energy dependence is not critical. We have

$$\sin^2\alpha_{33}=\frac{a^2q^6}{\omega^{*2}(1-r\omega^*)^2+a^2q^6}. \tag{1A}$$

To make the calculation easier we have used the following approximation for q:

$$q\simeq[(W+M)/2M](\omega^{*2}-1)^{\frac{1}{2}},$$

and for the denominator of Eq. (1) we have used

$$\omega^{*2}(1-r\omega^*)^2+a^2q^6\simeq\eta^2\omega^{*2}P_1P_2,$$

where $\eta^2=1/232.1$. P_1 and P_2 are quadratic expressions in ω^*. $P_1=(\omega^*-\alpha)(\omega^*-\alpha^+)$, $P_2=(\omega^*-\beta)(\omega^*-\beta^+)$, with $\alpha=-1.951+i7.431$; $\beta=1.951+i0.378$.

Integrals can now be evaluated by factoring the integrand. They are reduced to the form

$$\sum_1^n A_i\int_1^\infty\frac{(\omega^{*2}-1)^{\frac{1}{2}}}{\omega^*-a_i}d\omega^*,$$

where a_i are the roots of the denominator.

The A_i satisfy

$$\sum_1^n A_i=0,$$
$$\sum_1^n A_ia_i=0,$$
$$\cdots$$
$$\sum_1^n A_ia_i^{n-1}=1.$$

APPENDIX II

The expressions for the multipoles extracted from CGLN amplitudes are

$$M_{1+}=ef\sqrt{2}qk[-\tfrac{1}{3}F_M/(1+\omega^*/M)+(g_p+g_n)/6M\omega^* -\lambda h^{--}+\tfrac{1}{3}\lambda(h_{11}-h_{31})-1/gie^{i\alpha_{33}}\sin\alpha_{33}F_M]$$

$$M_{1-}=ef\sqrt{2}qk[\tfrac{2}{3}F_M/(1+\omega^*/M)-\tfrac{1}{3}(g_p+g_n)/M\omega^* +\tfrac{1}{3}\lambda(h_{11}-h_{31})]$$

$$E_{1+}=ef(\sqrt{2}/3)kq[F_Q/(1+\omega^*/M)+\tfrac{1}{3}ie^{i\alpha_{33}}\sin\alpha_{33}F_Q].$$

Notations are the same as in CGLN.

APPENDIX III

The algebraic system to be solved to obtain the five parameters S, X, Y, K, D, is reported here.

The three theoretical predictions (1)–(3) have been combined with the first two of Eqs. (4) using the expressions for S, X, Y, K, D, given by Eqs. (5). All multipoles can be supposed real in the energy region considered. Using Δ, Θ, and Γ for the values of the right-hand side of Eqs. (1)–(3) reported in Table I we find

$$S-[(M+\epsilon_2)/q](X+Y)+\tfrac{1}{2}D=(\Delta+\tfrac{1}{2}\Gamma)(q/kQ)^{\frac{1}{2}}$$
$$X=(\Theta/2)(q/kQ)^{\frac{1}{2}}$$
$$[2q/(M+\epsilon_2)](X+Y+K)-3D=-\Gamma(q/kQ)^{\frac{1}{2}}$$
$$S^2+X^2+Y^2+SD=a_0$$
$$-2SK=a_1.$$

Then

$$a_2=S^2-2SD+K^2-a_0.$$

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Mass Splittings within Baryon Charge Multiplets*

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We have calculated the effect upon the mass splittings within each isobaric multiplet of a phenomenological boson mass difference. It has been possible to sum the diagrams to all orders in the strong couplings but the results are only valid to first order in the mass difference. These results can be compactly expressed as derivatives with respect to the intermediate masses of a function related to the proper self energy. The second-order perturbation results are also calculated.

INTRODUCTION

OVER the past few years, there have been several attempts at explaining the mass differences within charge multiplets by means of the electromagnetic self-energy arising from electric charge and charge-magnetic moment interactions.[1] These attempts have been beset by many difficulties, not the least of which is the presence of divergent integrals which must be cut off in a customary but unsatisfactory manner. Moreover, in addition to this purely electromagnetic difficulty, there exist the extremely complex contributions from the combined effects of the electromagnetic and strong interactions—contributions which

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[1] See for example, R. Feynman and G. Speisman, Phys. Rev. 94, 500 (1954); Marshak, Okubo, and Sudarshan, Phys. Rev. 106, 599, (1957); J. Sakurai, Phys. Rev. 115, 1304 (1959); H. Katsumori, Progr. Theoret. Phys. (Kyoto) 17, 803 (1957).

previously have been approximated only through the use of a phenomenological magnetic moment. Recently, however, Bransden and Moorhouse[2] have suggested that a major part of the Σ^+-Σ^- mass difference might arise through the combined effect of the strong interactions and the K^+-K^0 mass splitting (presumably electromagnetic). By introducing the K^+-K^0 mass difference phenomenologically, they obtained a cutoff-independent answer in second-order perturbation theory, which, with reasonable values of the strong coupling constants and of the K^+-K^0 mass difference, was of the same order of magnitude as the observed splitting. It is the purpose of this paper to extend these calculations.

