Electron Scattering by the Quadrupole Charge Distribution of N^{14}†

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The recent data of Meyer-Berkhout on the elastic scattering of 420-Mev electrons by N^{14} have been analyzed in the first Born approximation. A detailed treatment is given of the effect of the quadrupole charge distribution of the nucleus in causing a filling-up of the diffraction minimum. The intermediate coupling ground-state wave function of Visscher and Ferrell with an admixture of the spheroidal type of collective wave function, suggested by Fallieros and Ferrell, has been used in calculating the quadrupole form factor. Instead of using the Born approximation monopole form factor, the exact monopole scattering curve computed by Ravenhall from phase-shift analysis has been made use of. The spin-flip type magnetic scattering has also been calculated and found to be extremely small. The experimental data show the importance of the quadrupole charge scattering in the neighborhood of the diffraction minimum of the monopole scattering. The quadrupole scattering for the intermediate coupling is, however, completely inadequate to explain the observed data. This gives strong evidence for a collective enhancement of the N^{14} quadrupole moment through core deformation. The model of Fallieros and Ferrell, which predicts a total quadrupole moment of 3.07×10^{-26} cm^2 (collective enhancement $=2.01\times10^{-26}$ cm^2), is found to give a fairly good fit with the data.

I. INTRODUCTION

THE elastic scattering data of 420-Mev electrons on N^{14} have been obtained by Meyer-Berkhout[1] and analyzed by Meyer-Berkhout, Ford, and Green.[1] These data reveal a striking dissimilarity with the corresponding data for two other p-shell nuclei,[2] namely C^{12} and O^{16}, in that the pronounced diffraction minimum observed in the latter cases is entirely filled up in the case of N^{14}.

A spherically symmetric nuclear charge distribution can give rise to only the monopole elastic scattering of the electrons. A first order Born approximation calculation for p-shell nuclei, having such a distribution, predicts an exact zero in the elastic form factor at a value of the momentum transfer $|\gamma^{-\frac{1}{2}}\mathbf{q}|$ given by $[6Z/(Z-2)]^{\frac{1}{2}}$, where γ is the scale parameter of the infinite oscillator well occurring in the Gaussian factor $\exp(-\frac{1}{2}\gamma r^2)$ of the single-particle wave functions.[3] This should be true for the even-even spin-zero nuclei, C^{12} and O^{16}. The exact phase-shift analysis[2] with the shell-model charge distribution for these nuclei shows that the simple Born approximation result is correct to a very good approximation; only the exact zero of the form factor is replaced by a partially filled minimum, in agreement with the experiments.

The case of N^{14} does not fit in with the above simple result because this nucleus has a spin of unity and a finite value of the quadrupole moment. The present author[4,5] applied the intermediate-coupling shell model to the analysis of electron scattering by the p-shell nuclei. It was pointed out[5] that nuclei having a non-vanishing quadrupole moment (i.e., with spin greater than one-half) will produce additional quadrupole scattering which will incoherently add to the monopole scattering and fill up the diffraction minimum. In particular, from a calculation based on the intermediate-coupling wave function for the N^{14} ground state given by Visscher and Ferrell,[6] it was suggested that this nucleus would be a very favorable case where an experiment might reveal the quadrupole scattering.

The experiment of M actually shows a marked quadrupole effect, much more than one would expect from the VF wave function. This wave function produces a quadrupole moment of 1.06×10^{-26} cm^2 for N^{14}, and the electron scattering calculated with it is found to be only about one-half of that observed at the angle where the monopole scattering would be expected to have its minimum. MFG have found that an intermediate-coupling quadrupole moment of about 1.8 $\times10^{-26}$ cm^2 can give agreement with the observed electron scattering. But as they point out, the maximum intermediate coupling quadrupole moment, though of this magnitude, corresponds to a very unrealistic wave function. These authors have, therefore, considered two other models, namely the deformed oscillator potential and the deformed p-shell models, and have found several sample combinations of the models which can reproduce the 420-Mev scattering data. In view of the phenomenological nature of their deformed models they

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[1] Meyer-Berkhout, Ford, and Green (to be published). In the rest of this paper the experimental part of this work done by the first author will be referred to as M and the theoretical part by MFG.

[2] Ehrenberg, Hofstadter, Meyer-Berkhout, Ravenhall, and Sobottka, Phys. Rev. **113**, 666 (1959).

[3] In the remainder of this paper we shall use r and q to denote the dimensionless quantities $\gamma^{\frac{1}{2}}r$ and $\gamma^{-\frac{1}{2}}q$, respectively.

[4] M. K. Pal and S. Mukherjee, Phys. Rev. **106**, 811 (1957); M. K. Pal and M. A. Nagarajan, Phys. Rev. **108**, 1577 (1957).

[5] M. K. Pal, Ph.D. thesis, 1958, Calcutta University (unpublished).

