

Range of 1–10 keV Electrons in Solids

CHARLES FELDMAN*

United States Naval Research Laboratory, Washington, D. C.

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The range of 1–10 keV electrons in metals and phosphors has been measured by recording the light output as a function of energy from a sample consisting of a metallic layer deposited on a transparent phosphor layer. The measurements lead to the development of the following range-energy expression:

$$R = 250(A/\rho Z^{n/2})E^n,$$

where $n = 1.2/(1 - 0.29 \log_{10} Z)$, E = energy in keV, R = range in angstroms, and A , ρ , Z , have their customary meaning. This differs from the usual range expression in the dependency of n on Z . Usually n is assumed to be independent of material. This dependency can be justified on the basis of Bethe's theory of electron stopping power.

I. INTRODUCTION

THE preponderance of electron range measurements in the last decade have been confined to the energy region above 100 keV where there is reasonable agreement with theory.¹ At lower electron energies, particularly 10 keV and below, both measurements and theory are inadequate. Even for aluminum, the material most often studied, numerical agreement is poor. A sufficient number of materials have not been studied by any investigator in order to relate the range expressions to the properties of the material. There is general agreement, however, that the range-energy expression has the form:

$$R = bE^n, \quad (1)$$

where b is a constant related to the material and n is a constant assumed to be independent of material.² Some typical range-energy expressions are listed in Table I.^{3–9} The Thompson-Whiddington square relation³ has some theoretical justification, based on the stopping power theory of Bohr¹⁰ and Bethe.¹¹ Although these theories were derived for impinging particle energies much greater than the energies of the atomic electrons in the material, it is often assumed, that the theory is valid at lower energies. The rest of the range expressions in the table are purely empirical.

The situation is complicated by the fact that the investigators measured different types of ranges. This

is indicated in the last column of Table I. Due to the straggling effect, large numerical differences can exist between the types of ranges measured. This is illustrated by the electron absorption curve in Fig. 1.¹² From this curve, a maximum range, R_0 ; a practical or extrapolated range, R_p ; and an average range, R_a , are defined. A maximum practical range can also be defined as being the maximum measured range limited by the experiment. That is, it is impossible to detect the range of that single electron which has traveled the farthest since a certain number of electrons is needed before detection is possible. These electrons have ranges smaller than the maximum. Hence the maximum practical range will be less than the true maximum and will lie somewhere between the maximum range and the extrapolated range. It will become obvious that the range measured in the experiments to be described here corresponds to the maximum practical range just defined. This range will be designated by R in this paper.

The method employed for the determination of range involves the use of transparent luminescent films. It is a combination of the procedures of Young⁸ who used a phosphor powder screen for the detection of range in aluminum and the method indicated by Koller and

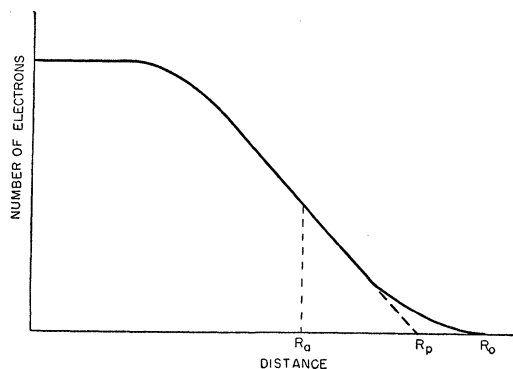


FIG. 1. Schematic electron absorption curve.

* Now at Melpar, Inc., Falls Church, Virginia.
¹ See H. A. Bethe and J. Ashkin, *Experimental Nuclear Physics* (John Wiley and Sons, New York, 1953), Vol. 1, p. 166; and B. Rossi, *High-Energy Particles* (Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1952).
² Throughout this paper range (R) will be expressed in angstrom units (\AA) and energy (E) in keV.
³ R. Whiddington, Proc. Roy. Soc. (London) **A86**, 360 (1912).
⁴ H. M. Terrill, Phys. Rev. **22**, 101 (1923).
⁵ L. Katz and A. S. Penfold, Revs. Modern Phys. **24**, 28 (1952).
⁶ W. Ehrenberg and J. Franks, Proc. Phys. Soc. (London) **66**, 1051 (1953).
⁷ R. O. Lane and D. J. Zaffarano, Phys. Rev. **94**, 960 (1954).
⁸ J. R. Young, J. Appl. Phys. **27**, 1 (1956).
⁹ A. Ia. Viatskin and A. F. Makhov, J. Tech. Phys. (U.S.S.R.) **28**, 740 (1958) [translation: Soviet Phys. (Tech. Phys.) **3**, 690 (1958)].
¹⁰ N. Bohr, Phil. Mag. **25**, 10, 1913; **30**, 581 (1915).
¹¹ H. A. Bethe, Ann. Physik **5**, 325 (1930).

