

$p'$ - $\gamma$  Angular Correlation in the Inelastic Scattering of 16.6-Mev Protons by  $Mg^{24}\dagger$ 

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(Received August 10, 1959)

The angular correlation between the inelastically scattered protons leaving the first excited state of  $Mg^{24}$  (1.37 Mev,  $2+$ ) and its de-excitation gamma rays produced by the bombardment of 16.6-Mev protons on a natural magnesium target has been investigated. For  $\theta_p' = 30^\circ$ ,  $42.5^\circ$ , and  $70^\circ$  the result shows that the correlation functions are represented fairly well by the simple Born approximation calculation,  $\sin^2 2(\theta_\gamma - \theta_0)$ . However, at larger angles,  $\theta_p' = 95^\circ$ ,  $120^\circ$ , and  $150^\circ$ , changes in the shapes of correlation curves are observed, introducing a contribution from a  $\sin^2(\theta_\gamma - \theta_0')$  term. This fact suggests the existence of exchange effect as well as the distortion effect in the present nuclear reactions.

## INTRODUCTION

OVER the last few years a considerable amount of experimental data has been accumulated concerning the direct process in the nuclear reactions.<sup>1</sup> The analysis indicates that the process occurs preferably at a high-energy region of the incident particles, where the existence of the compound nuclear state as an intermediary between initial and final state is less dominant. In 1956, it was first pointed out by Satchler<sup>2</sup> that, independently of the compound process, there should be a strong directional correlation between the recoil nucleus and the de-excitation gamma ray if the reaction took place through a direct mechanism. This is in contrast to the angular correlation through a compound process in which the correlation function is related to the spins and particles of the levels of the compound nucleus being formed. Consequently, by observing such a strong correlation as Satchler predicted, one is able to show another evidence of the direct process. In fact, the first ( $p, p'\gamma$ ) experiment of the angular correlation performed by Sherr and Hornyak on carbon with 16.6-Mev protons<sup>3</sup> clearly showed such a strong correlation with only little deviation<sup>4</sup> from the curve predicted by Satchler. The pioneering work on the angular correlation of  $Mg^{24}$  was done by Gove *et al.*<sup>5</sup> as early as 1952 at the energy of 7.3 Mev. However, since they fixed the gamma counter and

moved the proton counter in taking the data, we are unable to tell whether such a simple function as Satchler predicted would also hold at that energy. In the present experiment,  $Mg^{24}$  was chosen again as a subject of the study on the mechanism of the direct process because of its  $0^+$  ground state and  $2^+$  first excited state (as is the case with carbon), its large inelastic cross section, the wide energy spacing of the first excited state and ground state, the absence of appreciable yield of gamma rays other than the 1.37-Mev de-excitation gamma rays, the preparatory knowledge of the angular distribution of the inelastically scattered protons from the 1.37-Mev level in this energy region,<sup>6</sup> and the ready availability of the target as a natural magnesium foil. As long as we insist on the standpoint of the direct interaction theory, it was also expected that this experiment would shed light on the distorted wave effect at the higher atomic number and on the application of the collective model to this type of reaction. The result actually obtained showed some deviations from the Satchler curve in the backward proton angles. Since this work was begun, the same experiments at different energies have been done elsewhere.<sup>7</sup>

## APPARATUS

The 16.6-Mev proton beam produced by the Princeton FM cyclotron is collimated by a  $\frac{3}{16}$ -in. diameter collimator supplemented by three  $\frac{1}{4}$ -in. antiscattering collimators and is scattered by a magnesium foil held by a target holder 13 in. behind the last collimator. On the south side of the beam tube, centered to the target, a  $\frac{3}{4}$ -in.  $\times$  4-in. window has been made covered with a 2-mil aluminum diaphragm. The scattered protons emerge through this window to be detected by a proton counter. The unscattered proton beam passes through the beam tube and is collected by a Faraday cup at the end. The scintillation detector used to measure the scattered protons is mounted in a light tight aluminum housing held by two arms which project horizontally from the scattering center and are rotatable

<sup>†</sup> This work was supported by the U. S. Atomic Energy Commission and The Higgins Scientific Trust Fund. The paper is based on a thesis submitted in partial fulfillment of the requirements for the Ph.D. degree in Physics at Princeton University. Also refer to Bull. Am. Phys. Soc. 3, 200 (1958).

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<sup>1</sup> P. C. Gugelot, Phys. Rev. 81, 51 (1951); 93, 425 (1954); W. E. Burcham *et al.*, Phys. Rev. 92, 1266 (1953); G. E. Fisher, Phys. Rev. 96, 704 (1954); G. Schrank *et al.*, Phys. Rev. 96, 1156 (1954); I. E. Dayton and G. Schrank, Phys. Rev. 101, 1358 (1956); C. P. Browne *et al.*, Phys. Rev. 104, 1099 (1956); H. E. Conzett, Phys. Rev. 105, 1324 (1957); R. W. Peelle, Phys. Rev. 105, 1311 (1957); R. Sherr, *Proceedings of the University of Pittsburgh Conference on Nuclear Structure, 1957*, edited by S. Meshkov (University of Pittsburgh and Office of Ordnance Research, U. S. Army, 1957).

