

if the experiment were repeated independently, with some improvement in experimental technique. Special precautions were taken to prevent any possible sputtering from the sample holder to the sample. In addition, the use of metal valves, rather than stopcocks in the gas handling system, is considered an improvement in technique over that used by other investigators.

It will be noted that the general features of Handler's data for a typical surface<sup>4</sup> are similar to those reported here, but the details are considerably different. We have not noted any significant change in clean surface properties during exposure at pressures as low as  $10^{-7}$  mm, but these low pressure measurements on active gases are very difficult to make and hard to interpret.<sup>21</sup> Our field effect measurements on a cleaned surface are much higher than those previously reported. Handler's<sup>4</sup> published data are an order of magnitude lower and Autler<sup>22</sup> and Wallis<sup>23</sup> reported intermediate values for the field

<sup>21</sup> R. E. Schlier, J. Appl. Phys. **29**, 1162 (1958).

<sup>22</sup> Autler, McWhorter, and Gebbie, Bull. Am. Phys. Soc. **1**, 145 (1956).

<sup>23</sup> P. Handler, *Semiconductor Surface Physics*, edited by R. H.

effect mobility of a cleaned surface. On the basis of the data shown in Fig. 4, the discrepancy between the different published values for the field effect mobility of a cleaned surface can be understood. It is believed that the lower mobility values reported in the literature are associated with an incomplete anneal of the surface damage caused by the argon bombardment cleaning.

In addition, it is believed that the data reported on the anisotropic behavior of the (111) and (100) cleaned surface are one of the first pieces of experimental evidence to indicate the validity of the atomistic model of a cleaned surface as consisting of broken orbital bonds on surface atoms.

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Kingston (University of Pennsylvania Press, Philadelphia, 1957), p. 31.

## Aspects of the Theories of Dislocation Mobility and Internal Friction

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The theory of dislocation mobility is reconsidered, and it is concluded that the interaction between thermal waves and a moving vibrating dislocation causes a drag to the first order in  $v/c$ . Modifications of the Seeger-Donth theory for the Bordoni peak are suggested. When account is taken of the diffusion of kinks, general agreement with experiment is obtained. The theory of internal friction in the microwave region is briefly reviewed and discussed. No thorough comparison with experimental data has been performed.

### I. DISLOCATION MOBILITY

A DISLOCATION scatters elastic waves, and in an isotropic distribution of thermal waves a moving dislocation will experience a retarding force proportional to its velocity. This problem was first investigated by Leibfried,<sup>1</sup> who concluded that at room temperature and for ordinary stresses the dislocation velocity would be only a small fraction of the velocity of sound. The Leibfried work was later criticized and extended by Nabarro.<sup>2</sup> Nabarro pointed out that two separate scattering mechanisms should be considered:

(1) Scattering of sound waves by the strained matrix around the dislocation; and (2) Scattered waves radiated from the dislocation vibrating under the action of the impinging waves.

It was concluded that the scattering cross section for mechanism (1) was smaller than the cross section as-

sumed by Leibfried, and that in an isotropic flux of sound waves mechanism (2) does not lead to a drag to the first order in  $v/c$ , where  $v$ =dislocation velocity; and  $c$ =velocity of shear waves.

We shall only reconsider mechanism (2). For the sake of comparison we follow closely parts of the treatment given by Nabarro.<sup>2</sup> Consider the two-dimensional problem of a pure undissociated screw dislocation along the  $y$ -axis in a Cartesian system  $(x,y,z)$ , interacting with shear waves with propagation vectors in the  $x$ - $z$  plane. The solid is assumed to be elastically isotropic.

According to Eshelby,<sup>3</sup> this problem has a complete electromagnetic analogy:

$$\partial u_y / \partial t = H_y / (4\pi\rho)^{1/2}, \quad \sigma_{yz} = -(\mu/4\pi)^{1/2} E_z, \quad \sigma_{xy} = (\mu/4\pi)^{1/2} E_x,$$

$$E_y = H_z = H_x = 0, \quad e = (\mu/4\pi)^{1/2} b,$$

$$\mathbf{F} = e\mathbf{E} + (e/c)\mathbf{v} \times \mathbf{H},$$

<sup>1</sup> G. Leibfried, Z. Physik **127**, 344 (1950).