The general philosophy will be as follows: It will be assumed that the baryon mass spectrum is initially split into four levels, N,Λ,Σ, and Ξ (the members of the charge multiplets being degenerate), that the boson mass spectrum (the K's and the π's) initially has no charge multiplet splitting and that the electromagnetic field is present. However, we shall assume that the only effect of the electromagnetic field is to give rise to a fine structure of the boson mass multiplet, i.e., $\delta m_K = m_{K^+} - m_{K^0}$, $\delta m_\pi = m_{\pi^+} - m_{\pi^0}$. The effects of δm_K and δm_π on the baryon charge multiplets will then be calculated. It will be noted that the mass (or rather the electromagnetic self-energy) operators δm_π and δm_K correspond to the observed boson mass multiplet splittings only when the bosons are free, but not when they are in virtual states. The effects of the *e-m* field which will not be included in this calculation are those arising from the total vertex operators for the mesonic and electromagnetic fields and from the clothed Fermion propagators (the same techniques as is used here for the clothed boson propagators does not work for the clothed Fermion propagators because the resulting mass differences are still linearly divergent). Therefore, the contribution which is calculated here forms *only a part* of what should be included in any final determination of these multiplet splittings. Rigorously, of course, all these contributions should be considered simultaneously since their effects are interdependent.

In the first section, this philosophy will be stated in more explicit terms by introducing the Lagrangian which will be used as a guide in the calculation. In the second section, the results of a second-order perturbation calculation will be displayed—constituting a direct extension of the work of Bransden and Moorhouse to all the multiplet splittings. In the third section, some theorems will be proved which are dependent only upon certain general symmetries of the Lagrangian. In the fourth section, certain theorems on these mass splittings, valid to all orders in the strong-coupling constants, but depending on the form of the Lagrangian, will be proved.

[2] B. H. Bransden and R. G. Moorhouse, Phys. Rev. Letters **2**, 431 (1959).

GENERAL

As the basis for the calculation outlined in the Introduction, we now write down the most general Lagrangian which has a Yukawa form and is charge independent: where the symbol for a particle denotes

$$\begin{aligned}\mathfrak{L} = {} & \bar{N}(p-m_N)N + \Xi(p-m_\Xi)\Xi + \bar{\Sigma}_i(p-m_\Sigma)\Sigma_i \\ & + \bar{\Lambda}(p-m_\Lambda)\Lambda + g_{N\pi}\bar{N}\Gamma_{N\pi}\tau_i N\pi_i + i g_{\Sigma\pi}\epsilon_{ijk}\bar{\Sigma}_i\Gamma_{\Sigma\pi}\Sigma_j\pi_k \\ & + g_{\Lambda\pi}\bar{\Sigma}_i\Gamma_{\Lambda\pi}\Lambda\pi_i + g_{\Xi\pi}\Xi\Gamma_{\Xi\pi}\tau_i\Xi\pi_i + g_{N\Lambda}\bar{N}\Gamma_{N\Lambda}\Lambda K \\ & + g_{N\Sigma}\bar{N}\Gamma_{N\Sigma}\tau_i\Sigma_i K + g_{\Xi\Lambda}\Xi\Gamma_{\Xi\Lambda}\Lambda K^G + g_{\Xi\Sigma}\Xi\Gamma_{\Xi\Sigma}\tau_i\Sigma_i K^G \\ & + \tfrac{1}{2}[\partial_\mu\bar{K}\partial_\mu K - m_K{}^2\bar{K}K] + \tfrac{1}{2}[\partial_\mu\pi_i\partial_\mu\pi_i - m_\pi{}^2\pi_i\pi_i] \\ & + \text{H.c.}, \qquad (1)\end{aligned}$$

$$K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix}, \quad K^G = \begin{pmatrix} \bar{K}^0 \\ -\bar{K}^+ \end{pmatrix}, \quad N = \begin{pmatrix} p \\ n \end{pmatrix}, \quad \Xi = \begin{pmatrix} \Xi^0 \\ \Xi^- \end{pmatrix},$$

the field operator which destroys it and Γ represents the appropriate Dirac matrix for each of the couplings.