<sup>2</sup> G. R. Satchler, Proc. Phys. Soc. (London) A68, 1037 (1956).

<sup>3</sup> R. Sherr and W. F. Hornyak, Bull. Am. Phys. Soc. 1, 197 (1956).

<sup>4</sup> C. A. Levinson and M. K. Banerjee, Ann. Phys. 2, 471 (1957).

<sup>5</sup> H. F. Gove and A. Hedgran, Phys. Rev. 86, 574 (1952).

<sup>6</sup> P. C. Gugelot and P. R. Philips, Phys. Rev. 101, 1614 (1956).

<sup>7</sup> F. D. Seward, Phys. Rev. 114, 514 (1959); T. H. Braid (private communication); H. A. Lackner *et al.*, Phys. Rev. 114, 560 (1959).



shown the proton and gamma spectra under such conditions as required for the coincidence measurements. For both counters higher dc voltage is applied and a larger aperture ( $\frac{1}{2}$  in.) is used for the proton counter in order to get a proper counting rate. The results are less linearity and resolution. To separate the elastic and inelastic peaks, which would be otherwise nearly overlapping due to the high voltage, a routine way to use absorbers in front of the counters was employed. The Fig. 3 is the spectrum taken under such conditions. It is of interest to note that this procedure, in spite of reducing the incident proton energy at the crystal, gives more output voltage than reducing the high voltage in order to get the same amount of separation. The nonlinear characteristic of output voltage *vs* energy around the saturation level is responsible for this effect.

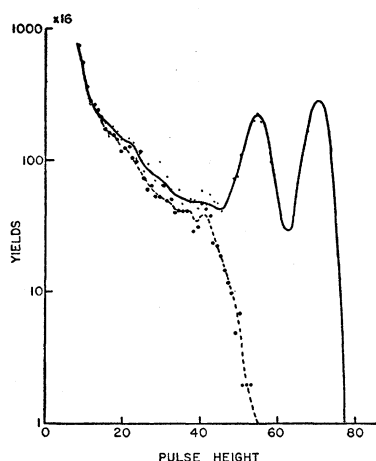


FIG. 3. A proton spectrum for the coincidence measurement. The applied high voltage to DuMont 6291 is 1750 volts. The aperture of the counter is  $\frac{1}{2}$  in. and 24-mil thick aluminum foil and is placed in front of it to separate the elastic and inelastic peaks. The dotted line represents the yield where the aluminum window is covered by a  $\frac{1}{8}$ -in. thick brass plate. It supposedly consists mainly of the gamma rays from the target.

A considerable number of lead bricks are used to shield the gamma counter from the gamma background originating in the Faraday cup, the collimator, and the beam tube. However, this introduces the possibility that the gamma rays from the target could be scattered from the shields into the crystal, increasing the effective solid angle of the gamma counter. In order to examine this effect, setting the energy geometry the same as in the experimental case, a  $Na^{22}$  source of a couple of microcuries was mounted at the exact scattering center on the target and we looked for the variation of 1.28-Mev spectrum which is sufficiently close to 1.37 Mev in energy. The answer was negative for several gamma angles measured. The effect only appeared in very low-energy gamma rays (0.12 Mev) showing up as a small peak which, of course, is irrelevant in the present experiment.

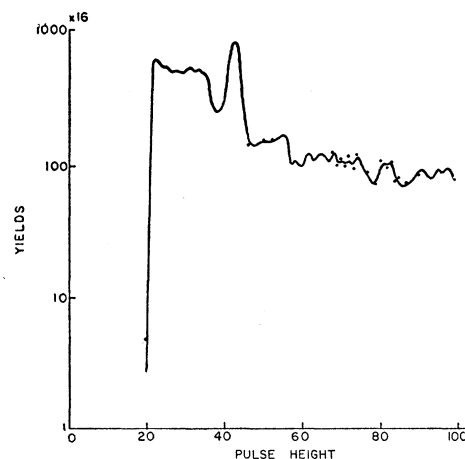


FIG. 4. A gamma-ray spectrum for the coincidence measurement. The applied high voltage to RCA 6655 is 1200 volts.

Since we cut off the strong low-energy gamma-ray background before putting the gamma pulses into the slow coincidence circuit (cutoff level  $\sim 0.61$  Mev, Fig. 4), it is important to maintain the gain of photomultiplier constant regardless of its position and of time; otherwise, the intensity of 1.37-Mev gamma rays after the discriminator would illegitimately vary and a further correction would become necessary. The same is true for the gain of the following amplifier. The photomultiplier tube 6655 reduces its gain by 20–30% when 5 gauss of magnetic field is applied to the direction of the axis of its dynode cage.<sup>9</sup> Therefore, although the counter is shielded by the mu-metal, the residual existence of this large effect deserves to be paid attention to. Such variations of the gain were checked by connecting the output of a linear amplifier to a multi-channel analyzer in coincidence with the discriminator output and taking the gamma spectra during the runs. It was found that there were no appreciable changes of gain with angle. As for the over-all gain instability during the long time interval in one particular run, this method also indicates such fluctuations by the degradation of the resolution of the spectrum. In that case, the data for that run was discarded.

