Changes in the Differential Rigidity Spectrum of Primary Cosmic Rays Associated with Long-Term and Short-Term Intensity Variations

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During June and July, 1958, the cosmic-ray nucleonic component was surveyed along the geographic longitude 147.5°E between geographic latitudes 10°S and 44°S ($\lambda=19°$S and 53°S) at atmospheric depth 475 g cm^{-2} with a neutron monitor mounted in an aircraft. This route is substantially the same as one covered during similar surveys in July and August, 1957. The data have been analyzed to yield long-term and short-term changes in the differential rigidity spectrum $j(N)$, for N in the range 2.0 to 9.0 Bv. A power law, $\Delta j(N)\propto N^{-\beta}j(N)$, with $\beta=1.3$, fits the observations for the long-term change between July, 1957 and July, 1958. For a single short-term variation in 1958 the value $\beta=1.2$ was obtained, with evidence that β decreases at higher rigidities ($N>7.0$ Bv). A 1.5° northward shift of the south latitude knee between the two series of flights is reported. Finally, high-latitude intensity variations of the nucleonic component at 475 g cm^{-2} are compared with variations observed with a neutron monitor at Hobart ($\lambda=52°$, 950 g cm^{-2}). A 1.0% change in intensity at Hobart is equivalent to a 1.8±0.3% change at 475 g cm^{-2}, independently of the time scale of the variations.

1. INTRODUCTION

THERE is evidence that spectrum changes in the primary cosmic rays associated with long-term intensity variations differ markedly from the spectrum changes during short-term variations such as the Forbush decrease and 27-day variations. From neutron data recorded at Mawson ($\lambda=73°$S), Hobart ($\lambda=52°$S), and Lae ($\lambda=16°$S), McCracken[1] found that the long-term variation in counting rate at high latitudes was 4.0 times greater than at low latitudes. For short-term variations the ratio was 2.5. These ratios are interpreted as indicating that long-term variations are more strongly dependent on rigidity than short-term variations. Comparison of meson telescope data and neutron data from Hobart supports the conclusion. A similar comparison of meson and neutron data for northern hemisphere stations by Fenton *et al.*[2] provided similar results. From a more extensive analysis, McCracken[3] has suggested that the spectrum change takes the form $\Delta j(N)\propto N^{-\beta}j(N)$ with $N=p/Z$ the particle rigidity, $j(N)$ the differential rigidity spectrum, and $\Delta j(N)$ the change in spectrum associated with the event considered. In this formula $\beta=0.9$ for short-term events and $\beta=1.3$ for long-term intensity variations.

Surveys of the nucleonic component at 310 g cm^{-2} atmospheric depth by Meyer and Simpson[4,5] during the years 1948, 1951, 1954, and 1956 revealed a marked "softening" in the primary spectrum in 1951 and 1954 compared with 1948 and 1956. The softening is correlated with increased total intensity in these years reaching a peak in 1954, the year of minimum solar activity. Representing the rigidity spectrum by the power law $j(N)=CN^{-\gamma}$, Meyer and Simpson found $\gamma=2.7$ in 1951 and 1954, and $\gamma=2.5$ in 1956 assuming that γ had the value 2.0 in 1948. By contrast, in a similar series of surveys made during a period of frequent cosmic-ray disturbances, Simpson[6] finds the intensity variations, $\delta R/R$, independent of latitude between $\lambda=45°$ and $\lambda=60°$, indicating that the mechanism responsiboe does not depend on rigidity below 7.5 Bv.

From a study of a single Forbush event, also with an airborne neutron monitor, Storey[7] found $\delta R/R$ to be independent of latitude between $\lambda=14°$S and $\lambda=44°$S, (geomagnetic cutoffs 13.2 Bv and 7.5 Bv).

In the present paper the study of spectrum changes associated with short-term and long-term intensity variations is extended by comparing a series of surveys made with an airborne neutron monitor in June and July, 1958 between $\lambda=19°$S and $\lambda=53°$S, with surveys made over a similar route in July and August, 1957.

The 1957 series contained a single Forbush decrease and a study of this has been published.[7] A small intensity change between two flights in the 1958 series is analyzed to yield a spectrum change for short-term events. Between the 1957 and 1958 series the general level of cosmic-ray intensity dropped, and the spectrum change responsible is obtained by comparing data from a flight made during a quiet period in 1957 with data obtained during a similar quiet period in 1958.