¹² See F. Rasetti, *Elements of Nuclear Physics* (Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1936), p. 50.

TABLE I. Range-energy expressions for 0-10 kev electrons.

Investigator	Range-energy expression (numerical values given for aluminum only) $E(\text{kev})$, $R(\text{A})$	Type of range measured
Whiddington ^a	$R = 167E^2$	Average range
Terrill ^b	$R = 91E^2$	Average range
Katz and Penfold ^c	$\left\{ \begin{aligned} R &= 113E^{(1.265-0.0984 \ln E_0)} (E \text{ in Mev}) \\ R &= 158E^{1.72} \text{ at 10 kev} \end{aligned} \right\}$	Extrapolated range
Ehrenberg and Franks ^d	$R = (V_0^2/C) + d$	Average range
Lane and Zaffarano ^e	$R = 210E^{1.67}$	Extrapolated range
Young ^f	$R = 420E^{1.3}$	Extrapolated range
Viatskin and Makhov ^g	$R = bE^{1.4}$	Maximum range

^a See reference 3. ^b See reference 4. ^c See reference 5. ^d See reference 6. ^e See reference 7. ^f See reference 8. ^g See reference 9.

Alden¹³ and the author¹⁴ for the determination of range in transparent luminescent films. In brief, the method consists in examining the light emitted from a luminescent film coated with a metallic layer as the electron beam energy in a cathode-ray tube is varied. This results in a brightness *vs* energy curve shown in Fig. 2. Light is first emitted from the phosphor when the electrons pass through the metallic layer plus any inactive phosphor layer (dead layer) that may be present. The brightness curve then rises and remains linear until the point E_b is reached, at which time some of the electrons begin to pass through the phosphor. E_a and $E_b - E_a$ when corrected for the phosphor dead voltage, are then taken as the penetration energies of the metallic and phosphor layer, respectively. By varying the thickness of both layers, the range-energy relations for a large number of materials may be determined.

It should be pointed out that the purpose of this investigation was not to obtain extremely accurate range measurements, but rather to explore the values of b and n in Eq. (1) over a wide range of materials in order to determine their relationship, if any, with the atomic constants of the material.

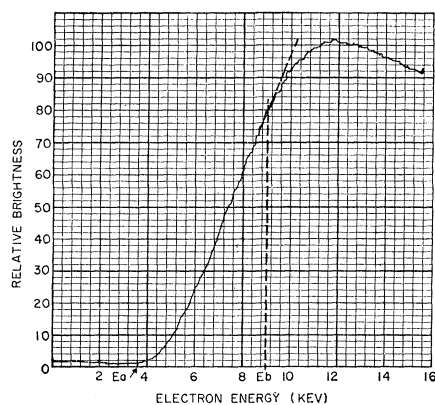


FIG. 2. Typical luminescent brightness *vs* electron energy recording.

¹³ L. R. Koller and E. D. Alden, Phys. Rev. **83**, 684 (1951).

¹⁴ C. Feldman, J. Opt. Soc. Am. **47**, 790 (1957).

II. EXPERIMENTAL

A. Preparation of Samples

The transparent phosphor samples were formed on Vycor substrates in a manner previously described.¹⁵ The metallic layers were deposited on the phosphor surfaces by standard vacuum deposition techniques. A few of the metallic layers were annealed in a vacuum at 500°C for one hour, but this did not affect the results and most of the layers were unannealed. The samples were placed in a demountable cathode-ray tube immediately after deposition of the metallic layers in order to limit oxidation and surface contamination. Thin aluminum layers, just thick enough to eliminate surface charge, were used when the range in phosphors was being studied. This prevented any undue electron straggling by the metallic layer. When the metal films were being studied, luminescent films of $\text{Zn}_2\text{SiO}_4(\text{Mn})$ were employed because of their high luminescent emission.

Pinholes in the luminescent films were no problem because they do not contribute to the emission and hence are not counted. Pin holes in the metal film could, of course, lead to errors. As far as possible, pinhole free samples were used. The effect of holes could, however, be detected and taken into account by inspection of the brightness-energy curves. That is, when electrons begin to penetrate the metallic layer there is a rapid change of slope in the brightness-energy curve, whereas the pinholes merely cause a linear background luminescence.