Not only the over-all gain, but also the bias and the counting efficiency of the gamma counter are not necessarily constant over the period of months. Consequently, these instabilities introduce fluctuations in apparent gamma-ray intensity presented to the slow coincidence circuit, resulting in the changes of final yield of coincidence events for a definite number of protons. Therefore, in order to secure a consistent result, a standard  $Na^{22}$  source (a few microcuries) was used to readjust the bias in such a way that one could get the same counts of 1.28-Mev gamma rays in a time interval of a certain length (say 200 sec). This procedure

<sup>9</sup> RCA 6655-5-55, Tube Division, RCA, Harrison, New Jersey.

was repeated not only at the beginning of a set of runs but between the runs as needed.

The scattering tube is aligned relative to the beam by means of two fluorescent screens. One of the screens is made of a Lucite disk with the zinc sulfide spattered on the surface, which replaces the Faraday cup during the alignment. The other is installed 2 in. before the target which can be turned up manually from outside, keeping the beam from hitting it during the actual measurement. Both screens have crosses at their centers so that one can align the beam tube in such a way that the beam goes through the centers. The alignment was examined every day.

#### ANALYSIS OF THE DATA

In order to obtain the genuine coincidences, one must estimate the accidental coincidences in the coincidence yields. The protons spectrum in coincidence with gamma rays displays a large peak corresponding to 1.37-Mev proton group accompanying a small peak corresponding to the elastic proton group. Since the elastically scattered proton does not produce gamma rays, the latter is interpreted as a peak formed by accidental events only. Therefore,

Number of accidentals in inelastic group

$$= \text{Number of accidentals in elastic group} \times \frac{\sigma_{\text{inelastic}}}{\sigma_{\text{elastic}}}, \quad (1)$$

when  $\sigma$ 's are the cross sections for both cases.

The errors hereafter discussed belong to systematic errors.

**Counting loss.**—The counting loss due to the dead time of the fast coincidence circuit must be estimated. For each run the correction for the counting loss has been made using Eq. (5) in Appendix I. The corrections are of the order of 10%.

**Geometry.**—Because of the finite dimensions of the counter, the measured angular distribution or the angular correction function will differ from those ideal cases in which the infinitesimal counters are used (Appendix 2). Assuming the ideal angular correlation function to be  $\sin^2 2\theta$ , where  $\theta$  is the angle of the gamma counter measured from the direction of the recoil nucleus, the measured correlation curve differs from the ideal one by 0.13,  $-0.07$ , and  $0.06$  at  $\theta=0, 45$  and  $95$  degrees in case the gamma counter is 3 in. away from the target. However, as will be shown, some of the correlation function differs from  $\sin^2 2\theta$  form, making the exact evaluation of the effect difficult. One expects smearing effects of about 10% at the optimum points of the correlation functions in the present case.

**Absorption of gamma rays.**—There is a gamma-ray absorption due to the thickness of the beam tube. From the thickness of the brass wall and the absorption

coefficient, the following formula has been obtained for 1.3-Mev gamma ray:

$$A = 1 - 0.014 \operatorname{cosec} \theta_\gamma, \quad (2)$$

where  $A$  is the absorption of the gamma ray, and  $\theta_\gamma$  is the direction of the gamma ray with respect to the incident beam direction. The validity of Eq. (2) was checked with a  $\text{Na}^{22}$  standard source. The correction has been made for each run using Eq. (2). However, the corrections have been made only for the positive gamma angles.

**Contribution from  $\text{Mg}^{25}$  and  $\text{Mg}^{26}$ .**—The contribution from natural isotopes,  $\text{Mg}^{25}$  (10.1%) and  $\text{Mg}^{26}$  (11.1%), to the coincidence events were examined by means of two-dimensional analyzer.<sup>10</sup> In particular, care was taken to detect any contribution from 0.98 Mev ( $3/2^+$ ) level of  $\text{Mg}^{25}$  and 1.83 Mev ( $2^+$ ) of  $\text{Mg}^{26}$ . The conclusion from the two-dimensional analysis is that there are no appreciable contributions from those levels.

#### PRESENTATION OF RESULTS

In most of the measurements the gain and the base of the twenty channel pulse-height analyzer was set in such a way that only the inelastic protons fall in the twenty channels. The accidental coincidences associated with the elastic protons are registered as the surplus. The base of an integral discriminator preceding the proton scaler was set exactly the same as that of the twenty-channel pulse-height analyzer. Each run was continued until the proton number registered in the scaler amounted to a definite number. The angle of the gamma counter was chosen randomly from one run to another, in order to average out the possible drift in the long time.

