  $\pi$ 's are small, this error may be quite important. Any quantitative analysis of the data in this range should be viewed in the light of the above remarks. The Hall values for samples 9-A-B were the same within 10%. These samples were also considered identical for the analysis of the piezoresistance data. The principal shear coefficients for the two sets of

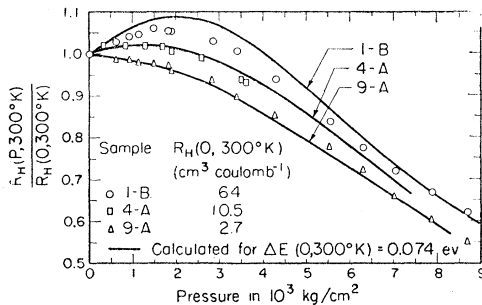


FIG. 5. Hall coefficient vs pressure data for three  $n$ -type GaSb samples. The curves are calculated by assuming  $b = \frac{1}{6}$ ,  $\Delta E(0, 300^\circ\text{K}) = 0.074$  eV and  $|d\Delta E/dP| = 1 \times 10^{-5}$  eV/atmos.

samples are given in Table III. It may be noticed that  $\pi_{11} \approx \pi_{12}$  and  $\pi_{44} \gg (\pi_{11} - \pi_{12})$  for both the samples.

The results of longitudinal piezoresistance measurements between 77°K and 300°K for samples 4 and 9 are given in Figs. 6 and 7, respectively. A plot of  $\pi_{44}$  vs  $T$  is also shown in these figures. The value of  $\pi_{44}$  at any temperature is calculated by assuming that  $\pi_{11} \approx \pi_{12}$  at all temperatures. It may be noticed that  $\pi_{44}$  decreases as the temperature is lowered and decreases more sharply for samples of higher purity (Samples 4-A-B-C) than for less pure samples (Samples 9-A-B).

### IV. DISCUSSION OF RESULTS

The increase in the value of  $R_H$  with increasing temperature as observed in Sec. III.1, Fig. 1, can be explained qualitatively by assuming the existence of two bands separated by the energy  $\Delta E$ , and characterized by different mobility values. The distribution of carriers in the two bands would depend on the temperature, more carriers going to the upper band as the temperature is raised. This would in effect increase

TABLE II. Piezoresistance data at 300°K (sign convention for  $\pi$ 's is the same as used by Smith<sup>11</sup>).

Sample	$R_H$ (at 300°K) $\text{cm}^3 \text{ coulomb}^{-1}$	Stress and current direction	Quantity measured	$\frac{1}{R} \frac{\partial R}{\partial P}$ ( $10^{-12} \text{ cm}^2/\text{dyne}$ )	$-\left(\frac{1}{P} \frac{\partial R}{\partial P}\right)_{P=0} = (\pi_{11} + 2\pi_{12})_{P=0}$ ( $10^{-12} \text{ cm}^2/\text{dyne}$ ) (by hydrostatic pressure)
4-A	10.5	[100]	$\pi_{11}$	-58.5	-168
4-B	10.1	[110]	$\frac{1}{2}(\pi_{11} + \pi_{12} + \pi_{44})$	-97.2	-173
4-C	...	[110]	$\frac{1}{2}(\pi_{11} + \pi_{12} + \pi_{44})$	-95	-168
9-A	2.7	[111]	$\frac{1}{3}(\pi_{11} + 2\pi_{12} + 2\pi_{44})$	-95.2	-107
9-B	2.85	[110]	$\frac{1}{2}(\pi_{11} + \pi_{12} + \pi_{44})$	-81	...

