PHYSICAL REVIEW VOLUME 117, NUMBER 2 JANUARY 15, 1960

Propagation of Plasma Waves across a Density Discontinuity*

ARNOLD H. KRITZ AND DAVID MINTZER
Laboratory of Marine Physics, Yale University, New Haven, Connecticut
(Received August 3, 1959)

The linearized plasma equations and the boundary conditions to be satisfied across a surface of density discontinuity are considered. It is shown that an incident longitudinal plasma wave generates reflected and transmitted transverse plasma waves, as well as reflected and transmitted longitudinal waves. The inverse process also occurs. The transverse waves are generated only when the longitudinal wave is almost normally incident; in the inverse process, the generated longitudinal waves are propagated normal to the boundary.

I. INTRODUCTION

We consider here the types of waves which result when pure longitudinal and pure transverse plasma waves are incident on a plasma density discontinuity. (Since we assume that the equilibrium pressure is the same on both sides of the boundary, a temperature discontinuity is associated with the density discontinuity.) In particular, we show that an incident longitudinal wave generates reflected and transmitted transverse waves, in addition to reflected and transmitted longitudinal waves. Thus, we describe a mechanism by which energy is transferred from compressional-type waves to electromagnetic-type waves. We also demonstrate that the inverse process occurs; transverse waves give rise to longitudinal waves when incident upon a boundary.

This problem, which bears upon the generation of radio noise in the solar atmosphere, as well as the generation of radio waves by plasma oscillations, has been partly treated by Field.[1] However, his work is restricted to the case of a plasma-vacuum boundary; this actually implies a gross velocity for the medium, which is not taken into account in his linearization of the plasma equations nor in his boundary conditions (which are incorrect for other reasons as well).

In Sec. II we discuss the plasma equations, and the types of waves which result. Since the boundary conditions are of considerable importance, they are considered in detail in Sec. III. Section IV contains the results for the cases of incident longitudinal and incident transverse waves; the reflection and transmission coefficients are calculated for all types of waves which appear in each case. Section V contains a discussion of the results, and some numerical calculations; in Sec. VI the work is summarized.

II. LINEARIZED PLASMA EQUATIONS

The plasma equations are obtained by applying the combined sets of hydrodynamic equations and Maxwell equations to a completely ionized gas. The assumption is made here that the ions are fixed in space, so that their only effect is to neutralize electrically the plasma when the electrons are uniformly distributed. Since we are thus considering a one-component (electron) gas, the parameters associated with the ions do not occur in the equations. The equations are linearized in the usual manner by assuming that the variations from equilibrium in the plasma variables are sufficiently small so that products of these variations can be neglected.[2]

The electron density and hydrostatic pressure are given, respectively, by $\rho_0+\rho$ and p_0+p; the ρ_0, p_0 are equilibrium values, and the ρ and p contain the space-time variations. It is assumed that there is no gross motion of the plasma and that no external fields are present; thus, the space-time variations of electron (fluid) velocity, electric field, and magnetic field are represented by **u**, **E**, **H**. The variables ρ, p, **u**, **E**, and **H**, as well as their space and time derivatives, are assumed to be of first-order smallness; products of first-order terms are neglected in the linearization. The linearized plasma equations are therefore:

$$\rho_0(\partial\mathbf{u}/\partial t)+\nabla p+(e\rho_0/m)\mathbf{E}=0, \tag{1}$$

$$\rho_0\nabla\cdot\mathbf{u}+\partial\rho/\partial t=0, \tag{2}$$

$$\nabla p=v^2\nabla\rho, \tag{3}$$

$$c\nabla\times\mathbf{E}=-\partial\mathbf{H}/\partial t, \tag{4}$$

$$\nabla\cdot\mathbf{E}=-(4\pi e/m)\rho, \tag{5}$$

$$c\nabla\times\mathbf{H}=-(4\pi\rho_0 e/m)\mathbf{u}+\partial\mathbf{E}/\partial t, \tag{6}$$

$$\nabla\cdot\mathbf{H}=0. \tag{7}$$

In the above equations, v is the adiabatic sound velocity, and c is the velocity of light; $-e$ and m are the charge and mass of the electron; ρ is the varying mass density, and $(e\rho/m)$ is the varying charge density; the current density is $-(\rho_0 e\mathbf{u}/m)$. Thus, the equation of conservation of mass, Eq. (2), is identical to the equation of conservation of charge.