To confirm the angular correlations which had been obtained by placing the gamma counter in the opposite semiplane to the proton counter, the measurements were extended to the negative gamma angles, placing the gamma counter on the same semiplane where the proton counter lay. The results were obtained for 42.5, 120, and 150 degrees of protons. The gamma counter had to be backed up to 4 in. away from the target so that it could get closer to the proton counter. This resulted in a larger statistical error after the correction of the distance. The results satisfactorily support the measured correlation function of the positive gamma angles within the limits of this statistical error.

In Fig. 5, the final results are plotted against the gamma angles for each proton angle. The distributions of gamma rays are also presented on the figures, except for the averaged cases. An attempt to fit the points empirically with the  $a + b \sin^2(\theta - \theta_0) + c \sin^2(\theta - \theta_0')$  for each proton angle was made, the result of which is shown by a curve and also given as a formula in each graph. The values of  $a, b, c, \theta_0$ , and  $\theta_0'$  are also tabulated in Table I together with the results of carbon.<sup>3</sup>

<sup>10</sup> Birk, Braid, and Detenbeck, Rev. Sci. Instr. 29, 203 (1958).

TABLE I. The relation between  $a$ ,  $b$ , and  $c$  and  $\Delta\theta_0$ ,  $\Delta\theta_0'$  observed<sup>a</sup> (see the text).

	$\theta_p$	$a$	$b$	$\Delta\theta_0$	$c$	$\Delta\theta_0'$
$Mg^{24}$	30°	0.17	1	3°		
	42.5°	0.26	1	0°		
	70°	0.15	1	8°		
	95°	0.43	1	5°	0.54	
	120°	0.17	1	-3°	0.55	
	150°	0.37	1	0	0.56	30°
$C^{12}$	30°	0.14±0.15	1	15±1°		
	45°	0.43±0.06	1	7±1°		
	60°	0.51±0.08	1	6.5±1°		
	80°	0.54±0.10	1	6.5±2°		
	90°	0.35±0.05	1	6.5±1°		
	110°	0.46±0.11	1	5±1°		
	150°	0.24±0.08	1	-0.5±1°		

<sup>a</sup>  $a$ ,  $b$ ,  $c$  are normalized to  $b=1$ ;  $\Delta\theta_0 \equiv \theta_0 - \theta_R$ ; and  $\Delta\theta_0' \equiv \theta_0' - \theta_R$ , where  $\theta_R$  is the angle of the recoil nucleus.

### DISCUSSION OF RESULTS

Suppose a nucleus with spin 0 has received a momentum transfer  $k-k'$  from an impinging spinless particle whose initial momentum is  $k$ , final momentum is  $k'$ , and excited to a state having spin  $j$ . From the conservation of magnetic quantum number, the magnetic quantum number of the new state remains zero. Therefore, the subsequent gamma radiation from the recoil nucleus is characterized by the transition from a state whose spin is  $j$  and magnetic quantum number is zero to the state having spin zero. The recoil direction  $k-k'$  is an axis of symmetry for the process. This implies that when the excited state is in  $j=2$ , the gamma distribution around  $k-k'$  gives the (2,0)-pole electromagnetic radiation,  $(\frac{5}{2})(6\cos^2\theta - 6\cos^2\theta_0)$ ,<sup>11</sup> which is equivalent to the well-known Satchler curve.

However, if the incoming and outgoing waves are not approximated by the plane waves, or the interaction Hamiltonian is more than the surface interaction, say having a spin-flip operator, the generalization of the above idea becomes necessary. In such a case the value of  $M$  of  $\alpha_{L,M}$  in Biedenharn and Rose's general theory<sup>12</sup> can range between -2 and 2. This gives a general correlation function  $W$ , in the case of  $0^+(p')2^+(\gamma)0^+$  transition, as

$$W = \sum_{\nu\mu\mu'} (-)^{\mu} T_{\mu\mu'} C(22\nu; \mu, -\mu') \times C(22\nu; 1-1)[\nu]^{-\frac{1}{2}} Y_{\nu, \mu'-\mu}^*(\theta_\gamma), \quad (3)$$

where  $\theta_\gamma$  is the angle between the recoil direction and the gamma ray. (Here  $T_{\mu\mu'} \equiv \alpha_{2, -\mu} \alpha_{2, -\mu'}^*$ .) This be-

TABLE II. The general expression for the angular correlation.

