Deuteron Magnetic Moment and Momentum Dependence of Two-Nucleon Potential*

MASAO SUGAWARA†
Brookhaven National Laboratory, Upton, New York
(Received August 12, 1959)

The correction to the deuteron magnetic moment [μ_d] is calculated, in the manner pointed out by Feshbach, for the potential derived recently by Sugawara and Okubo from pion field theory. This potential includes, besides an $\mathbf{L}\cdot\mathbf{S}$ potential, a quadratic term, $-V_2(\mathbf{r})(p^2/2\kappa^2)+$H.c. [$V_2(\mathbf{r})$ being the second-order static potential, p and κ the nucleon (relative) momentum and rest mass, respectively]. It is shown in particular that this new term gives a positive correction to μ_d. Numerical magnitudes are estimated using phenomenological deuteron wave functions fitted to all known deuteron data, the hard-core radius [r_C] and the D-state probability [P_D] being adjustable parameters. Results are shown graphically as functions of P_D for two values of r_C. It is seen that the corrections depend sensitively on these two parameters. If there were no other appreciable corrections to μ_d than those discussed here, *ps-ps* theory would lead to 6% for P_D, while μ_d would not be fitted in the *ps-pv* case as well as for the Gammel-Thaler potential, since the correction due to the quadratic term is not large enough to cancel the correction due to the $\mathbf{L}\cdot\mathbf{S}$ potential.

FESHBACH[1] has shown that the momentum-dependent term in the two-nucleon potential can appreciably affect the deuteron magnetic moment [μ_d]. The same consideration is, therefore, applied here to the pion-theoretical potential derived recently by the present author and Okubo.[2] The potential consists of three terms,

$$V=V(\text{static})+V_{LS}(r)\mathbf{L}\cdot\mathbf{S}-\{V_2(\mathbf{r})(p^2/2\kappa^2)+\text{H.c.}\},\quad(1)$$

where $V_2(\mathbf{r})$ is the second-order static potential, and p and κ are the nucleon (relative) momentum and rest mass, respectively.

The correction to μ_d in units of the nuclear magneton can be expressed in terms of the deuteron S- and D-wave functions, u and w, respectively, as

$$(\Delta\mu_d)_{p^2}=\frac{3\sqrt{2}}{2\kappa}\left[\int V_T(r)uwdr+\frac{\sqrt{2}}{4}\int V_C(r)w^2dr-\frac{\sqrt{2}}{2}\int V_T(r)w^2dr\right],\quad(2)$$

and

$$(\Delta\mu_d)_{LS}=\frac{\kappa}{6}\left[\int r^2V_{LS}(r)u^2dr-\frac{\sqrt{2}}{2}\int r^2V_{LS}(r)uwdr-\int r^2V_{LS}(r)w^2dr\right],\quad(3)$$

where[3]

$$V_2(\mathbf{r})\equiv-[V_C(r)+V_T(r)S_{12}],$$

and

$$\int[u^2+w^2]dr=1.\quad(4)$$

It is now obvious that $(\Delta\mu_d)_{p^2}$ is positive and $(\Delta\mu_d)_{LS}$ has the same sign as $V_{LS}(r)$ itself.

The above integrals have been numerically evaluated using phenomenological deuteron wave functions[4] fitted to all known deuteron data except the hard-core radius [r_C] and the D-state probability [P_D]. These two parameters are chosen as

$$r_C=0.4316 \text{ and } 0.5611\times10^{-13}\text{ cm},$$
$$P_D=3,\ 4 \text{ and } 5\%,\quad(5)$$

and the shape-dependent parameter is assumed to be zero, since it gives rise to only a minor change in the deuteron wave function.[4] The last term in (3) is neglected for simplicity. The results are plotted in Fig. 1 as functions of P_D for two values of r_C. The dashed lines indicate $(\Delta\mu_d)_{p^2}$ only. The solid curves are the sum of (2) and (3). The *ps-ps* curves refer to the *ps-ps* potential (the first paper of reference 2), while those designated as