The plane waves which may exist in the plasma are determined by assuming solutions to the plasma equations of the form $\exp[i(k\mathbf{n}\cdot\mathbf{r}-\omega t)]$; k is the propagation constant (or wave number) and, in general, is a function of the frequency, ω; **n** is a unit vector in the direction of propagation; and **r** is the vector to the observation point. By equating the

* Supported by the Office of Naval Research.
[1] G. B. Field, Astrophys. J. 124, 555 (1956).
[2] Cf., L. Oster, Revs. Modern Phys. 32, 141 (1960).

determinant of the coefficients of the plasma variables to zero (the condition for the solution of the simultaneous equations), an equation for k as a function of ω is found. This dispersion relation has three solutions. One solution corresponds to a longitudinal wave which has no magnetic field associated with it. The remaining solutions correspond to two transverse waves, of opposite polarization, which have no density variations associated with them. The propagation constants for these waves are:

$$\begin{aligned} k&=(\omega^2-\omega_e^2)^{\frac{1}{2}}/v, \quad \text{(longitudinal wave)} \\ K&=(\omega^2-\omega_e^2)^{\frac{1}{2}}/c, \quad \text{(transverse waves)} \\ \omega_e^2&=4\pi\rho_0 e^2/m^2, \end{aligned} \tag{8}$$

ω_e is the plasma frequency. If the propagation is along the x axis, the longitudinal wave has field variables u_x, E_x, p (or ρ); the transverse waves have variables u_z, E_z, H_y (and u_y, E_y, H_z). The relations between the variables are:

$$\left.\begin{aligned} \mathbf{u}_L&=-i\frac{m\omega}{4\pi e\rho_0}\mathbf{E}_L, \\ p&=-\frac{mv^2}{4\pi e}\nabla\cdot\mathbf{E}_L \end{aligned}\right\} \text{(longitudinal wave),}$$
$$\left.\begin{aligned} \mathbf{u}_T&=-\frac{ie}{\omega m}\mathbf{E}_T, \\ \mathbf{H}_T&=-\frac{ic}{\omega}\nabla\times\mathbf{E}_T \end{aligned}\right\} \text{(transverse waves).} \tag{9}$$

In general, all three waves may exist in the plasma; the boundary and initial conditions determine which waves exist for any physical problem.

Because the two transverse waves can act differently at a boundary, depending upon the direction of polarization with respect to the plane of incidence, they shall be denoted as follows: the transverse wave polarized with its magnetic vector perpendicular to the plane of incidence (electric vector in the plane of incidence) will be called a perpendicular magnetic (PM) wave; the wave with electric vector perpendicular to the plane of incidence will be called a perpendicular electric (PE) wave.

III. THE BOUNDARY CONDITIONS

Since the plasma equations are a set of coupled acoustic (or linearized hydrodynamic) and electromagnetic equations, the boundary conditions to be imposed are a combination of the usual acoustic and electromagnetic boundary conditions. It is to be noted that these sets of equations are coupled by the electronic charge. If e is set equal to zero, uncoupled acoustic and electromagnetic equations result, and the possible waves are then simply a longitudinal acoustic wave and transverse electromagnetic waves. This limiting case must be kept in mind when the conditions are imposed across a boundary.

The acoustic conditions to be imposed across the density discontinuity are the usual ones[3] of continuity of pressure and of the normal component of the fluid velocity:

$$p^i+p^r-p^t=0 \quad \text{at} \quad x=0, \tag{10}$$

$$\mathbf{i}_x\cdot(\mathbf{u}^i+\mathbf{u}^r-\mathbf{u}^t)=0 \quad \text{at} \quad x=0, \tag{11}$$

where the superscripts i, r, t refer to the incident, reflected, and transmitted waves, and $\mathbf{i}_x$ is the normal to the boundary, assumed at $x=0$. The continuity of pressure is a dynamic condition requiring the net force on the boundary to be zero. The continuity of the normal fluid velocity is a purely kinematic condition; if not imposed, then the fluids can separate. This condition therefore requires that the mass flow across the boundary be nonzero; this may be understood from the following considerations.