<sup>2</sup> F. R. N. Nabarro, Proc. Roy. Soc. (London) **209**, 278 (1951).

<sup>3</sup> J. D. Eshelby, *Solid State Physics* edited by F. Seitz and D. Turnbull (Academic Press, Inc., New York, 1956), Vol. 3.

where  $u_y$ =displacement in  $y$ -direction;  $\rho$ =density;  $\mu$ =shear modulus;  $\sigma$ =shear stress;  $b$ =Burger's vector;  $c$ =sound velocity;  $v$ =dislocation velocity;  $e$ =equivalent line charge density;  $\mathbf{F}$ =force per unit length of dislocation; and  $\mathbf{E}, \mathbf{H}$ =equivalent electric vectors. Nabarro<sup>2</sup> was the first one to demonstrate that the last term in  $\mathbf{F}$  has physical meaning for dislocations. This term is just what is needed to ensure "relativistic" invariance.

Consider a plane shear wave  $\partial u_y/\partial x$  impinging on the dislocation in the  $-x$ -direction and the resulting radiation force balanced by a uniform shear stress  $\sigma_{yz}(v=0)$  so that the dislocation on time average has zero velocity,  $\langle v_x \rangle_{Av} = v = 0$ .

The dislocation will vibrate in the  $y$ - $z$  plane, and when  $\langle v_x \rangle_{Av} = v = 0$ , the scattered radiation has no net "momentum." It is then permissible to calculate the radiation force from the time average of the Lorentz term in  $\mathbf{F}$ , with  $\mathbf{H}$  the vector of the impinging wave, i.e.,

$$\sigma_{yz}(v=0) = \frac{\mu}{2c^2} |v_z| \left| \frac{\partial u_y}{\partial t} \right| \cos \alpha, \quad (1)$$

where  $|v_z|$  is the amplitude of  $v_z$ , etc., and  $\alpha$  is the phase angle between  $v_z$  and  $\partial u_y/\partial t$ .

For not too short wavelengths the dislocation vibrations are "mass"-controlled by the effective dislocation mass per unit length,  $m$ . Nabarro<sup>2</sup> has shown that in this case  $\alpha$  is nearly frequency independent, corresponding to a scattering cross section proportional to  $\lambda$ , the wavelength. Now increase the uniform constant stress  $\sigma_{yz}(v=0)$  to  $\sigma_{yz}(v=v)$  so that the dislocation moves with a velocity  $\langle v_x \rangle_{Av} = v$ , while the impinging wave is unchanged.

To calculate the radiation force correctly by the previous method, we shall have to transform from the stationary system  $(x, y, z)$  to a system  $(x', y', z')$  moving with velocity  $\langle v_x \rangle_{Av} = v$ . Neglecting terms  $\sim v^2/c^2$ , we can write the following transforms for the wave by relativistic analogy,

$$\begin{aligned} \omega' &= (1+v/c)\omega, \\ |\partial u_{y'}/\partial t'| &= (1+v/c) |\partial u_y/\partial t|, \\ |\partial u_{y'}/\partial x'| &= (1+v/c) |\partial u_y/\partial x|, \\ e' &= e. \end{aligned} \quad (2)$$

Since the phase angle  $\alpha$  is constant,

$$m\omega' |v_{z'}| \propto |\partial u_{y'}/\partial x'|,$$

so that, from Eq. (2),

$$|v_{z'}| \propto 1/m |\partial u_y/\partial x|. \quad (3)$$

Now, applying Eq. (1) to the system  $(x', y', z')$  we find

$$\sigma_{y'z'}(v=v) \propto (1+v/c) (\mu/2mc^2) \times |\partial u_y/\partial x| |\partial u_y/\partial t| \cos \alpha. \quad (4)$$

Since for a uniform constant stress  $\sigma_{yz}$

$$\sigma_{yz} = \sigma_{y'z'}, \quad (5)$$

it follows immediately that

$$\sigma_{yz}(v=v) = (1+v/c) \sigma_{yz}(v=0). \quad (6)$$

Thus, compared with the radiation stress  $\sigma_{yz}$  on a stationary dislocation, a dislocation moving with velocity  $\langle v_x \rangle_{Av} = v$  experiences a radiation stress greater by  $(v/c) \sigma_{yz}$ . It appears that Nabarro<sup>2</sup> has not taken into account that the scattered radiation from a uniformly moving dislocation is not symmetric in the stationary system  $(x, y, z)$ .