It can easily be seen from this Lagrangian that each of the members of a charge multiplet has the same mass. This follows, formally, from the invariance of this Lagrangian and of the associated commutation relations under the set of charge-independent transformations (which interchange members of the same multiplet). Now, if the electromagnetic interaction Lagrangian were added, then the total Lagrangian would not be invariant under these charge-independent rotations and we would expect to find mass differences among the members of a charge multiplet. As stated in the Introduction, we will not make explicit use of the electromagnetic field, except insofar as it gives rise to the boson electromagnetic self-energy operators. Explicitly, by Lorentz invariance, the clothed boson propagator must be of the form $(\Delta')^{-1} = k^2 - m^2 + f(k^2)$. If $f(k^2)$ is considered as sufficiently small [or the difference between the $f(k^2)$ for the charged and neutral members of the multiplet if the neutral member can have an electromagnetic self-energy as has been suggested[3] for the K^0] compared with m^2, then we may consistently take it into account for our purpose by adding to the original Lagrangian terms of the form

$$\tfrac{1}{2}\bar{K}\delta m_K\tau_3 K + \tfrac{1}{3}\pi_i\delta m_\pi(\delta_{ij} - 3\delta_{3i}\delta_{3j})\pi_j, \qquad (2)$$

where δm_K and δm_π are the self-energy operators for the K and π mesons. It should be noted that these terms are not invariant under the charge independent transformations, and, therefore will lead, in the same way as the electromagnetic interaction does, to mass differences among the members of a charge multiplet. The effect of these terms, in the language of perturbation theory, is to insert into each boson line all the possible electromagnetic self-energy diagrams which are possible and then to take the correction they generate

[3] G. Feinberg, Phys. Rev. **109**, 1381 (1958).

as small enough (compared to the uncorrected mass) that only first-order terms need be retained.

It has been suggested that mass differences between the multiplets (e.g., between the N and Ξ) may arise from the interaction of a bare baryon with either the K or π field or with both fields.[4] This would imply that we should take all the bare baryon masses in our Lagrangian equal. It would then be necessary in the strong interactions to either take the coupling constants, g, to be unequal[5] or to adjust the interactions Γ_i (which is equivalent to assigning relative parities)[6] or to do both in such a way that this Lagrangian would still be capable of generating mass differences between the multiplets. An alternative suggestion[7] has been that the splitting between the multiplets may arise from some other source (e.g., mass quantization, etc.,), in which case we would be required to take all the baryon bare masses unequal in our Lagrangian. (This implicitly assumes a theory with appropriate form factors which cut off divergent integrals.) Our choices for the g's and Γ's in the strong interactions would then not be governed by a requirement of generating mass differences and hence might be chosen in a fairly symmetric way. In what follows, we shall explicitly state the conditions on the bare baryon masses, the g's and the Γ's for each of the results we find.

PERTURBATION CALCULATION

A standard second-order perturbation calculation with this Lagrangian leads to the following mass splittings when δm_π, δm_K are each considered to be small compared to m_π and m_K, respectively, and are taken to be constants equal to their mass shell values. By following the usual procedure, we find

$$m_n-m_p=\frac{-im_K\delta m_K}{2\pi^3}\bar{u}\frac{\partial}{\partial m_K^2}\{\Sigma^{(2)}(g_{n\Lambda},m_\Lambda,m_N,m_K)-\Sigma^{(2)}(g_{N\Sigma},m_\Sigma,m_N,m_K)\}u, \quad (3a)$$

$$m_{\Xi^0}-m_{\Xi^-}=\frac{-im_K\delta m_K}{2\pi^3}\bar{u}\frac{\partial}{\partial m_K^2}\{\Sigma^{(2)}(g_{\Xi\Lambda},m_\Lambda,m_\Xi,m_K)-\Sigma^{(2)}(g_{\Xi\Sigma},m_\Sigma,m_\Xi,m_K)\}u, \quad (3b)$$

$$m_{\Sigma^+}-m_{\Sigma^-}=\frac{-im_K\delta m_K}{\pi^3}\bar{u}\frac{\partial}{\partial m_K^2}\{\Sigma^{(2)}(g_{N\Sigma},m_N,m_\Sigma,m_K)-\Sigma^{(2)}(g_{\Xi\Sigma},m_\Xi,m_\Sigma,m_K)\}u, \quad (3c)$$

$$m_{\Sigma^+}-m_{\Sigma^0}=\tfrac{1}{2}(m_{\Sigma^+}-m_{\Sigma^-})+\frac{im_\pi\delta m_\pi}{2\pi^3}\bar{u}\frac{\partial}{\partial m_\pi^2}\times\{\Sigma^{(2)}(g_{\Lambda\pi},m_\Lambda,m_\Sigma,m_\pi)-\Sigma^{(2)}(g_{\Sigma\pi},m_\Sigma,m_\Sigma,m_\pi)\}u, \quad (3d)$$

where

$$\bar{u}\frac{\partial}{\partial m_B^2}\Sigma^{(2)}(g,m_2,m_1,m_B)u=-\frac{ig^2\pi}{4m_1}\bar{u}[\Gamma p\!\!\!/\Gamma I_2-im_2\Gamma\Gamma I_1]u,$$

$$I_n=\int_0^1 dx\frac{x^n}{x^2+bx+c};\quad b=-\frac{(m_1^2+m_2^2-m_B^2)}{m_1^2};\quad c=\frac{m_2^2}{m_1^2}.$$

It is obvious that these mass splittings can be further reduced to a second derivative by noting that if the coupling constants are the same, the $\Sigma^{(2)}$'s differ in only the intermediate Fermion mass. Thus, e.g.,

$$m_n-m_p=\frac{im_K\delta m_K\delta m_{\Sigma\Lambda}}{2\pi^3}\bar{u}\frac{\partial^2}{\partial m_K^2\partial m_{\Sigma\Lambda}}\Sigma^{(2)}(g,m,m_N,m_K)u, \quad (4)$$

$$(\delta m_{\Sigma\Lambda}=m_\Sigma-m_\Lambda),$$

and all others can be similarly expressed.