Suppose that the boundary is on the plane $x=0$ when there are no waves present; when a wave impinges upon the boundary, it moves with velocity $u_x^i+u_x^r=u_x^t$ (the linearization requires these velocities to be much smaller than the sound velocity, so that, to first order, the boundary conditions are taken at $x=0$). Across the plane $x=0$, there is a flow of mass per unit area, of

$$\Delta\rho_0 u_x=\rho_{01}(u_x^i+u_x^r)-\rho_{02}u_x^t, \tag{12}$$

where ρ_{01} and ρ_{02} are the equilibrium densities on the two sides of the boundary. There is therefore a transition layer, around $x=0$, of thickness $\xi=\int(u_x^i+u_x^r)dt=\int u_x^t dt=(i/\omega)u_x^t$ [assuming an $\exp(-i\omega t)$ time variation], which contains a "surface" distribution of mass per unit area of

$$\tau=(\rho_{01}-\rho_{02})\xi; \quad |\xi|\ll\lambda. \tag{13}$$

The condition on $|\xi|$ in Eq. (13) is just that $|\mathbf{u}|\ll v$, as is required for linearization. The continuity of pressure is insured since the rate of change of momentum per unit area, associated with this mass, is of second order.

There is associated with the "surface" mass distribution a "surface" charge distribution

$$\sigma\equiv(e/m)\tau=(e/m)(\rho_{01}-\rho_{02})\xi. \tag{14}$$

This "surface" charge distribution will give rise to a discontinuity in the normal component of electric field; however, the "surface" current density associated with this charge is of second order smallness and is therefore taken as zero in the boundary condition given by Eq. (16) below.

Since the two curl equations, Eqs. (4) and (6), and the equation of conservation of charge, (e/m) times Eq. (2), are a complete set of Maxwell's equations, the additional boundary conditions are derived from Eqs.

[3] R. B. Lindsay, *Concepts and Methods of Theoretical Physics* (D. Van Nostrand Company, Inc., New York, 1951), p. 369.

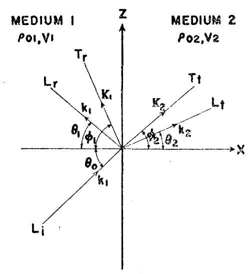

FIG. 1. Longitudinal wave (L_i: angle of incidence θ_0, wave number k_1) incident upon boundary (Case 1); reflected waves—longitudinal (L_r: θ_1, k_1), transverse (T_r: φ_1, K_1); transmitted waves—longitudinal (L_t: θ_2, k_2), transverse (T_t: φ_2, K_2).

(4) and (6) by applying the usual procedure[4]:

$$\mathbf{i}_x \times (\mathbf{E}^i + \mathbf{E}^r - \mathbf{E}^t) = 0, \tag{15}$$

$$\mathbf{i}_x \times (\mathbf{H}^i + \mathbf{H}^r - \mathbf{H}^t) = 0. \tag{16}$$

The discontinuity in the normal component of electric field can be obtained by taking the divergence of Eq. (6) and using the condition of continuity of the normal component of velocity. [This is equivalent to combining Eq. (5) and (e/m) times Eq. (2) indicating, of course, that Eq. (5) is not necessary for the complete set of Maxwell's equations.] Once again, applying the procedure for determining boundary conditions, we find that $\mathbf{i}_x \cdot \Delta \mathbf{E} = i(4\pi e/m\omega)\mathbf{i}_x \cdot \Delta \rho_0 \mathbf{u}$. Therefore, by comparing this with Eq. (14) and, Eq. (11), we see that the "surface" charge accounting for the discontinuity of normal electric field is the same as the surface charge derived from the transition layer considerations. The condition on the normal component of electric field is therefore

$$\mathbf{i}_x \cdot [\mathbf{E}^i + \mathbf{E}^r - \mathbf{E}^t] = 4\pi\sigma, \tag{17}$$

where σ is defined by Eq. (14). The boundary condition, on the normal component of magnetic field, derived from Eq. (7) is redundant (as is the equation itself).