The instrument used in the 1958 surveys was the two-counter neutron monitor used in 1957 and has already been described.[8] It was dismantled in the intervening period and installed in a different aircraft for the 1958 surveys. The aircraft was, however, of the same type, a Lincoln bomber, and the monitor was similarly placed. Although of the same nominal geometry small modifications were made and the

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[6] J. A. Simpson, Phys. Rev. **94**, 426 (1954).
[7] J. R. Storey, Phys. Rev. **113**, 302 (1959).
[8] J. R. Storey, Phys. Rev. **113**, 297 (1959).

problem of normalizing the 1958 data to the 1957 is discussed in a separate section.

2. METHOD OF ANALYSIS

The counting rate, $R(x,N_c)$, of a neutron monitor at atmospheric depth x, at a point where the geomagnetic cutoff rigidity is N_c is given by

$$R(x,N_c)=\sum_Z \int_{N_c}^{\infty} S_Z(x,N)j_Z(N)dN,$$

where $S_Z(x,N)$ is the specific yield function at atmospheric depth x for primaries with charge Ze and rigidity N, and $j_Z(N)$ is the differential rigidity spectrum. Denoting the total differential rigidity spectrum by $j(N)$, we get

$$R(x,N_c)=\int_{N_c}^{\infty} \sum_Z S_Z(x,N)f_Z(N)j(N)dN,$$

where $f_Z(N)$ is the fraction of primary particles at rigidity N with charge Ze. If now it is assumed that the mechanisms responsible for intensity changes depend only on particle rigidity so that $f_Z(N)$ is independent of time with respect to Z, we can write

$$S(x,N)=\sum_Z f_Z(N)S_Z(x,N),$$

as the average specific yield function, and then

$$R(x,N_c)=\int_{N_c}^{\infty} S(x,N)j(N)dN.$$

Time has not been included as an independent variable. When necessary, time will be indicated by a subscript to the function.

From a latitude survey made at time t_1 the function $R_1(x,N_c)$ is obtained. Here again the location of the monitor is specified by the local cutoff rigidity N_c. A similar survey at time t_2 provides $R_2(x,N_c)$. Let $j_1(N)$ and $j_2(N)$ be the differential rigidity spectra at t_1 and t_2, respectively.

Then

$$R_1{}' \overset{\text{def}}{=} \partial R_1(x,N_c)/\partial N_c=-S(x,N_c)j_1(N_c),$$

and

$$R_2{}' \overset{\text{def}}{=} \partial R_2(x,N_c)/\partial N_c=-S(x,N_c)j_2(N_c).$$

Therefore

$$\frac{R_2'-R_1'}{R_1'}=\frac{j_2(N_c)-j_1(N_c)}{j_1(N_c)},$$

or

$$\Delta j(N)/j(N)=\Delta R'/R'.$$

In principle this equation can be used to find $\Delta j(N)/j(N)$ for N in the range from equatorial cutoff to the low-rigidity primary cutoff or atmospheric absorption limit, whichever is greater. Unfortunately R' becomes small near the latitude knee and at low geomagnetic latitudes, and $\Delta R'/R'$ becomes too uncertain to be of much value. The method is therefore limited to rigidities which cut off along the steep portion of the intensity *vs* latitude plot.

3. NORMALIZATION OF THE 1958 DATA

As mentioned in the previous section, the reassembly of the monitor in a different aircraft for the 1958 flights could have changed the sensitivity and the problem of normalizing the 1958 data will now be considered.

Intensity variations are smaller and less dependent on altitude at low geomagnetic latitudes. It was therefore decided to attempt normalization at Port Moresby, the low-latitude limit of the 1958 flights, by comparing the change in aircraft monitor rate between 1957 and 1958 with the value estimated from ground observations. Port Moresby ($\lambda=19°$S, geographic latitude$=10°$S, longitude$=147°$E) is close to a sea-level monitor operating at Lae ($\lambda=16°$S, geographic latitude $=7°$S, longitude$=147°$E) and intensity variations observed at Lae were considered to apply at sea level, Port Moresby.