It is notable that these splittings do not depend upon any cutoff, i.e., they are finite even though each of the self-energies is itself infinite.

LAGRANGIAN-INDEPENDENT THEOREMS

In this section we derive some effects of the existence of a K- or π-mesonic mass difference which do not depend upon whether the interactions are of a local or nonlocal type, of a trilinear, quadrilinear or whatever form, or in fact anything else at all except that in the absence of these mass differences, the interactions are charge independent, i.e., that T^2, T_3 are good quantum numbers for the eigenstates of the Lagrangian neglecting mass differences among mesons.

These effects depend upon the fact that the mass differences, considered as a perturbation, can be written in such a form as to transform as a spherical tensor in isotopic spin space. The K-mesonic mass difference behaves as a vector while the π-mesonic mass difference transforms as a second rank tensor. More specifically,

$$H_{\delta m}=\tfrac{1}{2}\delta m_K T_1^0-\delta m_\pi T_2^0,$$

where T_1^0, T_2^0 are the integrals over space of the expression given in Eq. (2).

Then, if we only consider corrections to the masses, to first order in δm_π and δm_K, the mass differences between baryons defined by the large independent part of the Lagrangian are given by

$$m_A-m_B=\langle\mathbf{A}|H_{\delta m}|\mathbf{A}\rangle-\langle\mathbf{B}|H_{\delta m}|\mathbf{B}\rangle.$$

[4] See for example, J. Schwinger, Phys. Rev. **104**, 464 (1956).
[5] M. Gell-Mann, Phys. Rev. **106**, 1296 (1957); A. Pais, Phys. Rev. **110**, 574 (1958); J. Schwinger, Ann. Phys. **2**, 407 (1957).
[6] H. Katsumori, Progr. Theoret. Phys. (Kyoto) **19**, 342 (1958); S. Barshay, Phys. Rev. Letters **1**, 97 (1958); F. Gürsey, Phys. Rev. Letters **1**, 98, (1958); R. E. Behrends, Nuovo cimento **11**, 424 (1959).
[7] A. Pais, Physica **19**, 869 (1953).

Since the pion mass difference is a second rank tensor in I-space, it cannot contribute to the n-p or Ξ^--Ξ^0 mass difference. Thus

$$\begin{aligned} m_n - m_p &= \tfrac{1}{2}\{\langle \tfrac{1}{2},-\tfrac{1}{2},\alpha | T_1{}^0 \delta m_K | \tfrac{1}{2},-\tfrac{1}{2},\alpha\rangle \\ &\qquad - \langle \tfrac{1}{2},\tfrac{1}{2},\alpha | T_1{}^0 \delta m_K | \tfrac{1}{2},\tfrac{1}{2},\alpha\rangle\} \\ &= \tfrac{1}{2}\langle \tfrac{1}{2},\alpha \| T_1{}^0 \delta m_K \| \tfrac{1}{2},\alpha\rangle \{\langle 1,0;\tfrac{1}{2},-\tfrac{1}{2} | \tfrac{1}{2},-\tfrac{1}{2}\rangle \\ &\qquad - \langle 1,0;\tfrac{1}{2},\tfrac{1}{2} | \tfrac{1}{2},\tfrac{1}{2}\rangle\}, \end{aligned}$$

where α are all quantum numbers other than T^2, T_3 necessary to specify the state $\langle j_1,m;j_2,m' | j,m+m'\rangle$ are Clebsch-Gordan coefficients, and $\langle \tfrac{1}{2},\alpha \| T_1{}^0 \| \tfrac{1}{2},\alpha\rangle$ is a reduced matrix element. Evaluating the Clebsch-Gordan coefficient, we find

$$\begin{aligned} m_n - m_p &= \tfrac{1}{2}\langle \tfrac{1}{2},\alpha \| T_1 \delta m_K \| \tfrac{1}{2},\alpha\rangle \left\{ -\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{3}} \right\} \\ &= -\langle \tfrac{1}{2},\alpha \| T_1 \delta m_K \| \tfrac{1}{2},\alpha\rangle/\sqrt{3}. \end{aligned}$$

Similarly,

$$m_{\Xi^0} - m_{\Xi^-} = -\langle \tfrac{1}{2},\alpha' \| T_1 \delta m_K \| \tfrac{1}{2},\alpha'\rangle/\sqrt{3}.$$