A shear wave impinging in the  $-z$ -direction will make an angle  $\sim v/c$  with the  $z'$ -axis, and a radiation stress  $\sigma_{yz}$  opposing motion,

$$\sigma_{yz}(v=v) = (v/c) \sigma_{xy}(v=0), \quad (7)$$

results. This is a pure aberration effect, with no first order changes in frequency. It appears that also this effect has been neglected in previous treatments on dislocations, as all discussions have been carried out in terms of the Doppler shift  $\omega' = (1+v/c)\omega$ .

We will most frequently be interested in dissociated dislocations, constrained to move in one plane. In that case the Lorentz force is not operative as it is perpendicular to the slip plane. The radiation force can then be calculated to the first power of  $v/c$  from the term  $e\mathbf{E}$  by a generalization of the method employed by Leibfried<sup>1</sup> in his original paper. The results would not be much different from the case we have discussed.

Employing the cross section calculated by Nabarro,<sup>2</sup> it is found that near the Debye frequency, the cross section per unit length of dislocation is of the order  $\sim b$  such as assumed by Leibfried. The original Leibfried formulas are then probably the best estimate so far. It reads

$$v \sim (10c/\bar{\epsilon})\sigma, \quad (8)$$

where  $\bar{\epsilon}$  is the vibrational energy per atom, classically  $3kT$ . The zero-point energy should not be included in  $\bar{\epsilon}$ , as zero-point energy cannot be absorbed, i.e., it is not scattered.

The foregoing considerations apply to dislocations which are considered to be completely free in the slip plane, i.e., the Peierls barrier must be negligible or the dislocations must be so heavily kinked that they can be considered as effectively free.

For dislocations containing only a few kinks, we have to investigate the mobility of kinks separately. A kink is drawn in Fig. 1. The width  $d$  of a kink will typically

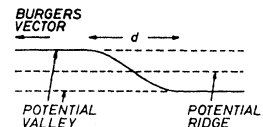


FIG. 1. A kinked screw dislocation.

be of the order  $10b$ . The mean thermal stress

$$(\sigma^2)^{1/2} \sim 2\mu kT/d^3,$$

over the kink will be much higher than ordinary applied stresses except at very low temperatures, so the motion of the kink will be of the diffusion-drift type. Any "Peierls" force against motion of the kink along the dislocation will be completely negligible.

For long wavelengths,  $\lambda > d$ , the problem is analogous to that of an electron in an electromagnetic field. For shear waves impinging about normally to the slip plane and of wavelength  $\lambda < d$ , the kink vibrations are largely radiation resistance controlled. Preliminary classical calculations have yielded for the kink-velocity the formula

$$v \simeq c\sigma ab^2/kT, \quad (9)$$

which is probably somewhat of an overestimate because of neglect of other scattering mechanisms.

Comparing Eq. (9) with the Einstein relation

$$v = D\sigma ab/kT, \quad (10)$$

where  $D$  is the diffusion coefficient for the kink, it follows that

$$D \simeq \nu_D b^2, \quad \nu_D \simeq c/b. \quad (11)$$

If the phonon mean free path is too short, calculations of the foregoing type are not valid. Thus, for short mean free paths, in order of magnitude smaller than the dimension of the scattering cross section, we should have to do thermoelastic or phonon "viscosity" calculations. More precisely, we can state two conditions:

- (1) The thermal waves are coherent over several cycles (Nabarro<sup>2</sup>).
- (2) The thermal waves impinging on the moving dislocation are isotropically distributed with respect to the crystal. [The stationary system  $(x, y, z)$ .]