A wave incident on any surface of discontinuity in a plasma can give rise to six possible waves, i.e., three reflected waves (one longitudinal and two polarizations of transverse waves) and three similar refracted waves. The six boundary conditions, Eqs. (10), (11), (15), and (16), will determine the amplitudes of the six waves. The "surface" charge distribution can then be determined by using Eq. (17). If the acoustic and electromagnetic systems are "uncoupled" by setting e equal to zero, then the electric field is no longer related to the fluid velocity, and the normal component of electric field becomes continuous as expected for dielectric media.

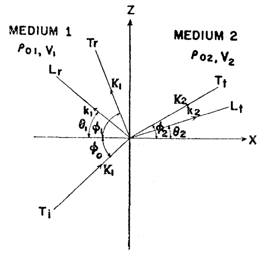

FIG. 2. Transverse wave (T_i: angle of incidence φ_0, wave number K_1) incident upon boundary (Case 2); reflected waves—longitudinal (L_r: θ_1, k_1), transverse (T_r: φ_1, K_1); transmitted waves—longitudinal (L_t: θ_2, k_2), transverse (T_t: φ_2, K_2).

[4] See J. A. Stratton, *Electromagnetic Theory* (McGraw-Hill Book Company, Inc., New York, 1941), p. 34 ff.

Since the requirement of continuity of normal particle velocity across a boundary gives rise to a discontinuity of normal electric field, one may expect that the presence of a boundary (e.g., of density discontinuity) will cause a coupling between the longitudinal and transverse waves. If a longitudinal wave, having a particle velocity $\mathbf{u}$ and associated electric field $\mathbf{E}$, impinges upon the boundary, the acoustic boundary conditions Eqs. (10) and (11) will "tend to" set up reflected and transmitted longitudinal waves; the electric fields associated with these longitudinal waves (alone) cannot satisfy Eq. (14) (unless the incident wave is normal to the boundary). Thus reflected and transmitted transverse waves are required in order to satisfy all the boundary conditions.

Similar effects hold for the case of an incident transverse wave with its electric field in the plane of incidence (PM wave). Moreover, since the longitudinal wave and the PM wave have their electric fields in the plane of incidence, the PE wave will not be generated, as seen from Eqs. (14) and (16). The PE wave, on the other hand, having its electric field parallel to the boundary, will act simply as does an electromagnetic wave (with the wave number K), satisfying Eqs. (14) and (16), and will not couple to the PM transverse or longitudinal waves.

Since the incident PM transverse wave does give rise to reflected and transmitted longitudinal plasma waves, the PM reflection (and transmission) coefficients are quite different from those obtained in the ordinary electromagnetic case. Therefore, the existence and properties of longitudinal plasma waves probably can be inferred from this difference in reflection coefficients.

It is to be noted that, allowing the electric charge to to to zero in the plasma equations, "decouples" the hydrodynamic and electromagnetic fields. In this case, the boundary conditions are no longer related to each other [σ goes to zero in Eq. (17)], and the separate acoustic and electromagnetic boundary conditions result.

IV. APPLICATION OF THE BOUNDARY CONDITIONS

Let us consider two semi-infinite media separated by the plane $x=0$. Medium 1, for $x<0$, will be characterized by an equilibrium density ρ_{01} and sound velocity v_1 (and plasma frequency ω_{e1}); medium 2, $x>0$, has equilibrium density ρ_{02}, sound velocity v_2 (plasma frequency ω_{e2}). We assume for the electric fields of the various waves the following form:

$$\begin{aligned} &\text{longitudinal wave} \quad E_L(\mathbf{i}_x \cos\theta + \mathbf{i}_z \sin\theta) \\ &\qquad\qquad \times \exp[ik(x\cos\theta + z\sin\theta) - i\omega t], \\ &\text{transverse PM wave} \quad E_T(-\mathbf{i}_x \sin\varphi + \mathbf{i}_z \cos\varphi) \\ &\qquad\qquad \times \exp[iK(x\cos\varphi + z\sin\varphi) - i\omega t], \\ &\text{transverse PE wave} \quad E_T'\mathbf{i}_y \exp[iK(x\cos\varphi \\ &\qquad\qquad + z\sin\varphi) - i\omega t], \end{aligned} \tag{18}$$

where the longitudinal wave propagates at an angle θ with respect to the positive x-axis and has wave number k; the transverse waves propagate at an angle φ with respect to the positive x-axis and have wave number K (see Figs. 1 and 2). The wave numbers k and K are given by Eq. (8). The other variables associated with the waves are found from Eq. (9).

Case I. Incident longitudinal waves.—We consider first a purely longitudinal wave, with wave number k_1, incident on the density discontinuity at angle θ_0. The following waves are assumed: reflected-longitudinal (angle θ_1, wave number k_1) and transverse (angle φ_1, wave number K_1) waves; transmitted-longitudinal (angle θ_2, wave number k_2) and transverse (angle φ_2, wave number K_2); see Fig. 1. Applying the boundary conditions, Eqs. (10), (11), and (14), (16), at $x=0$, we find that the following forms of Snell's law must be satisfied:

$$k_1 \sin\theta_0 = k_1 \sin\theta_1 = k_2 \sin\theta_2 = K_1 \sin\varphi_1 = K_2 \sin\varphi_2. \quad (19)$$

Also, the amplitude of the PE wave is identically zero; the longitudinal and transverse PM ($\mathbf{E}$ in plane of incidence) waves have reflected and transmitted amplitudes given by

$$\begin{aligned}
\frac{E_L^r}{E_L^i} &= \frac{1}{M}\left\{\left[1+\Delta\frac{\tan\varphi_1}{\tan\varphi_2}\right]\left[\frac{\Omega_1}{\Omega_2}\frac{\tan\theta_0}{\tan\theta_2}-1\right] + \Omega_1(1-\Delta)^2 \tan\theta_0 \tan\varphi_1\right\},\\
\frac{E_L^t}{E_L^i} &= \frac{1}{M}\left\{2\Delta\frac{\sin\theta_0}{\sin\theta_2}\left[1+\Delta\frac{\tan\varphi_1}{\tan\varphi_2}\right]\right\},\\
\frac{E_T^r}{E_L^i} &= \frac{1}{M}\left\{-2(1-\Delta)\frac{\sin\theta_0}{\cos\varphi_1}\right\},\\
\frac{E_T^t}{E_L^i} &= \frac{1}{M}\left\{-2\Delta(1-\Delta)\frac{\sin\theta_0}{\sin\varphi_2}\tan\varphi_1\right\},\\
M &= \left[1+\Delta\frac{\tan\varphi_1}{\tan\varphi_2}\right]\left[\frac{\Omega_1}{\Omega_2}\Delta\frac{\tan\theta_0}{\tan\theta_2}+1\right] + \Omega_1(1-\Delta)^2 \tan\theta_0 \tan\varphi_1,\\
\Omega_1 &= \omega_{e1}^2/\omega^2, \quad \Omega_2 = \omega_{e2}^2/\omega^2, \quad \Delta = (1-\Omega_1)/(1-\Omega_2).
\end{aligned} \quad (20)$$

These results will be discussed in Sec. V.

Case II. Incident transverse wave.—The case of an incident transverse PE wave (electric field perpendicular to the plane of incidence) is similar to that of an ordinary transverse electromagnetic wave, of wave number K_1, incident upon a boundary. Since there is no pressure wave associated with a transverse wave, Eq. (10) does not enter in the boundary condition; since the electric field and fluid velocity are in the y direction, Eqs. (11) and (15) do not enter. This incident PE wave is therefore not coupled by the boundary to longitudinal waves (or PM waves); the reflected and transmitted amplitudes are those given by Fresnel's law for the case of the electric field normal to the plane of incidence.[5]