Dorman[9] has shown that the change in counting rate of a monitor can be represented by the following integral:

$$\frac{\Delta R}{R}=\int_{N_c}^{\infty} W(x,N_c,N)\frac{\Delta j(N)}{j(N)}dN,$$

where $W(x,N_c,N)$, called the "coupling constant" by Dorman, is defined by

$$W(x,N_c,N)=S(x,N)j(N)/R(x,N_c).$$

Knowledge of the spectrum change and the coupling constant applicable to the particular instrument and locality enables the change in counting rate to be estimated.

Up to the rigidity of equatorial cutoff $W(x,N_c,N)$ can be determined from latitude surveys. For higher rigidities one must extrapolate, e.g., by the method proposed by Dorman. Of interest here are the coupling coefficients, provided by Dorman in graphical form, applicable to a neutron monitor at sea level, mountain altitudes ($x=680$ g cm^{-2}) and 10 km altitude ($x=310$ g cm^{-2}), and for the geomagnetic latitudes $\lambda=0°$ and $\lambda=30°$.

TABLE I. The change in counting rate for a neutron monitor calculated for a primary spectrum change $\Delta j(N)=KN^{-\beta}j(N)$ with $\beta=1.3$, using Dorman's coupling coefficients.

λ	$\Delta R/R$(sea level)	$\Delta R/R$(310 g cm^{-2})	$\Delta R/R$(310 g cm^{-2}) / $\Delta R/R$(sea level)
0°	1.05×10^{-2}K	1.08×10^{-2}K	1.03
30°	2.36×10^{-2}K	2.66×10^{-2}K	1.13

[9] L. J. Dorman, *Cosmic Ray Variations* (State Publishing House for Technical and Theoretical Literature, Moscow, 1958). Translation by United States Technical Documents Liaison Office.

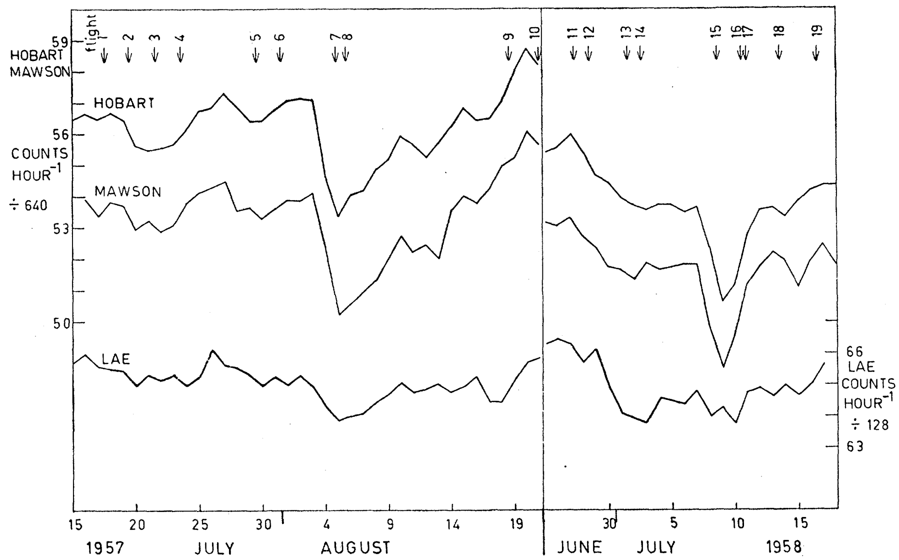

FIG. 1. Daily mean neutron intensities, corrected for atmospheric pressure variations, recorded at Mawson (λ=73°S), Hobart (λ=52°S), and Lae (λ=16°S) during the 1957 and 1958 flights. Times of flights are indicated.

By combining McCracken's formula for long-term spectral changes, written in terms of particle rigidity,

$$\Delta j(N)=KN^{-1.3}j(N),$$

and Dorman's coupling constants, Table I was prepared relating intensity changes at two latitudes and altitudes. Since it is the purpose of this paper to determine $\Delta j(N)/j(N)$ from the surveys of 1957 and 1958, it may appear invalid to assume a formula for the spectrum change in order to normalize data. However, in the present problem $\Delta j(N)/j(N)$ is required only for rigidities greater than the cutoff at Port Moresby, while the subsequent analysis yields $\Delta j(N)/j(N)$ for lower values of N.