\* See reference 7.

the value of  $R_H$  with increasing temperature, until the carriers in the two bands contribute equally to the conductivity, after which  $R_H$  would decrease with increasing temperature.<sup>11,12</sup> The data of Sec. III.1 exhibits such a behavior. The exact shape of the  $R_H$  vs  $T$  curve would depend on (a) the value of the energy separation  $\Delta E$ ; (b) scattering mechanisms and mobilities in the two bands; and (c) the densities of states in the two bands. The results of the piezoresistance experiment of Sec. III.5, provide some information about the characteristics of the two bands. The room temperature values of the shear coefficients  $[\pi_{44} \gg (\pi_{11} - \pi_{12})]$  in Table III, indicate that one of the bands is germanium like, with ellipsoidal constant-energy surfaces and minima along [111] directions in  $k$ -space. The temperature dependence of  $\pi_{44}$  (Figs. 6 and 7) suggests that below this (111) band there is another band, the electrons in which do not contribute to this shear coefficient. As the temperature is lowered, more

<sup>11</sup> C. S. Hung, Phys. Rev. **79**, 727 (1950).

<sup>12</sup> R. J. Sladek, J. Phys. Chem. Solids **5**, 157 (1958).

and more electrons occupy this lower band, decreasing the value of  $\pi_{44}$ . We recognize this lower band as the one observed by Zwerdling *et al.*<sup>3</sup> in their magneto-absorption experiment. They report that the edge of the conduction band in GaSb is in the center of the Brillouin Zone.

The study by Taylor,<sup>13</sup> of the optical gap under hydrostatic pressure (up to 2000 atmospheres) indicates that the conduction band of GaSb moves away from the valence band at a rate of  $1.57 \times 10^{-5}$  eV per atmosphere. On the basis of our discussion in the preceding paragraph, this would be the rate at which the (000) minimum of the conduction band moves away from the valence band. It may be further observed that this is about the rate at which the (000) minimum of germanium<sup>14</sup> moves away from the valence band. If we assume that the (111) minima in GaSb also move away from the valence band at about the same rate as do the (111) minima in germanium ( $0.5 \times 10^{-5}$  eV/atmos), we find that the two bands in GaSb approach each other at a rate of about  $1.1 \times 10^{-5}$  eV/atmos. This would mean an increase in the relative population in the (111) band compared to that in the (000) band on applying hydrostatic pressure. Our data of Sec. III.3

TABLE III. Shear and dilation coefficients at 300°K and atmospheric pressure.

Sample	$\frac{R_H}{\text{cm}^3}$ coulomb <sup>-1</sup>	$\pi_{44}$ (10 <sup>-12</sup> cm <sup>2</sup> /dyne)	$(\pi_{11} - \pi_{12})$ (10 <sup>-12</sup> cm <sup>2</sup> /dyne)	$(\pi_{11} + 2\pi_{12})$ (10 <sup>-12</sup> cm <sup>2</sup> /dyne)
4-A-B-C	10.3 ± 0.5	-78	-2.8	-170
9-A-B	2.7 ± 0.3	-89.3	-4.1	-107

show a big increase in the resistance of GaSb with pressure, indicating that the electrons in the (111) band have lower mobility than those in the (000) band. This is also expected from theoretical considerations and by comparison with mobility data on other materials.<sup>15</sup> The coefficient ( $d \ln R / dP$ ) tends to approach a small constant value ( $\sim 10 \times 10^{-12}$  cm<sup>2</sup>/dyne) at high pressures. The value of the pressure at which ( $d \ln R / dP$ ) approaches this constant value is higher for samples with higher carrier concentrations than for purer samples. This can be qualitatively understood on the basis of the above model as follows: The coefficient ( $d \ln R / dP$ ) approaches zero (neglecting minor effects) when the (000) band is raised high enough (by applying hydrostatic pressure) so that all the carriers in the (000) band are transferred to the (111) band. This condition will be achieved at a higher pressure for a sample with higher carrier concentration (its Fermi level being higher) than for a purer sample. Furthermore, the conductivity *vs* pressure data on sample 9-D are consistent with the data on other samples, in spite