For an order of magnitude estimate, we would not consider condition (1) to be critical, whereas condition (2) is the important one. In metals at room temperature the short wave phonon mean free path is probably only of the order  $\sim 10b$ . Phonon-phonon and phonon-electron collisions are about equally frequent. In pure metals the electrons have a mean free path of the order  $\sim 100b$ . Then, because of the phonon-electron interaction, we must expect that condition (2) is fairly well fulfilled even as high as ordinary temperatures for metals. For insulators our results are possibly valid only for quite low temperatures.

## II. THE BORDONI PEAK

Bordoni<sup>4</sup> first discovered an internal friction relaxation peak in metals in the temperature region  $\sim 100^\circ\text{K}$  in kilocycle experiments. Since then a number of careful experimental investigations resolved several peaks.

<sup>4</sup> P. C. Bordoni, J. Acoust. Soc. Am. **26**, 495 (1954).

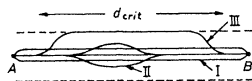


FIG. 2. Various stages of kink activation according to Seeger.

(Niblett and Wilks,<sup>5</sup> Caswell,<sup>6</sup> Thompson and Holmes,<sup>7</sup> and Paré.<sup>8</sup>) Activation energies are still quite uncertain, e.g., in copper the main peak has an activation energy in the region (0.12–0.18) ev. Mason<sup>9</sup> suggested that the Bordoni peak might be interpreted as thermal activation over the Peierls barrier. Seeger<sup>10</sup> and Donth<sup>11</sup> (see also Seeger, Donth, and Pfaff<sup>12</sup>) worked out this idea by applying the theory of stochastic processes to the activation of dislocation lines.

A brief outline of the Seeger-Donth theory is necessary. Consider a dislocation segment in its Peierls valley, pinned at two points  $A$  and  $B$ , and consider the ground vibration. (Fig. 2). For small oscillator energies, the dislocation vibrates essentially rigidly (I). For higher energies, a bulge will appear, similar to a pair of kinks (II). When the pair of kinks reaches a separation  $d_{crit}$  for which they may be torn apart by the applied stress, the activation is considered to be complete (III). Donth<sup>11</sup> calculates an activation energy in the region

$$W_A = (4.5 - 4.0)W_K, \quad (12)$$

depending on the applied stress.  $W_K$  is the kink energy. Analysis of experimental data by the Seeger-Donth theory yields typically

$$d_{crit} \sim 40b,$$

corresponding to a kink width  $20b$ . The Peierls stress will typically be

$$\sigma_P \sim 5 \times 10^{-4} \mu.$$

Frank<sup>13</sup> pointed out that kink pairs of separation greater than  $d_{crit}$  may recombine and annihilate by diffusion, and that the Seeger-Donth theory must be correspondingly revised. This idea has been applied by Lothe and Hirth<sup>14</sup> to a creep theory to explain some experiments by Lytton, Shepard, and Dorn.<sup>15</sup>

Actually the kinks must reach a separation

$$l > d_{crit} + l_{crit}, \quad (13)$$

where

$$\sigma ab l_{crit} = kT, \quad (14)$$

<sup>5</sup> D. H. Niblett and J. Wilks, Phil. Mag. **8**, 1427 (1957).

<sup>6</sup> H. Caswell, J. Appl. Phys. **29**, 1210 (1958).

<sup>7</sup> D. O. Thompson and D. K. Holmes, J. Appl. Phys. **30**, 525 (1959).

<sup>8</sup> V. K. Paré, thesis, Cornell University, July, 1955 (unpublished).

<sup>9</sup> W. P. Mason, J. Acoust. Soc. Am. **27**, 643 (1955).

<sup>10</sup> A. Seeger, Phil. Mag. **1**, 651 (1956).

<sup>11</sup> H. Donth, Z. Physik **149**, 111 (1957).

<sup>12</sup> Seeger, Donth, and Pfaff, Discussions Faraday Soc. **23**, 19 (1957).

<sup>13</sup> F. C. Frank (private communication).