In this expression, an identity  $T_{\lambda\mu} \equiv \alpha_{-\lambda, \mu} \alpha_{-\mu}^* = (-)^{\lambda+\mu} \alpha_{\lambda, -\mu}^*$  is used.  $T_{\lambda\mu}$  signifies  $T_{\lambda\mu} + T_{\mu\lambda}$ .  
 $W = A + B \cos 4\theta + C \sin 4\theta + D \cos 2\theta + E \sin 2\theta$ ,  
 $A = \frac{3}{16} T_{00} + \frac{1}{2} T_{11} + \frac{5}{16} T_{22} - \frac{3}{32} T_{2-2}' - \frac{1}{8} (\frac{3}{2})^{\frac{1}{2}} T_{02}'$ ,  
 $B = -\frac{3}{16} T_{00} + \frac{1}{8} (2T_{11} - T_{1-1}') - \frac{1}{32} (2T_{22} + T_{2-2}') + \frac{1}{8} (\frac{3}{2})^{\frac{1}{2}} T_{02}'$ ,  
 $C = -\frac{1}{4} (\frac{3}{2})^{\frac{1}{2}} T_{01}' + \frac{1}{8} (T_{12}' + T_{1-2}')$ ,  
 $D = \frac{1}{8} [2(T_{11} - T_{22}) + T_{1-1}' + T_{2-2}']$ ,  
 $E = \frac{1}{4} (T_{12}' + T_{1-2}')$ .

If  $\alpha_{-\mu} = (-)^{\mu} \alpha_{\mu}$  holds, as in many cases, those equations are more simplified.

$$A = \frac{3}{16} T_{00} + \frac{1}{2} T_{11} + \frac{5}{16} T_{22} - \frac{1}{8} (\frac{3}{2})^{\frac{1}{2}} T_{02}',$$

$$B = -\frac{3}{16} T_{00} + \frac{1}{2} T_{11} - \frac{1}{8} T_{22} + \frac{1}{8} (\frac{3}{2})^{\frac{1}{2}} T_{02}',$$

$$C = -\frac{1}{4} (\frac{3}{2})^{\frac{1}{2}} T_{01}' + \frac{1}{4} T_{12}', \quad D = E = 0.$$

If only diagonal elements exist,

$$A = \frac{3}{16} T_{00} + \frac{1}{2} T_{11} + \frac{5}{16} T_{22}, \quad B = -\frac{3}{16} T_{00} + \frac{1}{4} T_{11} - \frac{1}{16} T_{22},$$

$$D = \frac{1}{4} T_{11} - \frac{1}{4} T_{22}, \quad C = E = 0.$$

If only diagonal elements exist and  $\alpha_{-\mu} = (-)^{\mu} \alpha_{\mu}$  holds, only  $T_{00}$  is not from zero, therefore

$$A = -B = \frac{3}{16} T_{00}, \quad C = D = E = 0.$$

comes, in turn,

$$W = A + B \cos 4\theta_\gamma + C \sin 4\theta_\gamma + D \cos 2\theta_\gamma + E \sin 2\theta_\gamma$$

$$= a + b \sin^2 2(\theta_\gamma - \Delta\theta_0) + c \sin^2(\theta_\gamma - \Delta\theta_0'), \quad (4)$$

which was obtained by Sawicki.<sup>13</sup> (It was proved by Banerjee and Levinson<sup>4</sup> that if one takes the shell model, the spin-flip interaction, and a plane wave approximation,  $\Delta\theta_0$  equates to  $\Delta\theta_0'$ .) The coefficients  $A$ ,  $B$ ,  $C$ , and  $D$  are given in terms of  $T_{\mu\mu'}$  in Table II. In order to have  $\sin^2(\theta_\gamma - \Delta\theta_0')$  term,  $\alpha_{L, -\mu}$  must differ from  $(-)^{\mu} \alpha_{L, \mu}$ , and it is necessary to have the off-diagonal elements of the  $T$ -matrices.

The calculation of the angular correlation between the inelastic protons and the subsequent gamma rays on the basis of the compound process in this energy region is very complicated and there is no satisfactory theory at present. The reason for the difficulty is that the level spacing of the compound nucleus around 20 Mev is almost continuous, thus giving numerous intermediate states. The general mathematical expression of the correlation function was given by Kraus *et al.*<sup>14</sup> It is still unclear to us how much the compound processes are contributing to the nuclear reaction such as  $Mg^{24}(p, p')Mg^{24}$  with the incident energy around 16.6 Mev. It is convincing that the direct interaction dominates in the forward angles, but the naive theory of direct interaction does not explain the backward angle where the differential cross sections display a tail. However, some preliminary measurements of the backward angle cross section of the inelastic protons over the energy range of 16 to 19 Mev showed a less fluctuating spectrum than that of the elastic protons,<sup>15</sup>

<sup>11</sup> M. Yamada and M. Morita, Progr. Theoret. Phys. (Kyoto) 8, 431 (1952); H. A. Tolhoek and J. A. M. Cox, Physica 19, 101 (1953); see also J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952), p. 594.

<sup>12</sup> L. C. Biedenharn and M. R. Rose, Revs. Modern Phys. 25, 729 (1953).

<sup>13</sup> J. Sawicki, Nuclear Phys. 7, 503 (1958).