An incident transverse PM wave, however, does have a coupling to longitudinal waves; again, the PE wave does not enter. Taking the incident wave at angle φ_0, wave number K_1; reflected and transmitted transverse waves at angles φ_1 and φ_2, with wave numbers K_1 and K_2, respectively; reflected and transmitted longitudinal waves at angles θ_1 and θ_2, with wave numbers k_1 and k_2, respectively; we have Snell's laws:

$$K_1 \sin\varphi_0 = K_1 \sin\varphi_1 = K_2 \sin\varphi_2 = k_1 \sin\theta_1 = k_2 \sin\theta_2. \quad (21)$$

The various electric field amplitudes are given by

$$\begin{aligned}
\frac{E_T^r}{E_T^i} &= \frac{1}{N}\left\{\left[1-\Delta\frac{\tan\varphi_0}{\tan\varphi_2}\right]\left[\frac{\Omega_1}{\Omega_2}\Delta\frac{\tan\theta_1}{\tan\theta_2}+1\right] - \Omega_1(1-\Delta)^2 \tan\theta_1 \tan\varphi_0\right\},\\
\frac{E_T^t}{E_T^i} &= \frac{1}{N}\left\{2\Delta\frac{\sin\varphi_0}{\sin\varphi_2}\left[1+\frac{\Omega_1}{\Omega_2}\Delta\frac{\tan\theta_1}{\tan\theta_2}\right]\right\},\\
\frac{E_L^r}{E_T^i} &= \frac{1}{N}\left\{-2\Omega_1(1-\Delta)\frac{\sin\varphi_0}{\cos\theta_1}\right\},\\
\frac{E_L^t}{E_T^i} &= \frac{1}{N}\left\{2\Omega_1\Delta(1-\Delta)\frac{\sin\varphi_0}{\sin\theta_2}\tan\theta_1\right\},\\
N &= \left[1+\Delta\frac{\tan\varphi_0}{\tan\varphi_2}\right]\left[\frac{\Omega_1}{\Omega_2}\Delta\frac{\tan\theta_1}{\tan\theta_2}+1\right] + \Omega_1(1-\Delta)^2 \tan\theta_1 \tan\varphi_0.
\end{aligned} \quad (22)$$

V. DISCUSSION OF THE RESULTS

It has been shown that transmission of a longitudinal wave across a discontinuity can cause transverse waves to be generated. However, only a small cone of incident angles are allowed for transverse waves to be formed. For a normally incident longitudinal wave, Eqs. (20) show that no transverse wave appears (as was noted in the discussion of the boundary conditions); moreover, for a true transverse wave, the angle of propagation (φ_1 for the reflected wave, and φ_2 for the transmitted wave) must be less than 90°. The angles are given by Snell's laws, Eq. (19):

$$\sin\varphi_1 = \frac{k_1}{K_1}\sin\theta_0 = \frac{c}{v_1}\sin\theta_0, \quad (23a)$$

$$\sin\varphi_2 = \frac{k_1}{K_2}\sin\theta_0 = \Delta^{\frac{1}{2}}\frac{c}{v_1}\sin\theta_0. \quad (23b)$$

For all except very small θ_0, the angles φ_1 and φ_2 will be complex since (c/v_1) is much greater than unity. If the angle of propagation is complex, the transverse

[5] Reference 4, p. 495.

wave becomes a surface wave propagating along the boundary (z direction) and exponentially attenuating in the direction normal to the boundary (x direction). Moreover, the electric field, in this case, has a component in the direction of propagation.

Calculations have been performed assuming an electron density of 4×10^9 electrons per cubic centimeter and a temperature of 10^5 °K. These values are comparable to values of these parameters in the solar corona. The ratio of densities across the discontinuity is taken to be 2:1, with transmission into the less dense medium. The plasma frequencies are computed using Eq. (8): $\omega_{e1}=3.58\times10^9$ cps and $\omega_{e2}=2.53\times10^9$ cps. Since $c/v_1=1.89\times10^2$, reflected transverse waves will only occur for angles of incidence less than zero degrees eighteen minutes. Because the ratio of propagation constants, Eq. (23b), contains the factor $\Delta^{\frac{1}{2}}$, the transmitted waves will be generated over a slightly larger range of incidence angles; the allowed range increases as ω approaches ω_{e1}. In Fig. 3 are plotted the relative electric field amplitudes of the created transverse reflected and transmitted waves. Three different frequencies of the incident wave are assumed: I, $\omega=3.94\times10^9$ cps; II, $\omega=10\times10^9$ cps; III, $\omega=18.8\times10^9$ cps.