Both the π and K mass differences contribute to the Σ mass differences:

$$\begin{aligned} m_{\Sigma^+} - m_{\Sigma^0} &= \tfrac{1}{2}\langle 1,\alpha'' \| T_1 \delta m_K \| 1,\alpha''\rangle \{\langle 10;11|11\rangle \\ &\qquad - \langle 1,0;10|10\rangle\} - \langle 1,\alpha'' \| T_2 \delta m_\pi \| 1,\alpha''\rangle \\ &\qquad \times \{\langle 20;11|11\rangle - \langle -2,0;10|10\rangle\} \\ &= \tfrac{1}{2}\langle 1,\alpha'' \| T_1 \delta m_K \| 1,\alpha''\rangle/\sqrt{2} \\ &\qquad - \langle 1,\alpha'' \| T_2 \delta m_\pi \| 1,\alpha''\rangle [(10)^{-\frac{1}{2}} + 2(10)^{-\frac{1}{2}}] \\ &= \tfrac{1}{2}\langle 1,\alpha'' \| T_1 \delta m_K \| 1,\alpha''\rangle/\sqrt{2} \\ &\qquad - \langle 1,\alpha'' \| T_2 \delta m_\pi \| 1,\alpha''\rangle 3/(10)^{\frac{1}{2}}, \\ m_{\Sigma^0} - m_{\Sigma^-} &= \tfrac{1}{2}\langle 1,\alpha'' \| T_1 \delta m_K \| 1,\alpha''\rangle \{\langle 1,0;10|10\rangle \\ &\qquad - \langle 10;1-1|1-1\rangle\} - \langle 1,\alpha'' \| T_2 \delta m_\pi \| 1,\alpha''\rangle \\ &\qquad \times \{\langle 20;10|10\rangle - \langle 20;1-1|1-1\rangle\} \\ &= \tfrac{1}{2}\langle 1,\alpha'' \| T_1 \delta m_K \| 1,\alpha''\rangle/\sqrt{2} \\ &\qquad - \langle 1,\alpha'' \| T_2 \delta m_\pi \| 1,\alpha''\rangle 3/(10)^{\frac{1}{2}} \end{aligned}$$

On the other hand, the Σ^+-Σ^- mass difference does not depend on δm_π.

$$m_{\Sigma^-} - m_{\Sigma^+} = -\langle 1,\alpha'' \| T_1 \delta m_K \| 1,\alpha''\rangle/\sqrt{2}.$$

Thus, we see that in the absence of the π mass difference but with the K multiplet splitting, the Σ^+-Σ^- are split equally, but in opposite directions, from the Σ^0: the π's then displace the Σ^0 from the mean of the Σ^+-Σ^- mass.

Without a detailed theory, the different reduced matrix elements cannot be related theoretically.

LAGRANGIAN-DEPENDENT THEOREMS

In this section, expressions for the baryon mass splittings will be derived which hold to all orders in the strong couplings but which are valid only if certain relations exist among the relative parities and strong coupling constants. In each case, these will be stated explicitly. In order to derive these expressions, certain symmetry transformations of the strong couplings will be discussed.

Since the Lagrangian (and the commutation relations) are invariant under the charge symmetry transformations

$$\begin{array}{ll} p \leftrightarrow n, & \pi^\pm \rightarrow \pi^\mp, \\ \Sigma^+ \rightarrow \Sigma^-, & \pi^0 \rightarrow -\pi^0, \\ \Sigma^0 \rightarrow -\Sigma^0, & \\ \Xi^0 \leftrightarrow \Xi^-, & K^{+,0} \rightarrow K^{0,+}, \\ \Lambda^0 \rightarrow \Lambda^0, & \bar{K}^{+,0} \rightarrow -\bar{K}^{0,+}, \end{array} \tag{A}$$

then, if $\delta m_K = 0$, it is obvious that δm_π will not contribute to the n-p, Ξ^0-Ξ^- or Σ^+-Σ^- mass differences. Thus, if the expressions for the baryon multiplet mass splittings are restricted to just first order in either δm_K or δm_π (which is a good approximation since both δm_K and δm_π are very small), n-p, Ξ^0-Ξ^-, and Σ^+-Σ^- mass splittings can, at most, be proportional to δm_K.

p-n AND Ξ^0-Ξ^- MASS SPLITTING

It will now be assumed that

$$\begin{array}{lll} g_{N\Lambda} = g_{N\Sigma}, & g_{\Xi\Lambda} = g_{\Xi\Sigma}, & g_{\Lambda\pi} = g_{\Sigma\pi}, \\ \Gamma_{N\Lambda} = \Gamma_{N\Sigma}, & \Gamma_{\Xi\Lambda} = \Gamma_{\Xi\Sigma}, & \Gamma_{\Lambda\Sigma} = \Gamma_{\Sigma\Sigma}. \end{array} \tag{I}$$

The relations among the Γ's imply that the Λ and Σ have the same parity. The strong interaction is then invariant under the transformation

$$\begin{array}{ll} p \leftrightarrow n, & \pi^\pm \rightarrow \pi^\mp, \\ \Xi^0 \leftrightarrow \Xi^-, & \pi^0 \rightarrow -\pi^0, \\ \Sigma^+ \leftrightarrow (\Lambda^0 - \Sigma^0)/\sqrt{2} \equiv Y^0, & K^{+,0} \rightarrow K^{+,0}, \\ \Sigma^- \leftrightarrow (\Lambda^0 + \Sigma^0) \leftrightarrow \sqrt{2} \equiv Z^0, & \bar{K}^{+,0} \rightarrow \bar{K}^{+,0}. \end{array} \tag{B}$$