[6] W. M. Visscher and R. A. Ferrell, Phys. Rev. **107**, 781 (1957). We will refer to this work as VF in the rest of the paper.

have limited themselves to the qualitative conclusion that a combination of the undeformed shell model and the deformed oscillator potential model may be the true picture for the N^{14} ground state, and that its quadrupole moment is probably higher than 2.0×10^{-26} cm^2. For the details of the calculations based on a shell-collective model these authors have referred to the present work.

Here we have considered the predominant part of the ground-state wave function of N^{14} to be that given by VF. Using this shell-model wave function together with an admixture of a state that is obtained by coupling the $(1p)^{-2}$ ground state with the spheroidal excited state of the O^{16} core, Fallieros and Ferrell[7] have found an enhancement of the N^{14} quadrupole moment from the intermediate coupling value of 1.06×10^{-26} cm^2 to a value of 3.07×10^{-26} cm^2. The purpose of the present work is to see if the electron scattering data near the diffraction minimum can be accounted for in terms of the additional quadrupole effects of this spheroidal admixture to the intermediate-coupling shell-model wave function.

In Sec. II we have given the formula for the differential cross section in terms of the monopole and quadrupole form factors and have related these latter quantities to the monopole and quadrupole parts of the nuclear charge distribution. Section III contains explicit expressions for the charge density distributions and the resultant form factors. The numerical results are compared with the experimental data in Sec. IV. To avoid the error in the Born approximation result for the monopole scattering near the diffraction minimum, we have used the monopole scattering curve computed by Ravenhall[8] from exact phase-shift analysis. The small correction due to the spin-flip type magnetic scattering has also been added. Section V is a summary of the work. The paper contains an Appendix giving some of the details in the evaluation of the spin-flip scattering.

II. GENERAL THEORY

Following Schiff,[9] we define the charge density $\rho(\mathbf{r})$ as the expectation value of $\sum_i \frac{1}{2}[1+\tau_3(i)]\delta(\mathbf{r}-\mathbf{r}_i)$ in the nuclear ground state Ψ for the magnetic substate $M=J$, i.e.,

$$\rho(\mathbf{r})=\langle\Psi_J{}^J|\textstyle\sum_i \frac{1}{2}[1+\tau_3(i)]\delta(\mathbf{r}-\mathbf{r}_i)|\Psi_J{}^J\rangle. \quad (1)$$

The isotopic spin projection factor $\frac{1}{2}[1+\tau_3(i)]$ has the effect of restricting the above summation over all nucleons to that over the protons alone. Expanding $\delta(\mathbf{r}-\mathbf{r}_i)$ as[10]

$$\delta(\mathbf{r}-\mathbf{r}_i)=\frac{1}{r_i^2}\delta(r-r_i)\sum_{l,m} Y_m{}^{l*}(\theta_i,\varphi_i)Y_m{}^l(\theta,\varphi), \quad (2)$$

one obtains

$$\rho(r)=\rho_0(r)+\rho_2(r)Y_0{}^2(\theta)+\cdots, \quad (3)$$

where the monopole and quadrupole radial densities, $\rho_0(r)$ and $\rho_2(r)$, are given by

$$\rho_0(r)=(4\pi)^{-1}\left\langle\Psi_J{}^J\left|\sum_i \tfrac{1}{2}[1+\tau_3(i)]\frac{1}{r_i^2}\delta(r-r_i)\right|\Psi_J{}^J\right\rangle, \quad (4a)$$

$$\rho_2(r)=\left\langle\Psi_J{}^J\left|\sum_i \tfrac{1}{2}[1+\tau_3(i)]\frac{1}{r_i^2}\delta(r-r_i)Y_0{}^2(\theta_i)\right|\Psi_J{}^J\right\rangle. \quad (4b)$$

It may be pointed out that for a p-shell nucleus having a configuration $(1s)^4(1p)^n$ Eq. (3) for $\rho(\mathbf{r})$ terminates with the quadrupole term. In the case of N^{14} this is true, even when an admixture of higher configurations are considered, because the angular momentum J is equal to unity.

The mean square radius $\langle r^2\rangle$ and the quadrupole moment Q are related to the monopole and quadrupole radial densities, respectively, by the following integrals[3]:

$$\langle r^2\rangle=\gamma^{-1}\int_0^\infty \rho_0(r)r^4dr, \quad (5a)$$

$$Q=\gamma^{-1}\left(\frac{16\pi}{5}\right)^{\frac{1}{2}}\int_0^\infty \rho_2(r)r^4dr. \quad (5b)$$