<sup>14</sup> A. A. Kraus *et al.*, Phys. Rev. 104, 1667 (1956).

<sup>15</sup> R. Sherr (private communication).

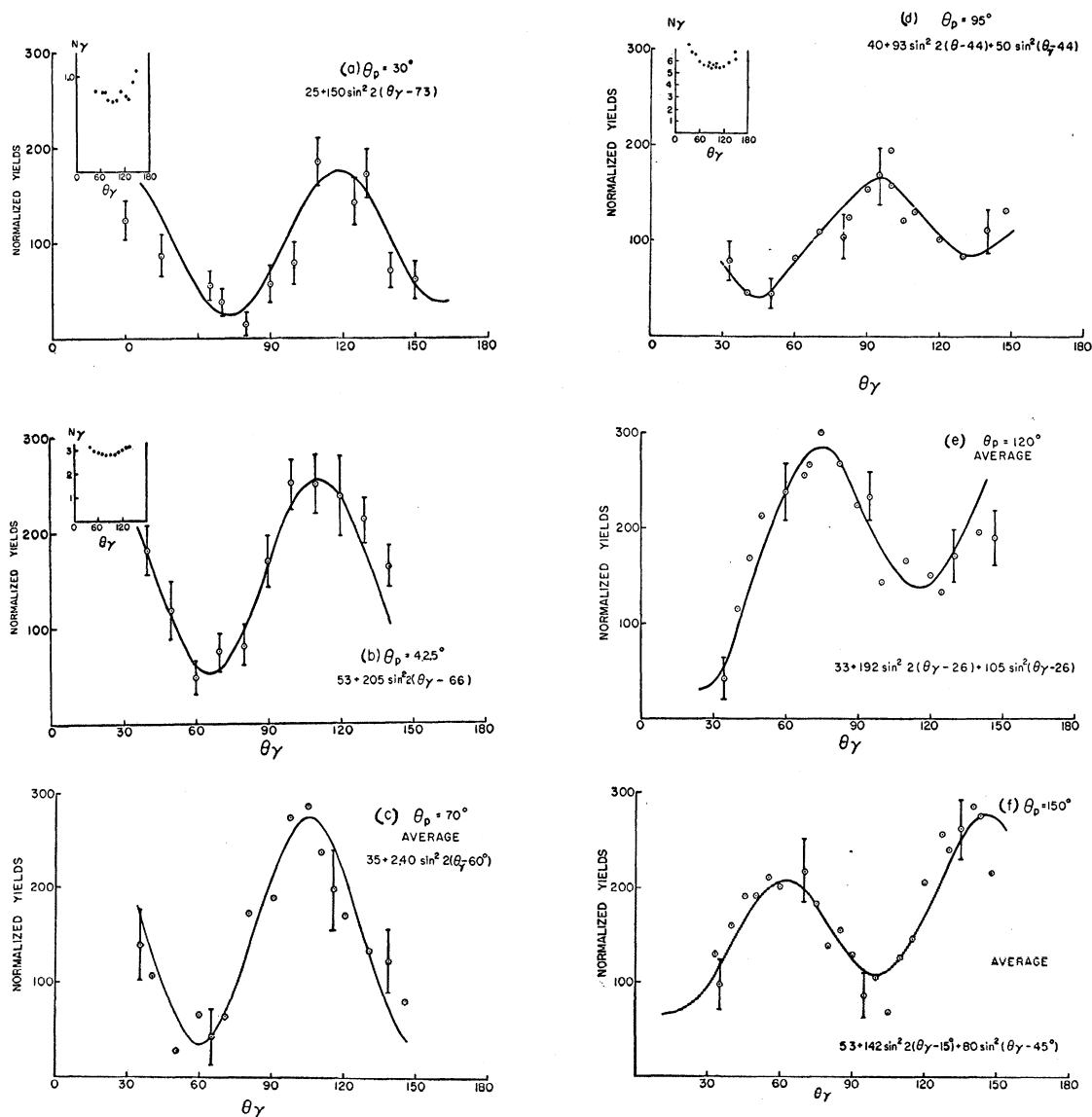
suggesting that the so-called broad resonances are less dominant than the case of elastic scattering.

Therefore it would be stated that the reaction taking place in the present experiment is dominated by the direct process and the deviations from the Satchler curve observed are not due to the compound effect, but to the direct effect as predicted,<sup>13</sup> even though the deviations show up only in the backward proton angles. In view of the close spacings of the compound levels, which are likely to cause the angular correlation function nearly isotropic, the strong correlations observed at all proton angles are supporting this statement.