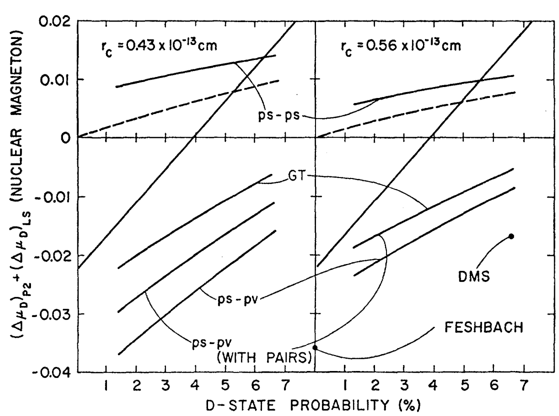

FIG. 1. The plot of the sum of (2) and (3) as functions of D-state probability for two values of the hard-core radius, r_C. The dashed curves represent our new term (2) only. Various curves correspond to several theoretical and phenomenological potentials.

* Supported by the U. S. Atomic Energy Commission and the National Science Foundation.
† On leave of absence during the summer of 1959 from Purdue University, Lafayette, Indiana.
[1] H. Feshbach, Phys. Rev. **107**, 1626 (1957).
[2] M. Sugawara and S. Okubo, this issue [Phys. Rev. **117**, 605 (1960)]; preceding paper [Phys. Rev. **117**, 611 (1960)].
[3] The last term in (3) differs from Feshbach's result [reference 1] by a numerical factor.
[4] These are tabulated by L. Hulthén and M. Sugawara, in *Encyclopedia of Physics*, edited by S. Flügge (Springer-Verlag, Berlin, 1957), Vol. 39, p. 92.

ps-pv and *ps-pv* (with pairs) refer to two versions of the *ps-pv* potential (the second paper of reference 2). The G-T curves correspond to the Gammel-Thaler $\mathbf{L}\cdot\mathbf{S}$ potential,[5] to which, however, our new correction (2) has been added. The two points on the figure show the previous calculations of $(\Delta\mu_d)_{LS}$ for the Gammel-Thaler potential[5] by Feshbach[1] and others.[6] In these calculations,[1,6] $r_C=0.4\times10^{-13}$ cm and P_D is fixed as placed on the figure.

Two conclusions are drawn from the figure. First, the correction depends sensitively on both r_C and P_D, or the internal detail of the deuteron wave function. Secondly, our new correction $(\Delta\mu_d)_{p^2}$ is not a large correction compared with $(\Delta\mu_d)_{LS}$ in case of the Gammel-Thaler[5] and the *ps-pv*[2] potentials.

The corrections (2) and (3) are supposed to be almost the sum of the relativistic and exchange current corrections to μ_d, which were previously estimated by the present author.[7] The nonadditivity correction is not included in (2) and (3), since all the self-energy diagrams are dropped in deriving the potential.[2] The conclusion reached in the previous work[7] that P_D seems to be slightly smaller than 4% according to pion field theory, does not agree with the present calculation since it suggests that P_D has to deviate from 4% appreciably. This difference is not due to the difference in approaches (previously[7] μ_d was evaluated directly from pion field theory, while it is now estimated from the momentum-dependent term in the potential), but rather to the difference in the approximations made; we did not previously[7] go far enough to match those momentum-dependent terms which are investigated in the present paper.

The straight line which completely traverses both parts of Fig. 1 is the plot of $\mu_d-\mu_p-\mu_n+\frac{3}{2}(\mu_p+\mu_n-0.5)P_D$ (μ_p and μ_n are proton and neutron moments in units of nuclear magneton); this is the line on which the correction $\Delta\mu_d$ should lie, where $\Delta\mu_d$ is the total correction to be added to the *S*- and *D*-state contributions. If there were no other significant corrections besides those given by (2) and (3), the figure indicates that *ps-ps* theory would lead to $\approx 6\%$ for P_D, while there is no way of fitting the deuteron magnetic moment in the other cases.

ACKNOWLEDGMENTS

The author wishes to express his gratitude to members of the Brookhaven National Laboratory, where the present work was done. All numerical calculations were done by the computor staff of this laboratory. Valuable discussions with Dr. M. Ross are highly appreciated.

[5] J. Gammel and R. Thaler, Phys. Rev. **107**, 291 (1957); **107**, 1337 (1957).
[6] DeSwart, Marshak, and Signell, Nuovo cimento **6**, 1189 (1957).
[7] M. Sugawara, Phys. Rev. **99**, 1601 (1955); and Progr. Theoret. Phys. (Kyoto) **14**, 535 (1955).