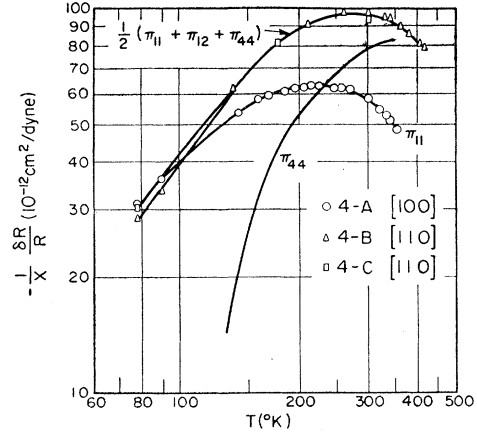


FIG. 6. Piezoresistance *vs* temperature for *n*-type GaSb. Detailed information about the sample is given in Table II. The curve  $\pi_{44}$  *vs*  $T$  was calculated from the piezoresistance data shown in this figure and by assuming  $\pi_{11} = \pi_{12}$  in the entire temperature range.

of its anomalous properties mentioned in Sec. III.2. The effects for which this sample shows anomalous behavior are primarily dependent on the scattering processes. This suggests that the anomalous behavior of this sample is probably due to the complicated scattering processes. The pressure effect, being mainly due to the inter-band transfer of carriers, does not show anomalous behavior for this sample.

The shift in the position of  $R_H(\text{max})$  towards lower pressures for samples with higher carrier concentrations, and the final disappearance of  $R_H(\text{max})$  for sample 9-A with the highest carrier concentration (Fig. 5), can be qualitatively understood on the basis of the above model as follows: The Fermi level is higher for samples with higher carrier concentrations, and there are proportionately more carriers in the (111) band. Thus

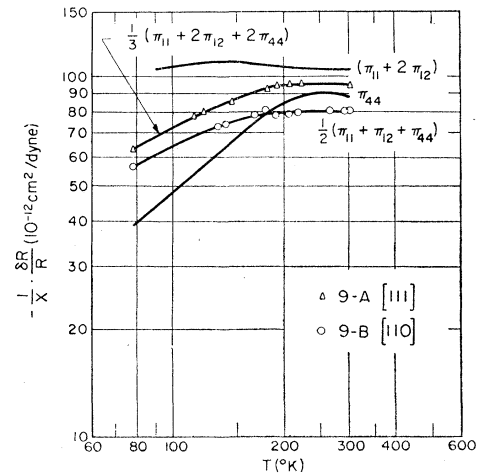


FIG. 7. Piezoresistance *vs* temperature for *n*-type GaSb. Detailed information about the samples is given in Table II. The curve  $\pi_{44}$  *vs*  $T$  was calculated from the piezoresistance data on these samples shown in this figure and by assuming  $\pi_{11} = \pi_{12}$  for the entire temperature range.

<sup>13</sup> J. H. Taylor, Bull. Am. Phys. Soc. 3, 121 (1958).

<sup>14</sup> W. Paul, J. Phys. Chem. Solids 8, 196 (1959).

<sup>15</sup> R. W. Keyes, J. Appl. Phys. 30, 454 (1959).

the condition of equal conductivity in the two bands [i.e., condition for  $R_H(\text{max})$ ] will be achieved at a much lower pressure for a sample with higher carrier concentration than for the one with lower carrier concentration. For sample 9-A the Fermi level is high enough to make  $\sigma_1 > \sigma_0$  even at normal pressure, and the condition for  $R_H(\text{max})$  cannot be achieved by applying hydrostatic pressure. Thus the data of Sec. III.4 on the change of the Hall effect with pressure are also in qualitative agreement with the above picture of the conduction band. The shape of the curves  $\sigma$  vs  $P$  and  $R_H$  vs  $P$  would depend strongly on the value of  $(d\Delta E/dP)$ .

Furthermore, this model explains qualitatively the discrepancy between the effective-mass value ( $m^* = 0.047m_e$ ) derived from the magnetoabsorption experiment<sup>3</sup> and the value ( $m^* = 0.20m_e$ ) derived from conductivity measurements<sup>4</sup> at high temperatures.

On the basis of the above discussion, the following model is proposed for the conduction band of GaSb: (a) The lowest minimum is in the center of the Brillouin Zone. It is characterized by spherical constant-energy surfaces with  $m^* = 0.047m_e$ . (b) The next minima are along (111) directions in  $k$ -space and are separated by an energy  $\Delta E$  from the (000) minimum. These minima are characterized by ellipsoidal constant-energy surfaces.