The ratios of the propagation constant for an incident transverse wave to the propagation constants for the generated longitudinal waves,

$$K_1/k_1=v_1/c,\quad K_1/k_2=\Delta^{\frac{1}{2}}v_2/c, \tag{24}$$

are much less than unity—of the order 10^{-2} in the calculations which were performed. Therefore, PM transverse waves incident at any angle give rise to reflected and transmitted longitudinal waves. As a result of Snell's law, though, the angles of reflection

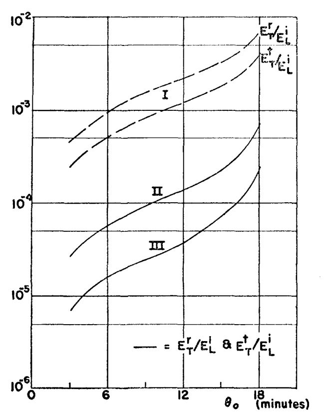

FIG. 3. Transverse wave reflection and transmission coefficients for incident longitudinal wave (Case 1) at incident angle of zero degrees, θ_0 minutes. Frequency of incident wave: I, $\omega=3.94\times10^9$ cps; II, $\omega=10\times10^9$ cps; III, $\omega=18.8\times10^9$ cps. Plasma frequencies: medium 1, $\omega_{1e}=3.58\times10^9$ cps; medium 2, $\omega_{2e}=2.53\times10^9$ cps.

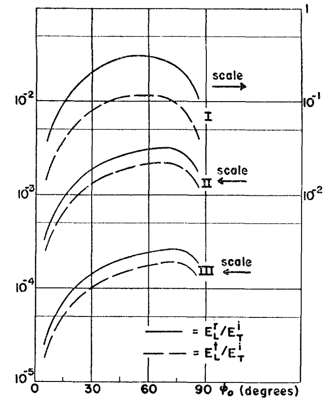

FIG. 4. Longitudinal wave reflection and transmission coefficients for incident transverse wave (Case II) at incident angle of φ_0 degrees. Frequency of incident wave: I, $\omega=3.94\times10^9$ cps; II, $\omega=10\times10^9$ cps; III, $\omega=18.8\times10^9$ cps. Plasma frequencies: medium 1, $\omega_{1e}=3.58\times10^9$ cps; medium 2, $\omega_{2e}=2.53\times10^9$ cps.

and transmission, θ_1 and θ_2 are approximately zero. Thus the created longitudinal waves propagate along the normal to the discontinuity.

The ratios of electric field amplitudes, of the created longitudinal waves, to that of the incident transverse wave, are plotted in Fig. 4. The parameters are assumed to have the same values as in the previous calculations.

VI. SUMMARY

We have investigated the problem of longitudinal and transverse waves incident on a plasma density (or temperature) discontinuity. Applying the boundary conditions, we have shown that energy can be converted from longitudinal waves to transverse waves. In addition, we have shown that the inverse process, transverse converted to longitudinal, also occurs provided that at least part of the incident transverse wave can be considered polarized with magnetic field perpendicular to the plane of incidence. The transverse waves are generated only when the longitudinal wave has a very small incidence angle; whereas, all angles of incidence for the transverse wave give rise to reflected and transmitted longitudinal waves. For each case calculations are made assuming three different values of frequency. As the frequency of the incident wave approaches the plasma frequency, the amplitudes of the generated waves increase. Comparing Figs. 3 and 4, one observes that the longitudinal waves are created more efficiently than the transverse waves. In addition to the difference in allowed range of incidence angle, the generated longitudinal waves have relative electric field amplitudes which are an order of magnitude greater than the relative electric field amplitudes of the generated transverse waves.