The ratio in the last column of Table I increases with altitude and latitude and 1.13 will be the upper limit for the normalizing point at λ=19°S. x=475 g cm^{-2}. Taking 1.10 as a reasonable value, it was deduced that all 1958 data should be multiplied by 1.028 before comparison with 1957 data.

A check on the validity of the foregoing analysis was provided by data from Ahmedabad (λ=11°N, sea level) and Huancayo (λ=1°S, 680 g cm^{-2}). The long-term variations at these stations are compared with Lae in Table II.

These figures can be compared with the long term decrease in the aircraft monitor at λ=19°S, x=475 g cm^{-2} estimated without normalization of data to be 4.8%.

TABLE II. Decrease in counting rates between the periods July 27 to August 2, 1957 and July 1 to July 7, 1958 for a selection of stations.

	Mawson (λ=73°S sea level)	Hobart (λ=52°S 950 g cm^{-2})	Lae (λ=16°S sea level)	Ahmedabad (λ=11°N sea level)	Huancayo (λ=1°S 680 g cm^{-2})
Decrease, percent	3.9	4.9	1.6	3.0	5.8

It is apparent from the Huancayo result that the altitude dependence is greater than the foregoing analysis predicts. The error probably lies in the coupling coefficients which at low latitudes are only extrapolations beyond the limit of experimental determination. The Huancayo figure also suggests strongly that there is no need for further correction to the aircraft data.

In view of the uncertainty in the normalizing factor is was decided to use the 1958 data without further correction. Care has been taken in the following analysis to indicate where the absence of a satisfactory normalizing factor could affect the conclusions.

4. THE CHANGE IN RIGIDITY SPECTRUM BETWEEN JULY, 1957 AND JULY, 1958

The 1958 series of flights consisted of four sweeps along the geographic longitude 147.5°E between the geographic latitudes 44°S and 10°S. This route took the aircraft over a neutron monitor operating at Hobart [geographic latitude (43°S)] and within 3° latitude of another operating at Lae (7°S). The longitude of Lae (147°E) is close to that of the survey. Most latitude sweeps were made in two flights.

The 1958 route was not identical with the route of the 1957 survey but the separation, at most 2° of longitude, was considered too small to be significant when the data were analyzed in geomagnetic latitudes.

In Fig. 1 are plotted the daily mean neutron intensities of Mawson (λ=73°S), Hobart (λ=52°S), and Lae (λ=16°S) for periods covering the 1957 and 1958 series of flights. Dates of flights relevant to the present analysis are indicated on the same figure. Each of the periods contains a Forbush decrease and each decrease is preceded by several days of steady cosmic ray intensity. These days were selected to represent the general level of cosmic-ray intensity for the two periods. Flights 1 and 2 provided most of the data for the July,

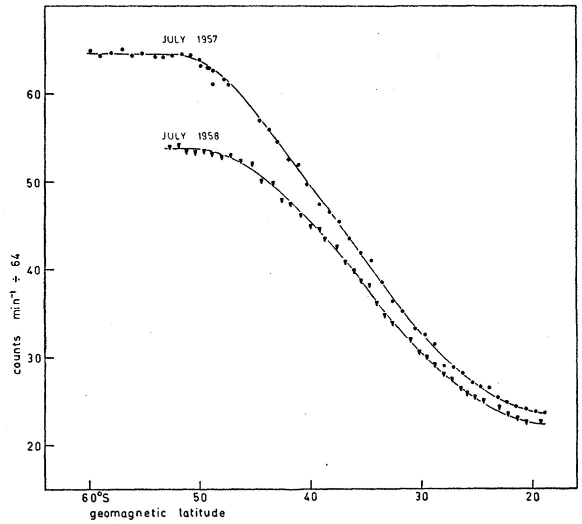

FIG. 2. Neutron intensity *vs* geomagnetic latitude at 475 g cm^{-2} atmospheric depth in July, 1957 (flights 1 and 2 with flights 9 and 10 adding a few high latitude points) and July, 1958 (flights 13 and 14 with 16 and 17 adding high-latitude points).

1957 latitude plot, with flights 9 and 10 adding a few high-latitude points. For July, 1958 flights 13 and 14 were chosen. It was found however that flights 16 and 17 produced an identical plot and data from these flights were used to add a few high-latitude points missed in flights 13 and 14.