The thickness of the layers was measured with a Zeiss interference microscope fitted with a Filar micrometer eyepiece. Measurements were accurate to about 100 Å. In order to eliminate any error due to film nonuniformity, the thickness and range measurements were made on the same area of the sample. A slight error is introduced in determining the true thickness of the silicate phosphor layers. These layers are fired to 1100°C and react with the substrate causing the measured thickness to be less than the true thickness by a small but as yet undetermined amount.

¹⁵ C. Feldman and M. O'Hara, J. Opt. Soc. Am. **47**, 300 (1957).

B. Apparatus

The experimental arrangement employed to obtain luminescent emission *vs* electron energy curves is illustrated schematically in Fig. 3. The light meter which consists of a photomultiplier tube, amplifier, and associated circuitry, is focused on a defocused electron beam image. The output of the meter is fed to the Y axis of an X-Y recorder. The X axis of the recorder is connected to the high voltage power supply through appropriate resistors. The power supply (ripple, 0.01%; maximum input voltage variation 0.2%) is connected between the metallic surface of the sample and a grounded nickel grid of 250 mesh and 70% open area. The grid is in electrical contact with a conducting tin oxide layer on the inside surface of the tube and serves to maintain the size of the electron beam spot constant as the voltage on the sample is varied. Thus as the voltage from the power supply is manually increased, a trace is obtained on the X-Y recorder. Figure 2 is a reproduction of such a trace. The portion of the curve from 0-2 kv was sketched by hand since the recorded trace only began at the cathode voltage. The source of electrons was a 5 AB electron gun wired in the customary manner with the cathode held at -2 kv. Occasionally for very thin samples, the cathode voltage was reduced to -1 kv, in order to make measurements at that voltage. The beam current, which was about $5 \mu\text{a}/\text{cm}^2$, was kept low in order to avoid phosphor saturation effects. Within the sensitivity of the apparatus, the voltage at which penetration commenced was not affected by the value of the current. At low currents the luminescent brightness is low, but an increase in amplification resulted in approximately the same curve as when the current was high and the amplification low.

The sensitivity of the apparatus under the conditions used was such that a change of 2-3% in the beam current corresponded to 1 division in the recording paper when the phosphor films were being studied. When the metallic layers are being studied, the amplification was 10 times higher so that 1 division corresponded to less than 1% change in current.

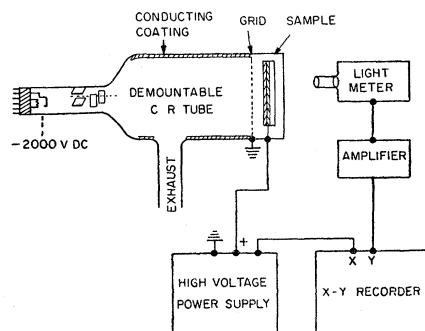


FIG. 3. Experimental circuit used to obtain recordings.

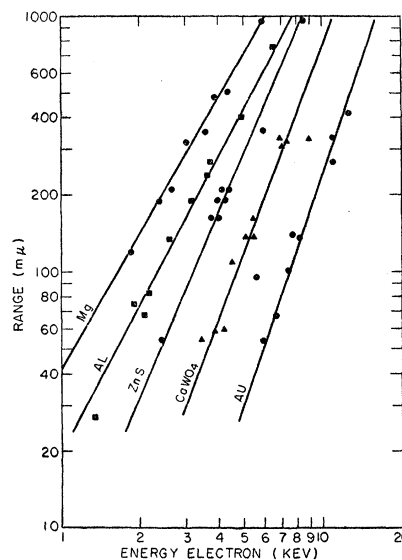


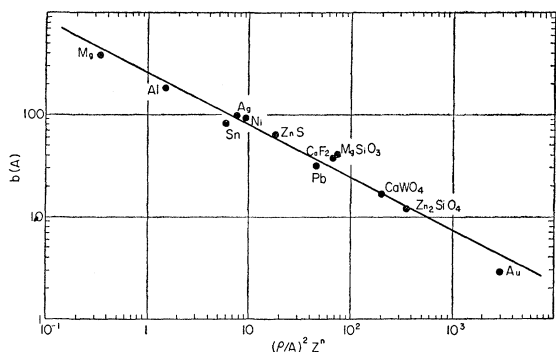
FIG. 4. Some representative measured range-energy curves.