If $m_\Sigma = m_\Lambda$, then the entire Lagrangian would be invariant under this transformation and again there would be no splitting of the n-p, Ξ^0-Ξ^-, independently of whether δm_K and δm_π are zero or not. These are the usual conditions for the doublet approximation which have been shown to be in contradiction with experiment.[5] This symmetry can be broken either by giving the Λ and Σ opposite parity,[6] by removing some of the coupling constant equalities, or by assuming the bare Λ and bare Σ masses differ. With the latter assumption, it is possible to derive some rather interesting theorems. Therefore, we will assume that $m_\Sigma = m_\Lambda + \delta m_{\Sigma\Lambda}$ and that $\delta m_{\Sigma\Lambda}$ is small compared with the average $\Lambda\Sigma$ mass.

Now consider the sum of the proper self-energy parts for the neutron and proton, $\Sigma_n{}^*$ and $\Sigma_p{}^*$, respectively. According to the transformation A, there is a term by term cancellation in $\Sigma_n{}^*$-$\Sigma_p{}^*$ of diagrams which differ only in that K^+ propagators are replaced by K^0 propagators, assuming the Σ^+,Σ^- masses to be the same. Similarly, transformation B states that there is an exact cancellation of terms which differ only in that

Σ^+ propagators are replaced by Y^0 propagators and Σ^- by Z^0 propagators. If δm_K and $\delta m_{\Sigma\Lambda}$ are considered sufficiently small $[m_{Y^0}=m_{Z^0}=\frac{1}{2}(m_\Lambda+m_\Sigma)]$ so that only the first terms in the Taylor series expansions need be retained, then this may be written

$$\Sigma_n^*-\Sigma_p^*\cong 2m_K\delta m_{\Sigma\Lambda}\frac{\partial^2}{\partial m_K{}^2\partial m_{\Sigma\Lambda}}\mathfrak{S}_N^* =\delta m_{\Sigma\Lambda}\frac{\partial^2}{\partial m_K\partial m_{\Sigma\Lambda}}\mathfrak{S}_N^* \quad (5)$$

where $m_{\Lambda\Sigma}$ is an average $\Sigma\Lambda$ mass and $\mathfrak{S}_N^*$ is an operator which if δm_K were on the mass shell would be δm_K times the sum of the proper self-energy parts for the nucleon (Σ_n^*) when the K^+K^0 and $\Sigma\Lambda$ mass differences are neglected. (The over-all sign may easily be determined from the second-order perturbation result.) Thus, the neutron-proton mass difference may be written [we explicitly include the masses and coupling constants upon which the $\mathfrak{S}^*$ depends] as

$$m_n-m_p=\frac{i}{4\pi^3}\delta m_{\Sigma\Lambda}\bar{u}'\frac{\partial^2}{\partial m_K\partial m_{\Sigma\Lambda}}\mathfrak{S}^*(g_{N\Lambda},m_N,m_{\Sigma\Lambda},m_K)u' \quad (6)$$

where u' is the renormalized wave function for the free nucleon ($\bar{u}'u'=1$).

Similar arguments may be applied to the Ξ^0-Ξ^- mass difference. Thus

$$m_{\Xi^0}-m_{\Xi^-} =\frac{i}{4\pi^3}\delta m_{\Sigma\Lambda}\left[\bar{u}'\frac{\partial^2}{\partial m_K\partial m_{\Sigma\Lambda}}\mathfrak{S}^*(g_{\Xi\Lambda},m_\Xi,m_{\Sigma\Lambda},m_K)u'\right]. \quad (7)$$

$\Sigma^+-\Sigma^-$ AND $\Sigma^+-\Sigma^0$ MASS SPLITTINGS

Now, we will alternatively assume that

$$\begin{aligned} &g_{N\pi}=g_{\Xi\pi}, \quad g_{\Xi\Lambda}=g_{N\Lambda}, \quad g_{\Xi\Sigma}=g_{N\Sigma}, \\ &\Gamma_{\Xi\Lambda}=\Gamma_{N\Lambda}, \quad \Gamma_{\Xi\Sigma}=\Gamma_{N\Sigma}. \end{aligned} \quad \text{(II)}$$

These relations among the Γ's imply equal Ξ-N parity.