The derivatives $\partial R/\partial N_c$ for flights 1 and 2; 13, 14, 16, 17 were obtained graphically from latitude curves, Fig. 2, and analytically from fitted functions. From the weighted means, $\Delta j(N)/j(N)$ for the period July, 1957 to July, 1958 was obtained by the method of Sec. 2. The function is plotted against N in Fig. 3. $\partial R/\partial N_c$ becomes small for large and for small values of N and the value of $\Delta j(N)/j(N)$, which depends on the difference between the values of $\partial R/\partial N_c$ for the two latitude curves, becomes inaccurate. Only for intermediate values of N (3.0 to 9.0 Bv) is the method satisfactory.

Two empirical functions were fitted to the points, one following an exponential and the other a power law. The exponential appears to provide a better fit but there is yet another test to be applied. A knowledge of $\Delta j(N)/j(N)$ and the coupling coefficient for a particular station and instrument enables $\Delta R/R$ to be calculated. The coupling coefficient for a neutron monitor at 475 g cm^{-2} atmospheric depth and $\lambda = 10°$ (Port Moresby) was obtained from the latitude curve of July, 1957 and extrapolated to high rigidities by the methods of Dorman. When used to find $\Delta R/R$ the exponential spectrum change failed badly, predicting a negligible drop in intensity at Port Moresby. The power law accounts for a fall of 2.8% which seems a reasonable value. It seems then that the power law $\Delta j(N) \propto N^{-1.3} j(N)$ provides a satisfactory fit and supports, though perhaps not strongly, the work of McCracken who also obtained a power law with exponent -1.3. The poor fit at high rigidities in Fig. 3 is probably due to the lack of normalization of the 1958 data.

5. SHIFT OF THE SOUTH LATITUDE KNEE BETWEEN JULY, 1957 AND JULY, 1958

From Fig. 2 it is apparent that there was a northward shift of the south latitude knee between July, August, 1957 and July, 1958. A change in curvature at the knee makes the exact value of the shift difficult to extimate but it appears to be about 1.5°. Defining the knee as the minimum latitude for which $\partial R/\partial \lambda = 0$, the shift is from $\lambda = 52.3°$S to 50.8°S.

6. THE CHANGE IN RIGIDITY SPECTRUM ASSOCIATED WITH SHORT-TERM VARIATIONS

The only significant short-term variation during the 1957 flights was a Forbush decrease commencing August 4. An investigation of this has already been

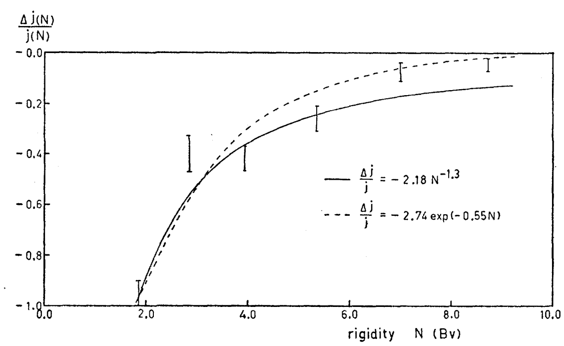

FIG. 3. The change in differential rigidity spectrum $\Delta_j(N)/_j(N)$ between July, 1957 and July, 1958 as a function of the primary particle rigidity (N).

published.[7] A Forbush decrease also occurred during the 1958 series but only one very short flight, (15), was made during the decrease. Of the four latitude sweeps between $\lambda = 19°$S and $\lambda = 53°$S the last three produced virtually identical data, but the first (flights 11 and 12), made when the intensity was appreciably higher, enables short-term variations to be investigated by the methods of Sec. 2. $\Delta j(N)/j(N)$ was obtained from flights 11, 12, and 13, 14, 16, 17, and plotted in Fig. 4.

A power law with exponent $\beta = 1.2$ provides a satisfactory fit up to $N = 7.0$. At higher rigidities the points are less accurate but there is strong evidence for hardening, i.e., β apparently decreases as N increases. There is no problem of normalization in this case as the monitor remained unchanged throughout the series.