## V. ANALYSIS OF THE DATA

### 1. Theory and Assumptions

In this section we will see how well our data of Sec. III agree quantitatively with our calculations based on the model proposed in the preceding section. The expression for conductivity and Hall coefficient for our model is given by

$$\sigma = e(n_0\mu_0 + n_1\mu_1), \quad (2)$$

$$R_H = \frac{1}{ec} \frac{(n_0\mu_0\mu_{0H} + n_1\mu_1\mu_{1H})}{(n_0\mu_0 + n_1\mu_1)^2}, \quad (3)$$

where  $\mu_0$  and  $\mu_1$  are the drift mobilities in the (000) and (111) bands, respectively;  $\mu_{0H}$  and  $\mu_{1H}$  are the Hall mobilities<sup>16</sup> in the (000) and (111) bands, respectively.

$$N = n_1 + n_0 = [4\pi(2kT)^{3/2}/h^3][m_0^{3/2}F_{3/2}(\eta) + \nu_1 m_{1l}^{3/2} m_{1t} F_{3/2}(\eta - \Delta)], \quad (4)$$

$$\frac{n_1}{n_0} = \frac{\nu_1 m_{1l}^{3/2} m_{1t} F_{3/2}(\eta - \Delta)}{m_0^{3/2} F_{3/2}(\eta)}, \quad (5)$$

$$\eta = E_F/kT; \quad \Delta = \Delta E/kT,$$

$$F_\alpha(\eta) = \int_0^\infty \frac{x^\alpha dx}{(e^{x-\eta} + 1)},$$

<sup>16</sup> C. Herring, Bell System Tech. J. 34, 237 (1955).

where  $N$ =total number of carriers,  $\nu_1$ =number of minima in the (111) band,  $m_{1l}$ =longitudinal mass in the (111) band,  $m_{1t}$ =transverse mass in the (111) band,  $m_0$ =effective mass in the (000) band, and  $E_F$ =Fermi energy measured from the edge of the (000) band. The subscript "0" is used to denote the properties of the (000) band and "1" the properties of the (111) band.

We will make the following assumptions about some of the parameters involved in the above expressions for  $\sigma$  and  $R_H$ :

(a) The (000) band is characterized by an effective mass  $m^* = 0.047m_e$ . This result is taken from the magnetoabsorption measurements by Zwerdling *et al.*<sup>3</sup>

(b) The characteristics of the (111) band in GaSb are similar to those of the (111) band in germanium, i.e.,  $(m_{1l}/m_{1t}) \gg 1$ , and the value of the density of states is similar to that in germanium. This leads to the ratio between the density of states in the (111) band and the (000) band  $\approx 40$ .

(c) The rate of change of energy separation with pressure  $|d\Delta E/dP| \approx 1 \times 10^{-5}$  eV/atmos. We have taken this number from our discussion in the preceding section.

(d) *About the relaxation times in the two bands:* In the absence of any sound understanding of the scattering mechanisms involved, we are unable to determine the energy dependence of the relaxation times in the two bands. We will assume these relaxation times to be independent of energy. This is a very crude assumption and is made only for the sake of mathematical simplicity and because of lack of any better assumption.

### 2. Analysis of Conductivity and Hall Data for Samples 1-B, 4-A, and 9-A

#### a. Hall vs Pressure

First we will analyze the data on sample 1-B. We will use classical statistics in our analysis for this sample. This is justified because in the temperature range of interest (77°K to 300°K) the sample is non-degenerate. The data for  $R_H$  as a function of temperature suggest a value for  $R_H(\text{max})/R_H(\text{min}) \approx 2$  where  $R_H(\text{min})$  is the value of  $R_H$  extrapolated to  $T=0$ . This leads to a value of  $b \equiv \mu_1/\mu_0 \approx \frac{1}{6}$ . Using the value of  $b \approx \frac{1}{6}$  and the previously determined value of  $|d\Delta E/dP| \approx 1 \times 10^{-5}$  eV/atmos, we determine the value of  $\Delta$  which gives the best fit to  $R_H(P, T)/R_H(0, T)$  vs  $P$ . The results are shown in Fig. 5 for  $\Delta(0, 300^\circ\text{K}) = 2.85$ . This value of  $\Delta$  corresponds to  $\Delta E(0, 300^\circ\text{K}) \approx 0.074$  eV. The total number of carriers calculated by using the above values for the parameters is found to be  $N = 2.1 \times 10^{17}/\text{cm}^3$ . Next we examine the Hall data for samples 4-A and 9-A, using the values of parameters derived from Hall data of sample 1-B. The effect of degeneracy has to be taken into account for these samples. The values of  $\eta(0, 300^\circ\text{K})$  and  $N$  were determined for each sample by