Then the strong interaction is invariant under the transformation

$$\begin{aligned} &p\leftrightarrow \Xi^-, \quad \pi^\pm\rightarrow \pi^\mp, \\ &n\leftrightarrow \Xi^0, \quad \pi^0\rightarrow -\pi^0, \\ &\Sigma^+\leftrightarrow \Sigma^-, \quad K^+\leftrightarrow \bar{K}^+, \\ &\Sigma^0\leftrightarrow -\Sigma^0, \quad K^0\leftrightarrow -\bar{K}^0, \\ &\Lambda^0\rightarrow \Lambda^0, \end{aligned} \quad \text{(C)}$$

If $m_\Xi=m_N$, then the entire Lagrangian would be invariant under this transformation and there would be no splitting of the Σ^+-Σ^- masses, independently, of whether δm_K or δm_π are zero. These are conditions for the AB symmetry of Feinberg and Behrends[8] which have not as yet been tested by experiment. It should be noted that if, in addition to (C), $A_\mu\rightarrow -A_\mu$ in a Lagrangian with the electromagnetic field included according to the principle of minimal electromagnetic coupling,[9] then $m_\Xi=m_N$ would imply that the Σ^+-Σ^- masses were degenerate and that $m_n-m_p=m_{\Xi^0}-m_{\Xi^-}$ to all orders in the strong *and* the electromagnetic coupling constants, e.g., including the contributions from magnetic moments, etc.[8]

As before, we will assume that this symmetry (C) is broken by the bare masses only, i.e., by assuming that $m_\Xi=m_N+\delta m_{\Xi N}$, where $\delta m_{\Xi N}$ is small compared with the average Ξ-N mass.

By arguments similar to those given for the n-p mass difference, it can be shown that

$$m_{\Sigma^+}-m_{\Sigma^-} =\frac{i}{2\pi^3}\delta m_{\Xi N}\bar{u}'\frac{\partial^2}{\partial m_K\partial m_{\Xi N}}\mathfrak{S}^*(g_{N\Sigma},m_\Sigma,m_{\Xi N},m_K)u'. \quad (8)$$

In order to find the Σ^+-Σ^0 mass splitting, consider the isotopic rotation in the 1–3 plane:

$$\begin{aligned} &p\rightarrow (p+n)/\sqrt{2}, && \pi^1=(\pi^-+\pi^+)/\sqrt{2}\rightarrow \pi^0, \\ &n\rightarrow (p-n)/\sqrt{2}, && \pi^2\rightarrow \pi^2, \\ &\Xi^0\rightarrow (\Xi^0+\Xi^-)/\sqrt{2}, && \\ &\Xi^-\rightarrow (\Xi^0-\Xi^-)/\sqrt{2}, && K^+\rightarrow (K^++K^0)/\sqrt{2}, \\ &\Sigma^1=(\Sigma^-+\Sigma^+)/\sqrt{2}\rightarrow \Sigma^0, && K^0\rightarrow (K^+-K^0)/\sqrt{2}, \\ &\Sigma^2=(\Sigma^--\Sigma^+)/\sqrt{2}i\rightarrow \Sigma^2, && \\ &\Sigma^0\rightarrow \Sigma^1. && \end{aligned} \quad \text{(D)}$$

If δm_π is zero, note that $m_{\Sigma^1}-m_{\Sigma^0}$ is zero because of transformation (A) (i.e., $\Sigma^1\rightarrow\Sigma^1$, $\Sigma^0\rightarrow -\Sigma^0$, $K^+\rightarrow K^0$) *and* transformation (D). Explicitly, transformation (D) states that the following terms exist:

$$\cdots\frac{1}{k^2-m_{K^+}{}^2}\cdots-\frac{1}{k^2-\frac{1}{4}(m_{K^0}+m_{K^+})^2},$$

while transformation (A) guarantees that concurrently, terms of the form

$$\cdots\frac{1}{k^2-m_{K^0}{}^2}\cdots-\frac{1}{k^2-\frac{1}{4}(m_{K^0}+m_{K^+})^2}$$

exist. The net result is zero to first order in δm_K. Similar arguments can be used for the isotopic spin rotations in the 2–3 plane. It is then possible to show, by the same general method as used for the p-n mass

[8] G. Feinberg and R. E. Behrends, Phys. Rev. **115**, 745 (1959).

[9] M. Gell-Mann, *Proceedings of the Sixth Annual Rochester Conference on High-Energy Nuclear Physics, 1956* (Interscience Publishers, New York, 1956); A. Pais, Phys. Rev. **86**, 663 (1952).

splitting, that

$$m_{\Sigma^1}-m_{\Sigma^0}=\frac{-i}{4\pi^3}\bar{u}'\frac{\partial}{\partial m_\pi}\mathfrak{S}^*(m_\pi)\bar{u}',$$

$$m_{\Sigma^2}-m_{\Sigma^0}=\frac{-i}{4\pi^3}\bar{u}'\frac{\partial}{\partial m_\pi}\mathfrak{S}^*(m_\pi)\bar{u}'.$$

Adding these equations, it follows that

$$(m_{\Sigma^1}+m_{\Sigma^2}=m_{\Sigma^+}+m_{\Sigma^-})$$

$$\frac{m_{\Sigma^+}+m_{\Sigma^-}}{2}-m_{\Sigma^0}=\frac{-i}{4\pi^3}\bar{u}'\frac{\partial}{\partial m_\pi}\mathfrak{S}^*(m_\pi)u'. \tag{9}$$

It should be emphasized that this result depends only upon the Lagrangian being charge independent and not upon any particular choices of coupling constants.