In the same way the form factor for electron scattering can be expressed in terms of radial integrals over $\rho_0(r)$ and $\rho_2(r)$. The matrix element $F_{M'M}$ of the form factor is defined to be

$$F_{M'M}=Z^{-1}\langle\Psi_{M'}{}^J|\textstyle\sum_i \frac{1}{2}[1+\tau_3(i)]\exp(i\mathbf{q}\cdot\mathbf{r}_i)|\Psi_M{}^J\rangle, \quad (6)$$

which can be written as

$$F_{M'M}=Z^{-1}\int d^3r\, e^{i\mathbf{q}\cdot\mathbf{r}} \times\langle\Psi_{M'}{}^J|\sum_i \tfrac{1}{2}[1+\tau_3(i)]\delta(\mathbf{r}-\mathbf{r}_i)|\Psi_M{}^J\rangle. \quad (7)$$

If the quantization axis is chosen along the recoil momentum $\mathbf{q}$ then the matrix element is nonvanishing only for $M=M'$. Expanding $e^{i\mathbf{q}\cdot\mathbf{r}}$ in terms of spherical harmonics,[11]

$$e^{i\mathbf{q}\cdot\mathbf{r}}=\textstyle\sum_l i^l[4\pi(2l+1)]^{\frac{1}{2}}j_l(qr)Y_0{}^l(\theta), \quad (8)$$

and using Eqs. (2) and (4), one obtains

$$F_{MM}=F_0+F_{MM}{}^{(2)}, \quad (9)$$

[7] S. Fallieros and R. A. Ferrell (to be published); this work will be referred to as FF subsequently.
[8] D. G. Ravenhall, see reference 1.
[9] L. I. Schiff, Phys. Rev. **96**, 765 (1954).
[10] The spherical harmonics $Y_m{}^l(\theta,\Phi)$ used in the present work are the same as used by E. U. Condon and G. H. Shortley, *Theory of Atomic Spectra* (The McMillan Company, New York, 1935), p. 50.
[11] See, for example, J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952), p. 785.

where

$$F_0=4\pi Z^{-1}\int_0^\infty j_0(qr)\rho_0(r)r^2dr, \tag{10a}$$

$$\begin{aligned}F_{MM}^{(2)}&=-(20\pi)^{\frac{1}{2}}Z^{-1}\int^\infty r^2dr\, j_2(qr)\\&\quad\times\langle\Psi_M{}^J|\sum_i \tfrac{1}{2}[\cdot+\tau_3(i)]\delta(r-r_i)Y_0{}^2(\theta_i)|\Psi_M{}^J\rangle\\&=-(20\pi)^{\frac{1}{2}}Z^{-1}\frac{\langle J2M0|J2JM\rangle}{\langle J2J0|J2JJ\rangle}\\&\qquad\times\int_0^\infty j_2(qr)\rho_2(r)r^2dr.\end{aligned} \tag{10b}$$

In Eq. (10b) $\langle j_1j_2m_1m_2|j_1j_2jm\rangle$ is a Clebsch-Gordan coefficient.[12] In expressing the M dependence of the matrix element through the Clebsch-Gordan coefficient use has been made of the Wigner-Eckart theorem.[13] The ratio of the two Clebsch-Gordan coefficients has the following simple expression:

$$\frac{\langle J2M0|J2JM\rangle}{\langle J2J0|J2JJ\rangle}=\frac{3M^2-J(J+1)}{(2J-1)J}. \tag{11}$$

The differential cross section for electron scattering is given by[14]

$$\frac{d\sigma}{d\Omega}=\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}}|F|^2, \tag{12}$$

where $(d\sigma/d\Omega)_{\text{Mott}}$ is the usual Mott scattering by a point nucleus and the quantity $|F|^2$ is the average of $|F_{MM}|^2$ over nuclear orientation:

$$\begin{aligned}|F|^2&=\frac{1}{2J+1}\sum_M|F_{MM}|^2\\&=\frac{1}{2J+1}\sum_M|F_0+F_{MM}^{(2)}|^2.\end{aligned} \tag{13}$$

The interference between the monopole and the quadrupole terms vanishes, leaving

$$|F|^2=|F_0|^2+|F_2|^2, \tag{14}$$

where F_0 is given by (10a) and F_2 is given by

$$F_2=(4\pi)^{\frac{1}{2}}Z^{-1}\langle J2J0|J2JJ\rangle^{-1}\int_0^\infty j_2(qr)\rho_2(r)r^2dr. \tag{15}$$

The quadrupole form factor F_2 can be formally related to the quadrupole moment Q by

$$F_2=Z^{-1}\frac{\sqrt{5}}{30}\langle J2J0|J2JJ\rangle^{-1}\gamma Qq^2\mathfrak{F}(q), \tag{16}$$

where $\mathfrak{F}(q)$ has the following expression:

$$\mathfrak{F}(q)=15\int_0^\infty\left[\frac{j_2(qr)}{q^2r^2}\right]\rho_2(r)r^4dr\bigg/\int_0^\infty\rho_2(r)r^4dr. \tag{17}$$

The function $\mathfrak{F}(q)$ has a behavior very similar to that of the monopole form factor. It equals unity for $q\to 0$ and approaches zero asymptotically in the limit $q\to\infty$. The q dependence of the quadrupole form factor has thus been reduced to the determination of the function $\mathfrak{F}(q)$, which depends upon the details of the radial distribution $\rho_2(r)$.