<sup>14</sup> J. Lothe and J. P. Hirth, Phys. Rev. **115**, 543 (1959).

<sup>15</sup> Lytton, Shepard, and Dorn, Trans. Am. Inst. Mining, Met. Petrol. Engrs. **212**, 220 (1958).

in order to be considered as effectively separated. With typically  $a \sim b$ ,  $\mu b^3 \sim 5$  eV,  $kT \sim 0.01$  eV, we deduce

$$\begin{aligned} (a) \quad & \sigma \sim 10^{-7} \mu, \quad l_{\text{crit}} = 2 \times 10^4 b, \\ (b) \quad & \sigma \sim 10^{-4} \mu, \quad l_{\text{crit}} = 20b. \end{aligned} \quad (15)$$

(15a) corresponds to the lowest amplitudes used in internal friction experiments. (15b) may be typical of internal stresses. Higher stresses than (15b) are probably not operative on dislocations contributing to the Bordoni peak as the position of the peak does not shift appreciably with increasing amount of cold work. As the kink will only move distances of the order one lattice distance before reversing direction when diffusing, the diffusion treatment is expected to be appropriate for all values of  $l_{\text{crit}}$ .

We shall assume that only a small fraction of the kink pairs reaching a separation  $d_{\text{crit}}$  separate beyond  $d_{\text{crit}} + l_{\text{crit}}$ . From the balance of nucleation and annihilation, it follows that the concentration of kink pairs of a separation in the region  $\sim d_{\text{crit}}$  is about the same as on a stress-free infinite dislocation line. The Bordoni relaxation is considered to take place by diffusion out of this reservoir. In particular it follows that the activation energy of the Bordoni peak is twice the energy governing the thermal concentration of kinks on a free dislocation line.

Define  $n(l)dl$  as the number of double kinks with a separation between  $l$  and  $l+dl$ , per unit length of dislocation. Consider an assembly of  $N$  dislocation segments each of length  $L \gg d_{\text{crit}}$ . The diffusion coefficient for a double kink in  $l$  space is  $2D$ . With no external stress on the dislocations and the boundary condition  $n(L) = 0$ , the time  $\tau_0$  needed for all dislocation segments to have diffused into a neighboring valley is determined by

$$n(d_{\text{crit}})LN(2D/L)\tau_0 = N, \quad (16)$$

i.e.,

$$\gamma_0 = 2Dn_0, \quad n(d_{\text{crit}}) \sim n_0, \quad \gamma_0 = 1/\tau_0. \quad (17)$$

$n_0$  is the constant value of  $n(l)$  for double kinks, imagined to be noninteracting, on an infinite dislocation under no stress.  $\gamma_0$  is the appropriate relaxation frequency of the assembly. If there are stresses  $\sigma$  so great that

$$\sigma abL \gg kT, \quad (18)$$

we find a relaxation frequency

$$\gamma = \gamma_0 L / l_{\text{crit}}. \quad (19)$$

A pure relaxation takes place only between states equally populated. Paré<sup>8</sup> has investigated theoretically the importance of internal stresses on the Bordoni peak. It is shown that with reasonable distribution functions for internal stresses and loop lengths, only those dislocations which have two configurations of nearly equal energy, within  $\sim kT$ , will contribute to the Bordoni peak. Thus, consider as a simple case a dislocation in its Peierls valley pinned at two points  $A$  and  $B$ , a dis-

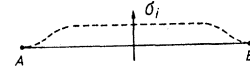


FIG. 3. A dislocation segment  $AB$  which can bow out by forming two kinks. When  $\sigma_i abL = 2W_K$ ,  $W_K$  = kink energy, the kinked configuration has the same energy as the unkinked configuration.

tance  $L$  apart, as shown in Fig. 3. This dislocation will contribute to the Bordoni peak only if it is acted upon by an internal stress  $\sigma_i$  so that

$$\sigma_i abL = 2W_K. \quad (20)$$

Combining Eqs. (19), (14), and (20), we find

$$\gamma = (2W_K/kT)\gamma_0. \quad (21)$$

Thus, our model leads to a relaxation frequency independent of internal stresses and loop lengths.<sup>16</sup>