C. Dead Voltage Correction

The dead voltage of the luminescent films was measured by inserting an uncoated phosphor layer pressed against a nickel mesh in place of the usual metallic coated sample. The mesh served to prevent surface charges being formed on the phosphor and to create a uniform field between the grounded grid and the sample. The polarity of the power supply was reversed and the negative side connected to the sample. The brightness was recorded as a function of voltage as before. As the voltage became increasingly negative, the brightness decreased to zero. The difference between the voltage at zero brightness and the point at which the screen voltage is equal to the cathode voltage is then called the dead voltage. The measurement made in this fashion represents a calibration of the entire apparatus, including, not only phosphor dead voltage, but differences in sensitivity of the light meter with wavelength and the recording instrument itself. This correction thus has the effect of shifting the measured range a little closer to the true maximum range. The

TABLE II. Measured values of b and n in the range expression $R = bE^n$.

Material	n	$b(\text{\AA})$
Mg	1.7	420
Al	1.9	183
Ni	1.8	92
Ag	1.7	97
Sn	1.9	105
Au	2.9	2.7
Pb	2.2	32
CaF ₂	2.9	38
ZnS	2.4	63
MgSiO ₃	2.7	40
Zn ₂ SiO ₄	3.0	12
CaWO ₄	2.7	17

FIG. 5. $\log_{10} b$ vs $\log_{10} (\rho/A)^2 Z^n$.

measured corrections were 150 v for Zn_2SiO_4 , 200 v for ZnS and 300 v for CaF_2 , MgSiO_3 , and CaWO_4 . Due to the variation between samples of the same material and the inaccuracy involved in reading voltages, these figures are only accurate to ± 25 v. The correction to the total penetration voltage is quite small.

III. RESULTS

The ranges of all the materials studied could be expressed as a function of energy according to Eq. (1). The values of b and n are listed in Table II. A few representative range curves, illustrating the typical experimental scatter, are shown in Fig. 4. The scatter and hence error in CaF_2 layers was larger than those illustrated because the films tended to darken or burn as the measurements were performed.

It is obvious, on examining this table and corresponding figure, that n as well as b depends on the material. In fact, it was determined that b is related to n according to the expression:

$$b = 250A/\rho Z^{n/2}, \quad (2)$$

where: ρ is the bulk density¹⁶; A , the atomic or molecular weight of the material; Z , the atomic number or the number of electrons per molecule in the case compounds. This can be seen from Fig. 5, which is a plot of $\log_{10} b$ vs $\log_{10} (\rho/A)^2 Z^n$. The resulting curve is a straight line with a slope of $-1/2$. Similarly it was found that n could be plotted as a function of $\log_{10} Z^n$, as shown in Fig. 6, leading to the following expression for n :

$$n = 1.2 / (1 - 0.29 \log_{10} Z). \quad (3)$$

Thus, inserting the relations for n and b into Eq. (1), the range expression becomes:

$$R = 250(A/\rho)(E/Z^{1/2})^n, \quad n = 1.2 / (1 - 0.29 \log_{10} Z). \quad (4)$$

IV. DISCUSSION

The above range equation indicates a stronger dependency on atomic number than had been previously

¹⁶ The film density is very close to the bulk density because the phosphor layers were fired at high temperatures and the metallic layers were annealed as stated above.

suspected. At $n = 2$ ($Z = 24$), the expression becomes

$$R = 250(A/\rho Z)E^2, \quad (5)$$

which is similar to the Thompson-Whiddington law and which, as mentioned in the introduction can be justified from Bethe's expression for the stopping power. The term $\rho Z/A$ is proportional to the density of electrons in the material.

At other values of Z , the square law is not strictly obeyed. For aluminum the expression becomes:

$$R = 260E^{1.7}, \quad (6)$$

which may be compared with the range equations listed in Table I. The expression is closest to the Lane and Zaffarano relation which is for R_p rather than R_0 . It is difficult to make a better comparison, at the present time, because so much depends on the experimental methods used.

By differentiating Eq. (4), an expression for the stopping power may be obtained:

$$-\frac{dE}{dR} = 3.3 \times 10^{-3} \frac{\rho Z^{n/2}}{AE^{n-1}} \log_{10} \left(\frac{10}{Z^{0.29}} \right). \quad (7)$$

Due to the electron straggling, one cannot, in general, obtain the stopping power from the range-energy relation.¹⁷ However, the range determined in the manner described here deals with those electrons which have straggled the least. Furthermore, complications arising from elastic reflections from both the metallic and phosphor surfaces and elastic scattering in the phosphor layer are automatically eliminated due to the nature of the phosphor excitation process. It is therefore, reasonable and informative to compare this experimental stopping power with Bethe's relation. Expressed in the same units; an equation quite similar to Eq. (7) is obtained:

$$-\frac{dE}{dX} = 7.8 \times 10^{-4} \frac{\rho Z}{AE} \ln \left(\frac{1.16E}{I} \right). \quad (8)$$

When I is defined as the excitation energy averaged over all the electron shells in the atom, I is sometimes taken as $13.5Z^1$.