In the next section we shall obtain explicit expressions for ρ_0 and ρ_2, and consequently for F_0 and F_2. The latter will be most conveniently specified in terms of Q and $\mathfrak{F}$.

III. CHARGE DENSITIES AND FORM FACTORS

The monopole charge density $\rho_0(r)$, given by Eq. (4a), can at once be written for a nucleus having a shell-model wave function as follows:

$$\rho_0(r)=(4\pi)^{-1}\sum_p R_p{}^2(r), \tag{18}$$

where R is the radial wave function of a proton and the summation runs over all the proton states. Using infinite well harmonic oscillator wave functions, one finds for a $1p$-shell nucleus of charge Z the following well-known result:

$$\rho_0(r)=\frac{2}{\pi^{\frac{3}{2}}}\exp(-r^2)[1+\tfrac{1}{3}(Z-2)r^2]. \tag{19}$$

The monopole form factor, calculated with this $\rho_0(r)$, from Eq. (10a) is given by

$$F_0=\left(1-\frac{Z-2}{6Z}q^2\right)\exp(-q^2/4). \tag{20}$$

As mentioned in the introduction this form factor has a zero corresponding to $q=[6Z/(Z-2)]^{\frac{1}{2}}$.

In the absence of configuration mixing from higher shells the quadrupole radial density $\rho_2(r)$, given by Eq. (4b), of a $1p$-shell nucleus will obviously arise only from the $1p$-shell protons, and hence will be proportional to $R_{1p}{}^2(r)$. We, therefore, write

$$\rho_2{}^s(r)=KR_{1p}{}^2(r), \tag{21}$$

where the constant K is determined by the details of the angular momentum coupling within the $1p$ shell. The label s on ρ_2 (and on other subsequent quantities) stands for the intermediate-coupling shell model.

The N^{14} ground state has been treated with the intermediate-coupling shell model by VF. One can verify by using their wave function in Eq. (4b) that

[12] E. U. Condon and G. H. Shortley, reference 10, p. 73.
[13] See, for instance, M. E. Rose, *Elementary Theory of Angular Momentum* (John Wiley and Sons, Inc., New York, 1957), p. 85.
[14] R. Hofstadter, *Annual Review of Nuclear Science* (Annual Reviews, Inc., Palo Alto, 1957), Vol. 7, p. 231.

the constant K has the following expression:

$$K=(20\pi)^{-\frac{1}{2}}\left(7/20C_D^2-\tfrac{1}{2}C_P^2+\frac{2}{\sqrt{5}}C_SC_D\right). \quad (22)$$

For an explanation of the symbols C_S, C_P, C_D, and the numerical values of these quantities the reader is referred to their work.

It is evident from Eq. (17) that this constant does not enter into the expression for $\mathfrak{F}_s(q)$, which is given by

$$\mathfrak{F}_s(q)=15\int_0^\infty R_{1p}^2(r)\left[\frac{j_2(qr)}{q^2r^2}\right]r^4dr\bigg/\int_0^\infty R_{1p}^2(r)r^4dr$$
$$=\exp(-q^2/4). \quad (23)$$

With the help of Eqs. (20), (23), and (16) one obtains from Eq. (14) the following expression for the squared form factor:

$$|F|^2=\left[\left(1-\frac{Z-2}{6Z}q^2\right)^2+\frac{1}{180}Z^{-2}\langle J2J0|J2JJ\rangle^{-2}Q_s^2q^4\right]\exp(-q^2/2). \quad (24)$$

For N^{14} the ground-state spin J is equal to unity, $\langle J2J0|J2JJ\rangle=(10)^{-\frac{1}{2}}$, and $Q_S=1.06\times10^{-26}$ cm^2, as calculated by VF.

We next consider the collective admixture $d(\Phi^2,\Psi^1)_M{}^1$ of FF into the intermediate-coupling ground-state wave function of N^{14}. d is the admixture coefficient; Ψ^1 is the intermediate-coupling ground-state wave function of the two $1p$ holes in N^{14}; and Φ^2 is the spheroidal excited state of the O^{16} core defined below. $(\Phi^2,\Psi^1)_M{}^1$ denotes a state of resultant angular momentum 1 and projection M, formed by the angular momentum coupling of Φ^2 with Ψ^1.