### SUMMARY

Thus, assuming that the presence of  $\sin^2(\theta_\gamma - \Delta\theta_0')$  term is due to the direct interaction only and not to

other effects (say, compound process) the implication of the presence of this term is summarized as follows: Firstly, if one assumes plane wave, zero range force, interparticle as well as particle surface, and no exchange in any coordinate, namely, space, spin, and isotopic spin coordinates, the result is the Satchler curve,  $\sin^2 2\theta$ . In this case, the minimum of the correlation coincides with the recoil nucleus direction. This is simply examined by putting all the  $T$ -components zero, except for  $T_{00}$  in Eq. (3). Secondly, if one assumes distorted wave and/or finite range interparticle force, with, again, no exchange effect, the function takes a form  $a + b \sin^2(\theta_\gamma - \Delta\theta_0)$ , as was the case in carbon.  $D = E = 0$  in this case. Thirdly, in case the exchange effect is taken into account in addition to the distorted wave and/or the finite range, the function will be expressed by the general



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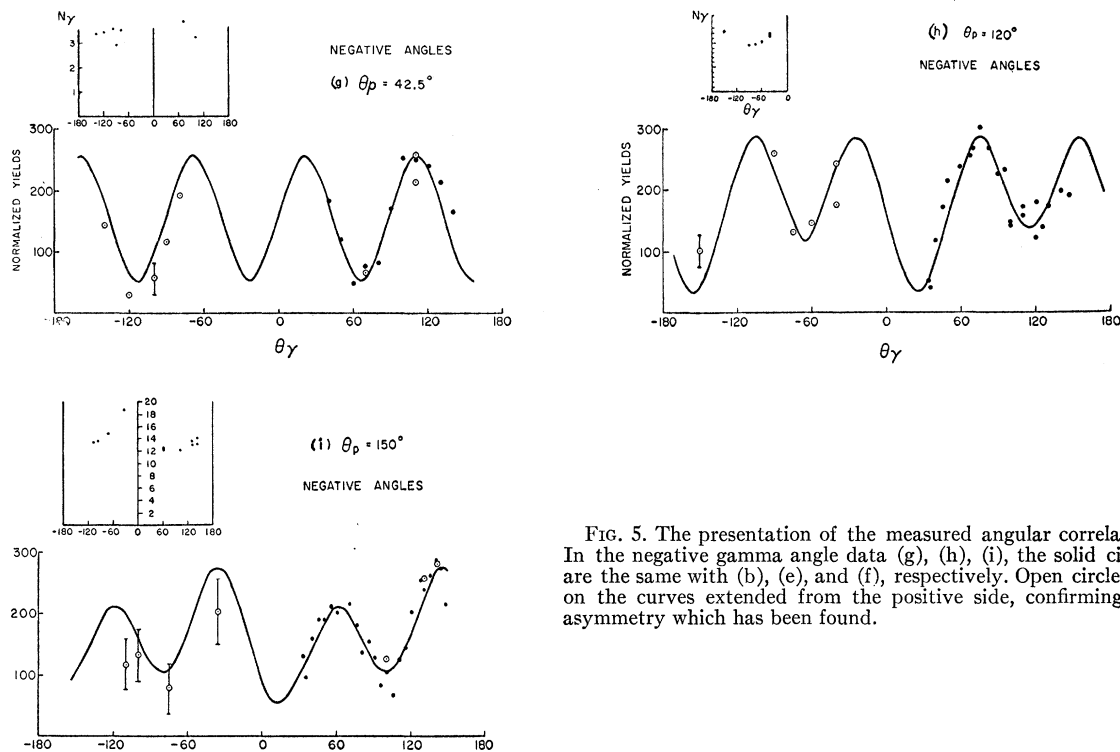


FIG. 5. The presentation of the measured angular correlation. In the negative gamma angle data (g), (h), (i), the solid circles are the same with (b), (e), and (f), respectively. Open circles are on the curves extended from the positive side, confirming the asymmetry which has been found.

form of Eq. (4), which is independent of the models. In the fourth place, however, if the exchange effect is present without the distorted wave and the finite range, namely, with the plane wave and zero range force (this means the off diagonal elements of  $T$ -matrices are zero), one can see the consequence of different models; namely, in collective excitation it gives the Satchler form, whereas individual particle excitation gives the general form, except  $\Delta\theta_0 = \Delta\theta_0'$ .

#### APPENDIX I

The present coincidence circuit has two independent circuits in series, the fast and slow, each of which has the individual dead time,  $\tau_f$  and  $\tau_s$ . However, in case  $\tau_f > \tau_s$ , one may consider the effect of  $\tau_f$  only. Suppose the total number of gamma rays  $N_\gamma$  and of protons  $N_p$  registered in a certain length of time  $T$ , the total counting loss  $\Lambda$  is given by

$$\Lambda = \frac{P}{f_M T} (N_\gamma \eta_\gamma + N_p \eta_p), \quad (5)$$

where  $P$  is

$$P = \frac{\int_0^{T_c - \tau_f} \int_{t_1}^{t_1 + \tau_f} dt dt_1 + \int_{T_c - \tau_f}^{T_c} \int_{t_1}^{T_c} dt dt_1}{\int_0^{T_c} \int_0^{T_c} dt_1 dt}. \quad (6)$$

Here  $f_M$  is the number of the modulation per unit time in FM cyclotron,  $T_c$  is the duration of the single pulse of the proton beam. Since the pulses are usually registered after being discriminated at a certain level, the number of gamma (proton) pulses which actually trigger the EFP60 is generally larger than  $N_\gamma(N_p)$ , let us call this  $N_\gamma \eta_\gamma(N_p \eta_p)$ , where  $\eta_\gamma(\eta_p)$  is a factor which is to be determined experimentally. The validity of Eq. (5) was checked experimentally with  $C^{12}$  using independently determined  $\tau_f$ ,  $\eta$ , and  $T_c$ . The consistency is very satisfactory and, therefore, Eq. (5) has been accepted for the correction.