Lothe and Hirth<sup>14</sup> have calculated the partition function for a kinked dislocation and deduce the double-kink density function

$$n_0 = 1.1(ab\sigma_P/kT)(\pi b\sigma_P/2aS)^{\frac{1}{2}}e^{-2W_K/kT}. \quad (22)$$

Here  $S$  is the line energy

$$S \sim \mu b^2. \quad (23)$$

So the final expression for the relaxation frequency becomes, by Eqs. (17), (21), and (22).

$$\gamma \simeq 2D(ab\sigma_P/kT)(\pi b\sigma_P/2aS)^{\frac{1}{2}}(2W_K/kT)e^{-2W_K/kT}. \quad (24)$$

Since the kink energy deduced from experiment will be about twice as high in our model as in the Seeger-Donth model [see Eq. (12)], we deduce higher Peierls barriers and narrower kinks. We shall use as typical values, kink width  $\sim 10b$ ,  $\sigma_P \sim 10^{-3}\mu$ . With  $a \sim b$ ,  $\mu b^3 \sim 5$  eV,  $kT \sim 0.01$  eV,  $2W_K \sim 0.1$  eV and  $\nu_D \sim 10^{13}s^{-1}$  as typical values, and using Eq. (11) for  $D$ , we deduce for the pre-exponential in the relaxation frequency the value  $\sim 3 \times 10^{12}s^{-1}$ , which is nicely within the range permitted by the uncertainties in the observed activation energies.

As additional support for our model it may be mentioned that Thompson and Holmes<sup>7</sup> have shown that the background internal friction may be analyzed by the statistics of thermal equilibrium of activated states, and they deduce an activation energy about twice the Seeger-Donth kink energy. In our model it would simply be the kink energy. Indeed, independent kinks would obey the statistics employed by Thompson and Holmes, while it is difficult to see how double-kink configurations could. But it must be said that this interpretation is only tentative, as for it to be possible it must be assumed that pinning points do not hinder

<sup>16</sup> Precisely Eq. (20) should be replaced by the condition that the free energy of the two configurations in Fig. 3 be equal. Because of the entropy contribution from the kinks, a term logarithmic in stress should be subtracted on the right-hand side of Eq. (20). For  $\sigma_i$  in the region  $(10^{-6} - 10^{-4})\mu$ , this term is estimated to amount to a few  $kT$ , and the consequent change in the pre-exponential cannot be more than 50%.

single-kink equilibrium to be attained during the experiment.

Which theory is the correct one, the Seeger-Donth theory or the one proposed in this paper, will depend on the length of the distance  $l_{crit}$ , and thus on the magnitude of the internal stresses. For short  $l_{crit}$  it may be that kink pairs which diffuse back into one-another's attraction and make up an oscillator, will have a higher probability to be reactivated, beyond  $l_{crit}$ , than to annihilate. Further investigations are needed to settle this point.

### III. MEGACYCLE INTERNAL FRICTION

Lücke and Granato<sup>17</sup> have suggested a vibrating string model, corresponding to pinned dislocations. A damping term proportional to velocity is assumed. The theory has had some success in the interpretation of experimental data. However, the only clear case of agreement is the experiment on germanium by Granato and Truell.<sup>18</sup> Typical stress amplitudes in megacycle experiments are  $\sigma \sim 10^{-7}\mu$ . The frequency is as a rule far below the resonant frequency of the loop.

A loop of length  $l$  bows out a distance  $A_\sigma$  under a stress  $\sigma$ . By the line tension argument