The spheroidal state with projection zero, Φ_0^2, has been defined by Ferrell and Visscher.[15] For the sake of convenience in later discussions we repeat the definition here. Consider the following transformation:

$$x_i(\alpha)=x_ie^{\alpha/2},\quad y_i(\alpha)=y_ie^{\alpha/2},\quad z_i(\alpha)=z_ie^{-\alpha}, \quad (25)$$

on the coordinates of the individual nucleons in the ground-state wave function, Φ_0^0, and denote the transformed wave function by $\Phi(\alpha)$, α being a parameter. The state Φ_0^2 is defined by

$$\Phi_0^2=N\frac{d\Phi(\alpha)}{d\alpha}\bigg|_{\alpha=0}=\tfrac{1}{2}N\sum_i(3z_i^2-r_i^2)\Phi_0^0, \quad (26)$$

where the normalization constant N is easily found to be

$$N=(18)^{-\frac{1}{2}}. \quad (27)$$

We shall calculate the enhancement of the charge distribution due to this collective admixture up to terms linear in the admixture coefficient d. The term proportional to d^2 gives rise to an additional monopole contribution, which will be neglected. The term linear in d causes an enhancement of the quadrupole charge distribution given by

$$\rho_2{}^c(r)=2d\left\langle(\Phi^2,\Psi^1)_1{}^1\left|\sum_i\tfrac{1}{2}[1+\tau_3(i)]\frac{1}{r_i^2}\times\delta(r-r_i)Y_0^2(\theta_i)\right|(\Phi^0,\Psi^1)_1{}^1\right\rangle. \quad (28)$$

Here (and subsequently) the label c denotes "collective." The total quadrupole radial density is, therefore,

$$\rho_2(r)=\rho_2{}^s(r)+\rho_2{}^c(r). \quad (29)$$

The contribution to the quadrupole charge distribution from the core is given by

$$\rho_2{}^c(r)Y_0^2(\theta)=2d\langle2101|2111\rangle\left\langle\Phi_0^2\left|\sum_i\tfrac{1}{2}[1+\tau_3(i)]\frac{1}{r_i^2}\times\delta(r-r_i)Y_0^2(\theta_i)Y_0^2(\theta)\right|\Phi_0^0\right\rangle$$
$$=\frac{2d}{(10)^{\frac{1}{2}}}\langle\Phi_0^2|\sum_i\tfrac{1}{2}[1+\tau_3(i)]\delta(\mathbf{r}-\mathbf{r}_i)|\Phi_0^0\rangle. \quad (30)$$

Using (26) one obtains

$$\rho_2{}^c(r)Y_0^2(\theta)=\frac{2d}{(10)^{\frac{1}{2}}}\frac{N}{2}\frac{d}{d\alpha}\langle\Phi(\alpha)|\sum_i\tfrac{1}{2}[1+\tau_3(i)]\times\delta(\mathbf{r}-\mathbf{r}_i)|\Phi(\alpha)\rangle|_{\alpha=0}. \quad (31)$$

In evaluating the matrix element we make a change in the variables of integration from $\mathbf{r}_i$ to $\mathbf{r}_i'$ defined by the transformation of Eq. (25):

$$\mathbf{r}_i'=\mathbf{r}_i(\alpha), \quad (32)$$

and obtain

$$\rho_2{}^c(r)Y_0^2(\theta)=\frac{2d}{(10)^{\frac{1}{2}}}\frac{N}{2}\left\langle\Phi_0^0\left|\frac{d}{d\alpha}\sum_i\tfrac{1}{2}[1+\tau_3(i)]\times\delta(\mathbf{r}-\mathbf{r}_i'(-\alpha))\right|\Phi_0^0\right\rangle\bigg|_{\alpha=0}$$
$$=\frac{2d}{(10)^{\frac{1}{2}}}\frac{N}{2}\left\langle\Phi_0^0\left|\frac{d}{d\alpha}\sum_i\tfrac{1}{2}[1+\tau_3(i)]\times\delta(\mathbf{r}(\alpha)-\mathbf{r}_i')\right|\Phi_0^0\right\rangle\bigg|_{\alpha=0}$$
$$=\frac{2d}{(10)^{\frac{1}{2}}}\frac{N}{2}\frac{d}{d\alpha}\rho_0(\mathbf{r}(\alpha))\bigg|_{\alpha=0}, \quad (33)$$

[15] R. A. Ferrell and W. M. Visscher, Phys. Rev. 102, 450 (1956).

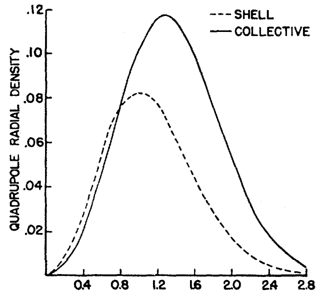

FIG. 1. Quadrupole radial density plotted against $\gamma^{\frac{1}{2}}r$. The ordinate is in units of $\gamma^{\frac{3}{2}}/\pi$. The dashed curve labeled "shell" corresponds to the intermediate-coupling wave function of Visscher and Ferrell.[6] The solid curve labeled "collective" includes the collective enhancement as well.