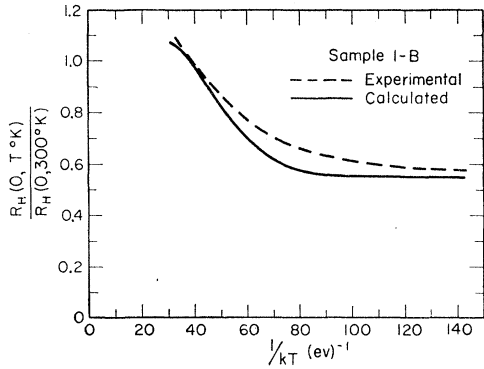


FIG. 8. Hall coefficient  $vs (kT)^{-1}$  between 370°K. to 77°K., for sample 1-B. The dashed curve is obtained by drawing a smooth curve through the experimental points in Fig. 1. The solid curve is calculated for  $b = \frac{1}{6}$  and  $\Delta E = 0.074$  eV, and by assuming that  $\Delta E$  is temperature independent.

forcing the Eqs. (3), (4), and (5) to give the experimentally observed value of  $R_H(0, 300^\circ\text{K})$ . We find  $\eta(0, 300^\circ\text{K}) = 0.7$ ;  $N = 1.4 \times 10^{18}/\text{cm}^3$  for sample 4-A and  $\eta(0, 300^\circ\text{K}) = 2.1$ ;  $N = 4.6 \times 10^{18}/\text{cm}^3$  for sample 9-A. The  $R_H vs P$  curves calculated for these two samples are also shown in Fig. 5. The agreement seems to be quite good for all the three samples.

#### b. Hall Effect vs Temperature

The  $R_H vs T$  curve was calculated for sample 1-B, assuming the energy separation  $\Delta E$  to be independent of temperature. The results are shown in Fig. 8. Again the agreement seems quite good. Similar agreement was obtained in case of other samples.

#### c. Conductivity vs Pressure

The results of similar calculations of  $\sigma vs P$  for these three samples are shown in Fig. 9. The agreement does

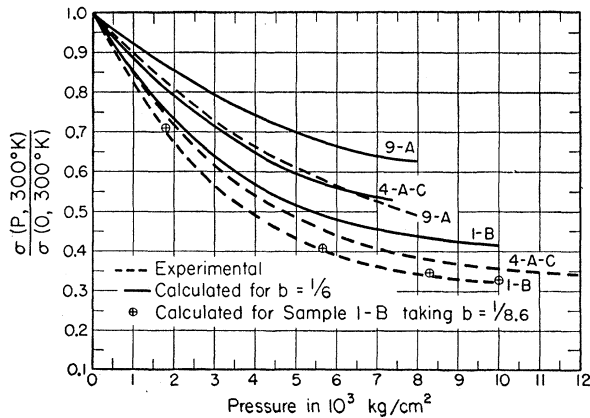


FIG. 9. Conductivity  $vs$  pressure for  $n$ -type GaSb samples. The dashed curves are obtained by drawing smooth curves through the experimental points of Fig. 3. The solid curves are calculated by assuming  $b = \frac{1}{6}$ ,  $\Delta E = 0.074$  eV and  $|d\Delta E/dP| = 1 \times 10^{-5}$  eV/atmos. The points are calculated for sample 1-B using  $b = 1/8.6$ .