A comparison of Eqs. (7) and (8) indicates that I is a function of energy as well as atomic number. This appears reasonable since at the low energies used here the degree of excitation is a function of energy. For materials with atomic number greater than 30, for example, there is not sufficient energy to excite the K shells. One would therefore expect the empirical data to fit the theory better at low Z than at high Z . That this is true can be seen in Fig. 7, which shows the empirical and theoretical stopping power curves for aluminum and gold using $I = 150$ ev and 520 ev, respectively.¹ A more detailed comparison of Eqs. (7)

¹⁷ T. J. Wang, *Nucleonics* 7, No. 2, 55 (1950).

TABLE III. Electron range at 10 kev.

Material	Measured range (μ)	Calculated from Eq. (4) (μ)	Computed theoretical (μ)
Mg	2.46	2.3	1.91
Al	1.75	1.42	1.27
Ni	0.57	0.62	
Ag	0.51	0.66	~ 0.55
Sn	0.92	0.94	0.89
Au	0.24	0.36	0.45
Pb	0.49	0.59	0.79
CaF ₂	2.74	1.77	
ZnS	1.51	1.46	
MgSiO ₃	1.75	1.73	
Zn ₂ SiO ₄	1.37	1.46	
CaWO ₄	0.81	0.97	
H ₂ O		3.15	2.47
AgBr		0.93	0.88

and (8) is not warranted because as mentioned previously Eq. (8) is not meant to be valid at these energies. Furthermore, other effects such as the density effect, are not included in Eq. (8).

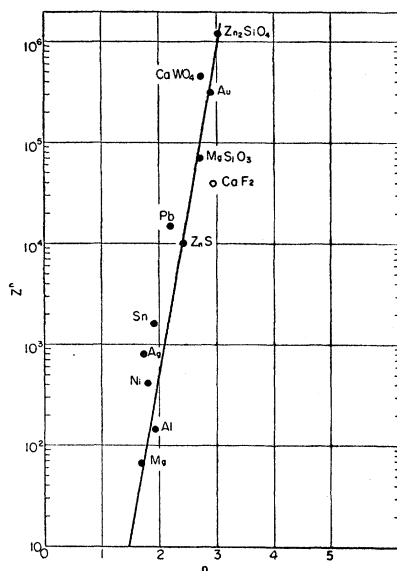
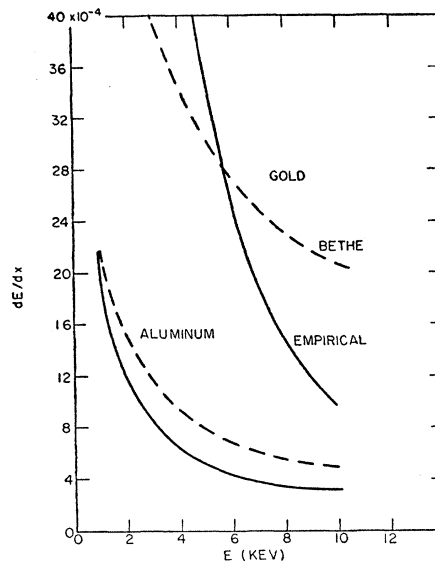
FIG. 6. n vs $\log Z^n$.

FIG. 7. Comparison of theoretical and empirical (solid curve) stopping power curves.

The density effect is, however, included in the range computations of Nelms.¹⁸ The ranges at 10 kev (the lowest energy values computed), are listed in Table III along with the figures obtained here. Considering the uncertainty in the values of I , the invalidity of the Bethe type equation, and the experimental errors in the measurements reported here, there is remarkable agreement between the two sets of figures for the pure materials as well as the compounds.

V. ACKNOWLEDGMENT

It is a pleasure to thank Miss Margaret O'Hara for her cooperation in all phases of this work and in particular for forming the necessary samples.

¹⁸ A. T. Nelms, Suppl. National Bureau of Standards Circular No. 577 (U. S. Government Printing Office, Washington, D. C., 1958).