where $\rho_0(\mathbf{r}(\alpha))$ is obtained from $\rho_0(\mathbf{r})$ by making the transformation (25). Carrying out the differentiation implied in Eq. (33) and using the value of N given by (27), we obtain

$$\rho_2{}^c=\frac{4}{15\pi}d(2r^4-r^2)\exp(-r^2). \tag{34}$$

The collective enhancement of the quadrupole moment, Q_c, is therefore, given by

$$Q_c=\left(\frac{16\pi}{5}\right)^{\frac{1}{2}}\gamma^{-1}\int_0^\infty \rho_2{}^c(r)r^4dr=\frac{6}{\sqrt{5}}d\gamma^{-1}. \tag{35}$$

Equating this to the value of $Q_c=2.01\times10^{-26}$ cm^2 obtained by FF and using $\gamma^{-\frac{1}{2}}=1.68\times10^{-13}$ cm, we obtain

$$d=0.26. \tag{36}$$

The above choice of the oscillator well parameter γ will be discussed in the next section.

Using (29) in (17), we obtain

$$\mathfrak{F}(q)=\frac{Q_s}{Q}\mathfrak{F}_s(q)+\frac{Q_c}{Q}\mathfrak{F}_c(q), \tag{37}$$

where Q is the net quadrupole moment $(=Q_s+Q_c)$ and $\mathfrak{F}_c(q)$ is obtained from (17) by putting $\rho_2{}^c(r)$ instead of $\rho_2(r)$. Note that $\mathfrak{F}_c(q)$ does not depend on the value of the admixture coefficient. The dependence of $\mathfrak{F}(q)$ on this coefficient is contained in the value of Q_c.

A straightforward evaluation of the radial integrals in (17) with $\rho_2{}^c(r)$ given by (34) yields the following result for $\mathfrak{F}_c(q)$:

$$\mathfrak{F}_c(q)=(1-\tfrac{1}{12}q^2)\exp(-q^2/4). \tag{38}$$

Equations (23), (37), and (38) complete the derivation of the quadrupole form factor F_2.

IV. COMPARISON WITH EXPERIMENT

In this section the numerical results for the differential cross section obtained from the formulas of the previous two sections will be compared with experimental data of M.

Figure 1 shows plots of the quadrupole radial density $\rho_2(r)$. The dashed curve labeled "shell" corresponds to the intermediate-coupling expression [Eq. (21)]. The solid curve labeled "collective" includes the collective enhancement [Eq. (34)] as well.

In Fig. 2 the calculated differential cross sections are compared with the experimental data. Note that the abscissa is the momentum transfer, instead of the usual angle of scattering. The experimental points are shown with their errors.

The lowest curve labeled "monopole" is the differential cross-section curve calculated with the monopole form factor alone. It also includes the spin-flip type magnetic scattering,[16] which contributes an extremely small fraction of the total scattering. Instead of using our Born approximation Eq. (20) to calculate the monopole scattering we have made use of the results of exact phase-shift analysis of Ravenhall.[8] This accounts for the replacement of the exact zero of the Born approximation by the partially filled-up minimum.

We have determined the oscillator well parameter γ by equating the momentum transfer corresponding to the zero of the Born approximation monopole scattering [Eq. (20)] to the momentum transfer corresponding to the middle of the flat plateau region of the experimental points. Obviously the choice of the middle point for this purpose is arbitrary and the value of $\gamma^{-\frac{1}{2}}=1.68\times10^{-13}$ cm determined in this way is liable to some uncertainty. However, this choice has been tested by calculating the monopole form factor from our Eq. (20) with this γ and verifying that this reproduces the observed differential scattering cross sections for small momentum transfers. Since the Born approximation overestimates the nuclear radius it is understandable that Ravenhall's exact calculations determined a slightly smaller value, namely $\gamma^{-\frac{1}{2}}=1.625\times10^{-13}$ cm. This value of the oscillator well parameter is in agreement with that determined[17] from the Coulomb energy difference of the neighboring mirror nuclei $N^{15}-O^{15}$.

We have included the magnetic scattering in the

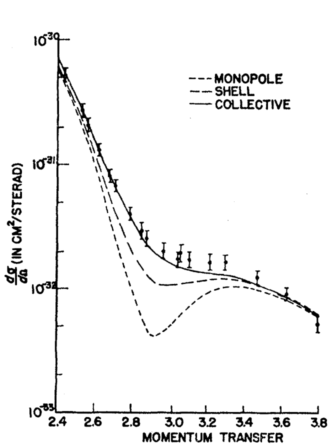

FIG. 2. Differential cross section $d\sigma/d\Omega$ (in cm^2/sterad) *versus* the momentum transfer $(\gamma^{-\frac{1}{2}}q)$. The lowest curve labeled "monopole" has been drawn after adding the magnetic scattering [Eq. (A11)] to the monopole curve computed by Ravenhall[8] from phase-shift analysis. The central curve labeled "shell" includes the quadrupole scattering due to the intermediate-coupling wave function of Visscher and Ferrell.[6] The solid curve labeled "collective" contains the additional quadrupole scattering due to collective enhancement of the quadrupole charge distribution.