#### APPENDIX II

The effect of the finite geometry on the measured angular functions has been investigated by Rose,<sup>16</sup> Feingold *et al.*<sup>17</sup> However, in the present experiment it is the recoil nucleus direction around which the gamma distribution is symmetric and not the proton direction which is actually detected in the solid angle made by the proton counter aperture and the scattering center. In general, if the proton counter aperture is a circle, the possible direction of the associated recoil nucleus direction is a distorted cone. We approximate, therefore, this distorted cone with such a solid as given by  $\beta = \gamma(1 + \eta \sin \varphi)$ , where  $\beta$ ,  $\gamma$ ,  $\eta$ , and  $\varphi$  are such angles as illustrated in Fig. 6. In this case, the two factors

<sup>16</sup> M. E. Rose, Phys. Rev. **91**, 610 (1953).

<sup>17</sup> A. M. Feingold and S. Frankel, Phys. Rev. **97**, 1025 (1955).

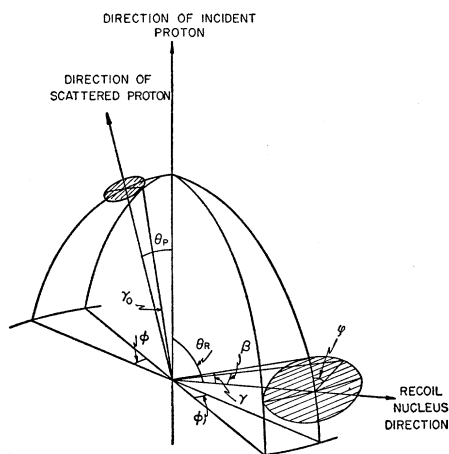


FIG. 6. Mapping of the recoil nucleus direction related to the direction of protons.

appearing in Eq. (5) in reference (16),  $J_l^p$  and  $J_l^\gamma$ , are given as follows.

$J_l^\gamma$ 's are to be numerically calculated according to the recipe in reference (16).  $J_l^p$ 's are, under the assumption of the previous paragraph, written into the form of,

$$J_0^p = 1 - \cos\gamma J_0(\gamma\eta),$$

$$J_2^p = \frac{1}{8} [\cos\gamma J_0(\gamma\eta) - \cos 3\gamma J_0(3\gamma\eta)],$$

$$J_4^p = (1/128) [2 \cos\gamma J_0(\gamma\eta) + 5 \cos 3\gamma J_0(3\gamma\eta) - 7 \cos 5\gamma J_0(5\gamma\eta)],$$

where  $J_0$  is the zeroth order of Bessel function. We neglected the higher order contributions of  $m$  of the associated Legendre polynomials in the calculations.

TABLE III. The attenuation coefficients for gamma geometry  $J_l^\gamma/J_0^\gamma$ .

	Energy of gamma ray (Mev)	$J_2^\gamma/J_0^\gamma$	$J_4^\gamma/J_0^\gamma$
$h=3$ in. <sup>a</sup>	4.43	0.966	0.878
	1.37	0.956	0.869
	0.85	0.957	0.868

<sup>a</sup>  $h$  is the distance from target to the front surface of the crystal. The dimension of the crystal is  $1\frac{1}{2}$  in. in diameter and 2 in. in height.

TABLE IV. The attenuation coefficients for proton geometry  $J_l^p/J_0^p$ .

$J_2^p/J_0^p (l=2)$		$J_4^p/J_0^p (l=4)$	
$\eta=0.000$	$\eta=0.800$	$\eta=0.000$	$\eta=0.800$
0.9914	0.9888	0.9716	0.9412

Contributions of such terms are one thousandth or less of the zeroth-order term. Numerical results of  $J^p$ 's and  $J^\gamma$ 's for the present experiment are presented in Tables III and IV.

#### ACKNOWLEDGMENTS

The author wishes to express his deep appreciation to Professor R. Sherr. Without his active guidance and cordial encouragement, the present work would not have been completed. Thanks are also due to Mr. R. W. Detenbeck, and Dr. H. A. Hill for frequent continual discussions which were most valuable. Several talks with Dr. J. Sawicki and Dr. J. Blair about the theory were very enlightening. He is also obliged to Professor G. Schrank, and Professor W. F. Hornyak and all the other members of the cyclotron group.