not seem as good as for the Hall data. The conductivity data seem to agree better for  $b \approx 1/8.6$  (the points shown in Fig. 9 were calculated for sample 1-B using the value of  $b \approx 1/8.6$ ). This is not surprising, in view of the fact that the quantity  $b$  derived from Hall measurements is the ratio of Hall mobilities whereas  $b$  derived from conductivity data represents the ratio of drift mobilities. We have assumed throughout our calculations that the relaxation times are independent of energy. The difference in the values of  $b$  derived from Hall and conductivity data indicates that this assumption about the relaxation times is probably not quite true. As the expression for conductivity involves an additional unknown parameter (i.e., energy dependence of the relaxation times individually), the values of the parameters derived from the Hall data would be considered more reliable.

#### 3. Room Temperature Mobility Calculation for Samples 1-B, 4-A, and 9-A

We calculate the mobility  $\mu_1$  in the (111) band for these samples, using the experimental values of  $\sigma$  at 300°K (Table I), and the results of calculations in Sec.

TABLE IV. Room temperature mobility in the (111) band calculated from experimental value of  $\sigma$  in Table I.

Sample	$n_1/N$	$\sigma_1/\sigma^a$	$\mu_1$ $\text{cm}^2 \text{ volt}^{-1} \text{ sec}^{-1}$
9-A	85%	47.8%	645
4-A	78%	36.6%	574
1-B	70%	27.8%	430

<sup>a</sup>  $\sigma_1$  = conductivity due to carriers in the (111) band and  $\sigma$  = total conductivity.

V.2. The mobility in the (000) band is given by  $\mu_0 = 6\mu_1$ . The results are shown in Table IV. It may be noticed that these values for  $\mu_1$  are of the same order of magnitude as those for germanium<sup>17</sup> with similar carrier concentrations. No significance should be attached to the variation of  $\mu_1$  for different doping, as our calculations are very crude.

#### 4. Analysis of the Piezoresistance Data

The piezoresistance coefficient  $\pi_{44}$  for the case of band minima along [111] directions is given by<sup>18</sup> (assuming that the relaxation time is independent of energy):

$$\pi_{44} = - \frac{\Xi_u}{3C_{44}kT} \frac{\mu_1 - \mu_{11}}{2\mu_1 + \mu_{11}} \times \frac{1}{2} \frac{F_{-3}(\eta')}{F_{\frac{1}{2}}(\eta')}, \quad (6)$$

where  $\eta' = (E_F'/kT)$ ,  $E_F'$  is measured from the edge of the (111) band,  $\Xi_u$  is the deformation potential constant,  $C_{44}$  is the elastic constant,  $\mu_1$  and  $\mu_{11}$  are the transverse and longitudinal mobilities with respect to the axis of the ellipsoid. In our case, only a fraction of

<sup>17</sup> R. Bowers, Phys. Rev. **108**, 683 (1957).

<sup>18</sup> M. Pollak, Phys. Rev. **111**, 798 (1958).

TABLE V. Shear coefficient  $m_{44}$  for  $n$ -type GaSb at room temperature with corresponding values for  $n$ -type germanium.

Samples	$N$ (cm $^{-3}$ )	$n_1/N$	$\sigma_1/\sigma$	$\pi_{44}$ (experimental) <sup>a</sup> (10 $^{-12}$ cm $^2$ /dyne)	$m_{44} = \pi_{44}C_{44}\sigma/\sigma_1$	$m_{44}$ for Ge <sup>b</sup> with similar carrier concentration
4	$1.37 \times 10^{18}$	78%	36.6%	-78	-92	-100
9	$4.6 \times 10^{18}$	85%	47.8%	-89.3	-80.5	-93

<sup>a</sup> Values used for elastic constants at room temperature [see R. F. Potter, J. Phys. Chem. Solids 3, 223 (1957)].  $C_{44}(\text{GaSb}) = 0.432 \times 10^{12}$  dynes/cm $^2$ ;  $C_{44}(\text{Ge}) = 0.671 \times 10^{12}$  dynes/cm $^2$ .