MASS RELATIONS

We will now attempt, through the use of more restrictive assumptions, to obtain some mass relations which are independent of the propagators. The additional set of assumptions that will prove useful is that which guarantees invariance under the combined set of transformations (B) and (C), i.e., assumptions I and II, or

$$\Gamma_{\Lambda N}=\Gamma_{\Sigma\Lambda}=\Gamma_{\Lambda\Xi}=\Gamma_{\Sigma\Xi}\equiv\Gamma,\ \Gamma_{\Lambda\Sigma}=\Gamma_{\Sigma\Sigma};$$
$$g_{\Lambda KN}=g_{\Lambda K\Xi}=g_{\Sigma KN}=g_{\Sigma K\Xi}\equiv g_K,$$
$$g_{N\pi}=g_{\Xi\pi},\ g_{\Lambda\pi}=g_{\Sigma\pi}. \tag{III}$$

Let us first consider the sum of Eqs. (6) and (7) under these conditions.

$$m_n-m_p+m_{\Xi^0}-m_{\Xi^-}=\frac{i}{4\pi^3}\delta m_{\Sigma\Lambda}\bar{u}'\frac{\partial^2}{\partial m_K\partial m_{\Sigma\Lambda}}[\mathfrak{S}^*(g,m_N,m_{\Sigma\Lambda})+\mathfrak{S}^*(g,m_\Xi,m_{\Sigma\Lambda})]\bar{u}'.$$

From what we have done before, this expression is valid only to first order in either $\delta m_{\Sigma\Lambda}$ or $\delta m_{N\Xi}$. Therefore using the expansion

$$\mathfrak{S}^*(g,m_N,m_{\Sigma\Lambda})+\mathfrak{S}^*(g,m_\Xi,m_{\Sigma\Lambda})=2\mathfrak{S}^*(g,m_{\Xi N},m_{\Sigma\Lambda})+\delta m_{\Xi N}\frac{\partial}{\partial m_{\Xi N}}\mathfrak{S}^*(g,m_{\Xi N},m_{\Sigma\Lambda})+\cdots,$$

and neglecting higher order terms, we obtain

$$m_n-m_p+m_{\Xi^0}-m_{\Xi^-}=\frac{i}{2\pi^3}\delta m_{\Sigma\Lambda}\bar{u}'\frac{\partial^2}{\partial m_K\partial m_{\Sigma\Lambda}}\mathfrak{S}^*(g,m_{\Xi N},{}_{\Sigma\Lambda})\bar{u}'. \tag{10}$$

If we now form the ratio $(m_n-m_p+m_{\Xi^0}-m_{\Xi^-})/(m_{\Sigma^+}-m_{\Sigma^-})$ from Eqs. (8) and (10) and notice that the average Σ-Λ mass and the average Ξ-N mass are almost the same, we find that

$$\frac{m_n-m_p+m_{\Xi^0}-m_{\Xi^-}}{m_{\Sigma^+}-m_{\Sigma^-}}=\frac{\delta m_{\Sigma\Lambda}}{\delta m_{\Xi N}} \tag{11}$$

This relation is, of course, only true for those contributions which arise from clothed boson propagators. The total mass splittings will be made up of these contributions plus those from clothed Fermion propagators and total vertex operators for both the meson and electromagnetic fields. These remaining contributions have not been considered.

SUMMARY

The perturbation calculation of Bransden and Moorhouse has indicated that for reasonable values of the coupling constants, the K^+-K^0 mass difference induces a Σ^+-Σ^- mass splitting of the same order of magnitude as that experimentally observed. This seems reasonable when one notes that if the magnetic moments of the Σ^+ and Σ^- were equal and opposite, the usual electromagnetic self-energies (e.g., as calculated for the n-p mass difference) would cause no mass splitting. Therefore any Σ^+-Σ^- mass difference, if due to the electromagnetic field, would have to arise from the total vertex operator and the total boson and Fermion propagators. Similarly, it might be expected that these vertex and propagator effects might be at least as important as the usual electromagnetic self-energy effects in the calculations of the n-p and Ξ^0-Ξ^- mass differences. On this basis, a mass relation, Eq. (11), arising solely from the effects of the boson propagators, was derived which is valid to all orders in the strong-coupling constants, providing certain relative parity and coupling constant assignments are made.

In addition, theorems were proved which demonstrated the qualitative effect of the clothed boson propagators on the various splittings within the baryon charge multiplets. Namely, it was shown that to first order in either δm_π or δm_K and either $\delta m_{N\Xi}$ or $\delta m_{\Sigma\Lambda}$ that the n-p, Ξ^0-Ξ^-, and Σ^+-Σ^- mass differences were independent of the pion mass splitting δm_π, that the K-meson mass splitting δm_K split the Σ^+ and the Σ^- masses equally but in opposite directions from the Σ^0 while the effect of δm_π was to move the Σ^0 level closer to either the Σ^+ or the Σ^-.

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