[16] For some details of the calculation of this quantity, see the Appendix.
[17] B. C. Carlson and I. Talmi, Phys. Rev. 96, 436 (1954).

"monopole" curve in order to emphasize the effect of the quadrupole scattering. The large discrepancy of the experimental data with the "monopole" curve in this region of momentum transfer has to be understood in terms of the quadrupole scattering.

To this end we have plotted the remaining two curves in Fig. 2. The central curve labeled "shell" contains, over and above the monopole and magnetic scattering, the quadrupole scattering calculated with the intermediate-coupling wave function of VF [Eq. (24)]. It is found that in the region of the minimum this curve predicts a scattering which is only about one-half of the observed scattering. This clearly demonstrates the existence of an additional quadrupole moment of N^{14} in addition to the usual shell-model quadrupole moment.

The curve labeled "collective" in Fig. 2 includes the effect of the collective enhancement of the quadrupole form factor. This curve agrees fairly well with the experimental data except for a small region on the high momentum transfer side where it still underestimates the scattering by a few percent. In view of the fact that the Born approximation has been applied in calculating the quadrupole scattering and also that the term of the order of d^2 has been neglected, we did not try to improve the fit further by altering the value of Q_c given by FF.

V. SUMMARY AND CONCLUSIONS

The electron elastic scattering data for N^{14} give definite evidence of quadrupole charge scattering. With the intermediate-coupling ground-state wave function of VF, one predicts a differential cross section which is still about one-half of the observed value near the diffraction minimum. By choosing the coefficients (C_S, C_P, C_D) of the S, P, D states in the intermediate-coupling wave function so that the maximum value of the quadrupole moment ($\approx 1.80\times 10^{-26}$ cm^2) is produced, it is possible to secure agreement with the observed electron scattering. But this would be in complete disagreement with the cancellation of the $C^{14}-N^{14}$ β-decay matrix element, which was the requirement satisfied by VF in choosing their wave function. The electron scattering data, therefore, definitely indicate a core deformation of N^{14} as the source of its additional quadrupole moment.

We have, therefore, used the spheroidal core deformation of FF and found that the collective enhancement Q_c of the quadrupole moment of N^{14} by 2.01 $\times 10^{-26}$ cm^2 found by these workers is in very good agreement with the observed electron scattering data. At the present state of the experimental data slightly different combinations of the intermediate-coupling quadrupole moment Q_s and the collective enhancement Q_c can be chosen and still one can obtain a good fit with the data. However, since one believes in the VF intermediate-coupling wave function on the ground of the cancellation of the $C^{14}-N^{14}$ β-decay matrix element, we can assume that the value of $Q_s = 1.06\times 10^{-26}$ cm^2 used in the present work is fairly reliable. Restricting ourselves to this value of Q_s, our conclusion in the present paper is that the 420-Mev electron elastic scattering data of M require a collective enhancement, $Q_c \approx 2\times 10^{-26}$ cm^2, of the quadrupole moment of N^{14} through core deformation.

We would like to point out an approximation made in writing the collective enhancement of the quadrupole charge distribution and the related quantities in Sec. III. The expressions given there correspond to the contribution to these quantities by the core, had there been no effect of the Pauli exclusion principle between the $(1p)$ holes in the intermediate coupling state Ψ^1 and the $(1p)$ hole present in the spheroidal state Φ^2. Such effects would cause some correction to the expressions given in Sec. III. One would expect, however, the correction to be small and of the order of $1/Z$ since this is caused by the effect of one proton lacking from the closed shell. This correction was explicitly calculated using the creation and annihilation operators to write the states and then applying the standard algebra of such operators. The correction found in this way was actually small and has, therefore, been omitted in the paper for the sake of simplicity and clarity of presentation.

In conclusion we remark that the quadrupole moment of N^{14} estimated from the quadrupole resonance experiments[18] is much smaller than the estimate made here from the electron scattering data. It would, therefore, be desirable to re-examine the uncertainties involved in the calculation of the molecular field, on which the value of the quadrupole moment found in the resonance experiment depends.

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APPENDIX

In this Appendix we give some details about the calculation of the spin-flip type magnetic scattering. Though the results have been found to be extremely small we prefer to include them here in view of the considerable interest in the magnitude of the spin-flip type scattering among the workers in this field. (See, e.g., MFG.)

[18] T. P. Das and E. L. Hahn, *Nuclear Quadrupole Resonance Spectroscopy* (Academic Press, Inc., New York, 1958), p. 153.