<sup>b</sup> Values taken from results of Pollak (see reference 18). Pollak's results are about 10% higher than those of Smith (see reference 7).

the total number of carriers contributes to  $\pi_{44}$ , so that

$$\pi_{44}(\text{experimental}) = -\frac{\sigma_1}{\sigma} \frac{\Xi_u}{3C_{44}kT} \frac{\mu_1 - \mu_{11}}{2\mu_1 + \mu_{11}} \times \frac{1}{2} \frac{F_{-1/2}(\eta')}{F_{3/2}(\eta')}, \quad (7)$$

where  $\sigma_1$  = conductivity due to carriers in the (111) band and  $\sigma$  = total conductivity.

Our data on the temperature dependence of  $\pi_{44}$  (Figs. 6 and 7) do not agree with Eq. (7), if we assume that  $\Delta E$  is temperature independent. However the theoretical results can be brought into agreement with our data by assuming that the energy separation between the two bands changes with temperature at a rate  $(d\Delta E/dT) \approx -3 \times 10^{-4}$  eV/°C. This assumption does not seem unreasonable in view of the experimental evidence<sup>19</sup> which indicates similar temperature dependence of the energy gaps in other materials. Furthermore, this assumption is not inconsistent with our Hall data on GaSb.

The shear coefficient  $m_{44}$  for the case when all the carriers are in the (111) band is given by

$$m_{44} = (\sigma/\sigma_1)C_{44}\pi_{44}(\text{experimental}).$$

The value of  $m_{44}$  at room temperature is calculated for samples 4 and 9 using the values of  $\pi_{44}$  and  $\sigma/\sigma_1$ , from Tables III and IV, respectively. The results are shown in Table V. The corresponding numbers for  $n$ -type germanium<sup>18</sup> with similar carrier concentrations are also shown. The shear coefficient  $m_{44}$  in GaSb seems to be of the same order of magnitude as in germanium, indicating that the deformation potential  $\Xi_u$  is probably not very different in the two materials.

## VI. SUMMARY AND COMMENTS

Our experiments indicate a double conduction band for GaSb with the lowest band-minimum in the center

of the Brillouin zone and the next minima along the (111) directions in  $k$ -space. Our data can be interpreted by assuming that the deformation potentials for the two bands in GaSb are similar to those of the corresponding bands in  $n$ -type germanium.<sup>14,20</sup> The energy separation  $\Delta E$  between the two bands is estimated to be about 0.074 eV at room temperature. This estimate for  $\Delta E$  is based on the assumption that the density of states for the (111) band in GaSb is similar to that of the corresponding band in  $n$ -type germanium. Since we do not have the experimentally observed values for the effective mass of electrons in the (111) band in this material, this assumption may introduce some uncertainty in our estimates for the value of  $\Delta E$ . However, a change in the density of states mass in the (111) band by a factor of two, would bring about a change in the estimated value of  $\Delta E$  by only 0.03 eV, without appreciably changing the theoretical curves in Figs. 5 and 9.

The data on the temperature dependence of piezo-resistance could be explained qualitatively by assuming that the energy separation  $\Delta E$  between the two bands changes with temperature at a rate  $(d\Delta E/dT) \approx -3 \times 10^{-4}$  eV/°C. The Hall and conductivity data were in fair agreement with our calculations based on this model for the conduction band.

Our calculations suffer from lack of proper understanding of the scattering mechanisms involved.

## ACKNOWLEDGMENTS

The author is grateful to Dr. R. W. Keyes for directing this research. He wishes to thank Dr. M. Pollak and Dr. R. J. Sladek for various suggestions in the course of this work, and the latter for his advice in the low-temperature measurements. He also wishes to thank R. G. Seidensticker, Dr. A. J. Cornish, and Dr. J. W. Faust for their advice and help in sample preparation.

<sup>19</sup> Fan, Shepherd, and Spitzer, *Proceedings of the Conference on Photoconductivity, Atlantic City, 1954*, edited by R. G. Breckenridge et al. (John Wiley and Sons, New York, 1955).

<sup>20</sup> C. Herring and E. Vogt, Phys. Rev. **101**, 944 (1956).