$$A_\sigma \sim 4l^2\sigma b/\pi^3S, \quad S \sim \mu b^2. \quad (25)$$

The potential energy of the loop at a bow-out  $A$  is

$$W = (\pi^2 S/4l)A^2. \quad (26)$$

Equating this to  $kT$ , we find the mean thermal amplitude

$$A_{th,Av} = (2/\pi)(lkT/S)^{1/2}. \quad (27)$$

During a time  $t = 1/\Omega$ , the dislocation will bow out as far as

$$A_{th,t} = (2/\pi)[(lkT/S) \ln(\gamma/\Omega)]^{1/2}, \quad (28)$$

where  $\gamma$  is the natural frequency of the loop

$$\gamma \sim (b/l)\gamma_D, \quad \gamma_D \sim b/c. \quad (29)$$

Let us insert the typical values  $\mu b^2 \sim 5$  ev,  $kT \sim 0.02$  ev,  $\Omega \sim 10^6 s^{-1}$ ,  $\gamma_D \sim 10^{13} s^{-1}$  and  $\sigma \sim 10^{-7}\mu$  for the cases  $l = 10^3b$ ,  $l = 10^4b$  and  $l = 10^5b$ .

$$\begin{aligned} (a) \quad l = 10^3b & \begin{cases} A_\sigma \sim (1/100)b \\ A_{th,Av} \sim b \\ A_{th,t} \sim 3b, \end{cases} \\ (b) \quad l = 10^4b & \begin{cases} A_\sigma \sim b \\ A_{th,Av} \sim 3b \\ A_{th,t} \sim 9b, \end{cases} \\ (c) \quad l = 10^5b & \begin{cases} A_\sigma \sim 100b \\ A_{th,Av} \sim 10b \\ A_{th,t} \sim 30b. \end{cases} \end{aligned} \quad (30)$$

It is seen that the validity of the Lücke-Granato treatment is obvious only in case (30c), where  $A_\sigma > A_{th,Av}$ .

With the Leibfried mobility Eq. (8) the natural ground frequency of a dislocation loop will be overdamped if

$$l > 10^3b.$$

It may then be more appropriate to consider the dislocation to diffuse between the limits  $\pm A_{th,Av}$  rather than to vibrate. In the interval  $l = (10^3 - 10^4)b$  we may then attempt to replace the Lücke-Granato theory by a relaxation model, in which dislocation segments of length  $\sim l$  diffuse freely between the limits  $\pm A_{th,Av}$ . The appropriate diffusion coefficient would be  $D = (b/l)D_0$ , where  $D_0$  is the diffusion coefficient for a segment one lattice spacing long, to be deduced from the Leibfried mobility and the Einstein relation. It turns out, however, that this model yields results practically identical with the Lücke-Granato theory, with the same dependence on temperature, frequency and loop length. So it is concluded that the Lücke-Granato theory may be considered valid when the pinning points are firm.

For the vibrating string model to be valid, it must at least be required that in a monatomic layer of area  $\sim lA_{th,t}$  or  $lA_\sigma$ , whichever is greatest, no impurities are present. Thus, for a free loop  $l \sim 10^4b$  to be used, the purity must at least be 99.999 at %.

### IV. CONCLUDING REMARKS

Kilocycle background internal friction is generally much higher than what would be expected from extrapolation of megacycle data by the Lücke-Granato theory. In Section III it was clearly shown that vibrating dislocations with high probability will engage themselves in impurity interactions. With binding energies between dislocations and impurities typically in the range (0.3–0.05) ev and a break-away frequency  $\sim 10^9 e^{-E/kT} s^{-1}$ , it is realized that one must usually expect a great number of dislocations to take part in relaxations with a relaxation frequency in the vicinity of the applied frequency, both in kilocycle and megacycle experiments. Megacycle experiments on cold worked superpurity aluminum carried out in this laboratory<sup>19</sup> do not agree with the Lücke-Granato theory. A theory for the amplitude-independent internal friction resulting from thermal break-away from impurities and subsequent recapture is urgently needed.

### ACKNOWLEDGMENT

The author is indebted to Dr. J. P. Hirth for correspondence on problems relevant to this paper.

<sup>17</sup> K. Lücke and A. Granato, *Dislocations and Mechanical Properties of Crystals*, edited by Fisher, Johnston, Thomson, and Vreeland, Jr. (John Wiley and Sons, Inc., New York, 1956), p. 425.

<sup>18</sup> A. Granato and R. Truell, *J. Appl. Phys.* **27**, 1219 (1956).

<sup>19</sup> I. Holwech, *J. Appl. Phys.* (to be published).