The spin-flip type scattering gives rise to an additional form factor, whose square adds up incoherently to that for charge scattering. This is given by[5]

$$|F_m|^2=Z^{-2}(2J+1)^{-1}\tfrac{1}{2}\cos^{-2}(\theta/2)\times[1+\sin^2(\theta/2)]|\mathfrak{M}|^2, \quad \text{(A1)}$$

where

$$|\mathfrak{M}|^2=\sum_{M,M',\mu}|\langle JM'|\mathfrak{J}_\mu|JM\rangle|^2, \quad \text{(A2)}$$

and

$$\mathfrak{J}=\frac{i\hbar}{2Mc}\{\tfrac{1}{2}[1+\tau_3(i)]\exp(i\mathbf{q}\cdot\mathbf{r}_i)\boldsymbol{\nabla}(i)+(\tfrac{1}{2}[1+\tau_3(i)]\mu_p+\tfrac{1}{2}[1-\tau_3(i)]\mu_n)\times(\mathbf{q}\times\boldsymbol{\sigma}_i)\exp(i\mathbf{q}\cdot\mathbf{r}_i)\}. \quad \text{(A3)}$$

Here μ_n and μ_p are the magnetic moments of neutron and proton, respectively, in units of the nuclear magneton and $\boldsymbol{\sigma}$ is the spin operator for the nucleon. The summation index μ takes the values ±1, and by the ±1 components of the vector operator $\mathfrak{J}$ we mean

$$\mathfrak{J}_{\pm1}=\mp\frac{1}{\sqrt{2}}(\mathfrak{J}_x\pm i\mathfrak{J}_y). \quad \text{(A4)}$$

In evaluating the above matrix element the z axis should be chosen in the direction of $\mathbf{q}$.

For the intermediate-coupling wave function we obtain the following expression for the matrix element:

$$\langle 1M'|\mathfrak{J}_{\pm1}|1M\rangle=\mp(E/Mc^2)\langle 11M,\pm1|111M'\rangle\frac{1}{\sqrt{2}}\mathfrak{C}\sin(\theta/2), \quad \text{(A5)}$$

where

$$\mathfrak{C}=\left\{\left(\frac{3}{4}C_D^2+\frac{1}{2}C_P^2\right)+(\mu_n+\mu_p)\left[\left(C_S^2-\frac{1}{2}C_D^2\right)+\left(-\frac{1}{6}C_S^2+\frac{17}{120}C_D^2-\frac{1}{6\sqrt{5}}C_SC_D+\frac{1}{8}\left(\frac{6}{5}\right)^{\frac{1}{2}}C_PC_D\right)q^2\right]\right\}\exp(-q^2/4). \quad \text{(A6)}$$

We will next calculate the collective enhancement to this matrix element. This is given by

$$2d\langle 21\mu M|211M'\rangle\langle\Phi_\mu^2|\mathfrak{J}_\mu|\Phi_0^0\rangle, \quad \text{(A7)}$$

where the state Φ_μ^2 is obtained by generalizing the second form for Φ_0^2 in Eq. (26) of the text as follows:

$$\Phi_0^2=\tfrac{1}{2}NQ_0^2\Phi_0^0,$$

where Q_0^2 is the mass quadrupole operator $\sum_i(3z_i^2-r_i^2)$, and hence

$$\Phi_\mu^2=\tfrac{1}{2}NQ_\mu^2\Phi_0^0. \quad \text{(A8)}$$

When the z axis is chosen in the direction of $\mathbf{q}$, one obtains

$$(\mathbf{q}\times\boldsymbol{\sigma})_{\pm1}=\pm iq\sigma_{\pm1}, \quad \text{(A9)}$$

which, operating on the single-particle states, should change the spin part of them. According to Eq. (A8) the single-particle states in Φ_μ^2 differ from those in Φ_0^0 only in their spatial parts and therefore, that part of $\mathfrak{J}_\mu$, which contains the magnetic moments, will not contribute to the matrix element (A7). Evaluating this matrix element with the rest of the operator $\mathfrak{J}_\mu$, we obtain

$$-\langle 21\mu M|211M'\rangle\left(\frac{E}{Mc^2}\right)\frac{1}{2\sqrt{6}}d\times\sin(\theta/2)(12+q^2)\exp(-q^2/4). \quad \text{(A10)}$$

Using (A5) and (A10) in (A2) and the resultant expression in (A1) we arrive at the following expression:

$$|F_m|^2=|F_m{}^s|^2+|F_m{}^c|^2, \quad \text{(A11)}$$

where

$$|F_m{}^s|^2=\tfrac{1}{6}f(\theta)\mathfrak{C}^2, \quad \text{(A12)}$$

$$|F_m{}^c|^2=\tfrac{1}{10}d^2f(\theta)(1+\tfrac{1}{12}q^2)^2\exp(-q^2/2), \quad \text{(A13)}$$

and

$$f(\theta)=Z^{-2}(E/Mc^2)^2\cos^{-2}(\theta/2)\sin^2(\theta/2)\times[1+\sin^2(\theta/2)]. \quad \text{(A14)}$$