7. COMPARISON OF HIGH LATITUDE INTENSITY VARIATIONS NEAR SEA LEVEL AND AT 475 g cm^{-2} ATMOSPHERIC DEPTH

A flight south on August 21, 1957 to locate the south latitude knee passed twice over Hobart at times 5

hours 20 min. apart. During this interval the cosmic-ray intensity decreased slightly and from the meager data it was concluded that a 1.0% change in intensity at Hobart corresponded to a 2.6±0.9% change at 475 g cm^{-2}. Four flights made over Hobart in the 1958 series has enabled this figure to be revised. In August, 1957 Hobart was about 0.5° north of the knee as determined at aircraft altitude, but by June, July, 1958 the knee had shifted leaving Hobart about 1.0° south. It will be assumed that the intensity variations observed at Hobart in 1957 and 1958 are high-latitude variations.

In Fig. 5 are plotted the intensities at 475 g cm^{-2} over Hobart against the corresponding Hobart intensities recorded at 950 g cm^{-2} atmospheric depth. Three types of intensity variation are represented in the figure. There is the long-term variation between August, 1957 and June-July, 1958 as well as the short-term variations during each of the flight series. The lowest intensity point was obtained from a flight during the Forbush decrease commencing July 8, 1958.

A straight line fits all the points reasonably well.

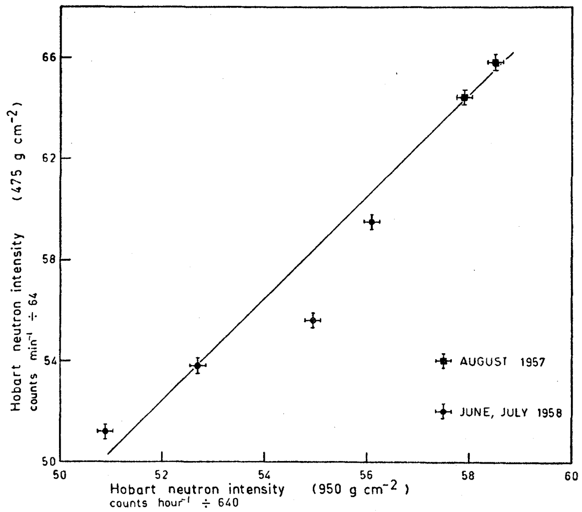

FIG. 5. Neutron intensity at 475 g cm^{-2} over Hobart plotted against intensity recorded at 950 g cm^{-2} by the permanent neutron monitor at Hobart.

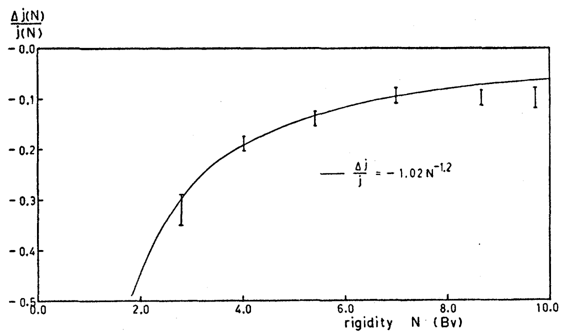

FIG. 4. $\Delta_j(N)/_j(N)$ for a decrease in intensity occurring between June 27–29, 1958 (flights 11 and 12) and July 2–3, 1958 (flights 13 and 14).

The slope of the line indicates that a 1.0% intensity change near sea level at Hobart corresponds to 1.8 ±0.3% change in intensity at 475 g cm^{-2} atmospheric depth. The standard error was estimated after allowing a reasonable error due to the absence of a normalizing factor relating the 1957 and 1958 aircraft data.

8. CONCLUSIONS

McCracken's proposed formula for the spectrum change associated with cosmic ray intensity variations,

$$\Delta j(N) \propto N^{-\beta} j(N),$$

with $\beta = 1.3$ for long-term variations and 0.9 for short-term events, receives support in the first case from the investigations of Sec. 4. The support is somewhat weakened by the difficulty in normalizing accurately the 1958 flight data to the 1957 data. For a single short-term event β was found to be 1.2 in contrast to the value 0.9 suggested by McCracken. It is likely, however, that short-term spectra changes differ markedly from one another. For example, the Forbush decrease investigated in 1957 showed no dependence on rigidity, corresponding to $\beta = 0$. The value $\beta = 0.9$ might well be a satisfactory average value for a large number of events.

9. ACKNOWLEDGMENTS

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