Positive Pion Production by Polarized Bremsstrahlung*

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By selecting bremsstrahlung produced in a 0.003-in. aluminum radiator at a small angle from the original electron direction, a beam of polarized bremsstrahlung has been obtained from the Stanford linear accelerator. The variation of the polarization and intensity with angle has been studied and compared with theoretical predictions. The polarized beam has been used to study π^+-meson production at 90° c.m. angle and photon energies of 242, 296, 337, and 376 Mev. The ratio of meson production along and at right angles to the electric field vector has been measured and compared with the values predicted by the relativistic dispersion relation.

INTRODUCTION

ADDITIONAL information can be obtained from meson photoproduction if the polarization of the photons is known. Terms in the cross section which have the same polar but different azimuthal variations can be separated.

In the experiment to be described much of the effort was directed to solving the problems of producing the polarized photons and determining that the measured asymmetry of meson production as a function of azimuthal angle could be produced only by the polarization of the photons and not by some insufficiently understood background.

After evidence had been obtained that a partly polarized beam of photons existed, the qualitative nature of the polarization and the bremsstrahlung angular distribution were checked. These measurements can be interpreted either as a check on the bremsstrahlung calculation or a check on our experimental ability to satisfy the conditions for producing polarized bremsstrahlung. We choose to consider it the former.

Having checked that the bremsstrahlung is at least qualitatively as calculated, we used the quantitative calculations to estimate the polarization of the beam used to study pion production. We concede that the information obtained concerning meson production would be more reliable if we were able to measure the amount of polarization experimentally. We were unable to do this and argue that, however poor our calculated value may be, it is probably more reliable than the meson theory we are checking.

PRODUCTION OF POLARIZED BREMSSTRAHLUNG

The possibility of producing polarized bremsstrahlung was pointed out when the bremsstrahlung process was first investigated. Sommerfeld[1] made estimates of considerable validity in the low-energy region and several investigators checked his and other earlier predictions.

In the low-energy region, electrons easily suffer a longitudinal deceleration and the polarization is primarily in the plane containing the initial electron direction and the direction of emission. As the electron becomes relativistic, transverse acceleration dominates. Although again the polarization must be in the plane containing the acceleration and the direction of emission, it no longer contains the initial electron direction but is at right angles to the plane containing it and the direction of emission. This polarization we shall call transverse while we call the former radial.

Classically it can be shown that this polarization reaches a maximum at an angle mc^2/E_0 (E_0 is the initial electron energy) while it must be zero in the forward direction for symmetry reasons. Calculations by May and Wick,[2] and May,[3] have derived the polarization to be expected in high-energy bremsstrahlung. The latter calculation is the more accurate, starting from the basic Bethe-Heitler[4] formula (not integrated over outgoing electron angles) and using a relativistic small-angles approximation. Gluckstern, Hull, and Breit[5] have calculated the dependence of the cross section on polarization to the same approximation as the usual Bethe-Heitler formula and have evaluated their results for the unscreened low-energy case.

The following cross sections for radial and transverse photons are obtained by May:

$$dN_{\parallel}=2\frac{\phi}{\pi}\frac{d\epsilon}{\epsilon}\frac{d\psi dx_0}{(1+x_0)^2}\left\{\left[1-\epsilon+\frac{\epsilon^2}{2}-4(1-\epsilon)\frac{x_0}{(1+x_0)^2}\right]\times\ln\frac{1+x_0}{f}-\frac{\epsilon^2}{4}+(1-\epsilon)\left[1-2\left(\frac{1-x_0}{1+x_0}\right)^2\right]\right\}, \quad (1)$$

$$dN_{\perp}=2\frac{\phi}{\pi}\frac{d\epsilon}{\epsilon}\frac{d\psi dx_0}{(1+x_0)^2}\left\{\left[1-\epsilon+\frac{\epsilon^2}{2}\right]\times\ln\frac{1+x_0}{f}-(1-\epsilon)-\frac{\epsilon^2}{4}\right\}, \quad (2)$$

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[1] A. Sommerfeld, Ann. Physik 11, 257 (1931).

[2] M. May and G. C. Wick, Phys. Rev. 81, 628 (1951).

[3] M. May, Phys. Rev. 84, 265 (1951).

[4] H. Bethe and W. Heitler, Proc. Roy. Soc. (London) A146, 83 (1934).

[5] Gluckstern, Hull, and Breit, Phys. Rev. 90, 1026 (1953); R. L. Gluckstern, and M. H. Hull, Phys. Rev. 90, 1030 (1953).

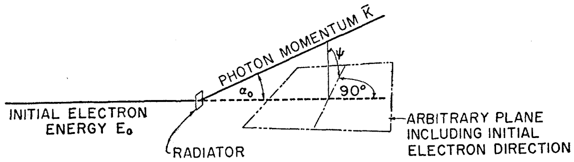

FIG. 1. Bremsstrahlung angular relations.

where $dN_{\parallel}$ is the cross section for radial and $dN_{\perp}$ is the cross section for transverse photons; $\phi = Z^2e^4/137$; $\epsilon = k/E_0$; k is the photon momentum; E_0 is the initial electron energy; ψ is the angle between the plane of the initial electron and photon and some fixed plane; $x_0 = E_0^2 \sin^2\alpha_0$; α_0 is the angle between initial electron and the emitted photon; and $f = Z^{\frac{1}{3}}/108$. (See Fig. 1.) All energies and momenta are in units of the rest energy of the electron with the velocity of light c equal to unity.

May's predictions of the polarization $(dN_{\perp} - dN_{\parallel})/(dN_{\perp} + dN_{\parallel})$ are shown in Fig. 2. It can be seen that the polarization reaches a maximum at an angle mc^2/E_0 and is larger the lower the photon energy. The largest value of 56% is obtained with zero-energy photons when classically one would expect the transverse accelerations to dominate completely.

The major problems of producing polarized bremsstrahlung are caused by the effects of multiple scattering. Since the maximum polarization is at an angle mc^2/E_0, roughly 0.05° at our energy, the foils in which the bremsstrahlung is produced must be so thin that the multiple scattering angle is not appreciably larger than this. Light elements are slightly more efficient than heavy in producing the polarization.

It can be shown that the effects of finite beam size and angular divergence are almost identical to those of multiple scattering at these small angles. In Fig. 3 we trace the paths of two electrons through a foil. Electron 1 is incident along the center line of the beam and has no angular divergence or displacement. It is multiply scattered in the foil, and in the absence of a magnetic clearing field would continue to the point P. Electron 2 has some angular divergence and displacement and is in addition multiply scattered so that it also arrives at the point P. If the electrons radiate just as they leave the foil, then the angles α_1 and α_2 between the directions of electrons 1 and 2 after leaving the foil, and point R in the region where the polarized bremsstrahlung is being observed, determine the polarization and intensity of the radiation from electrons 1 and 2 at point R. If the angles involved are small, then $\alpha_1 = \alpha_2$, and all electrons which arrive at point P produce the same polarization and intensity at R. The angle is primarily determined by the distance RP and the distance to the radiator, while the point at which the electron passes through the radiator has very little effect.

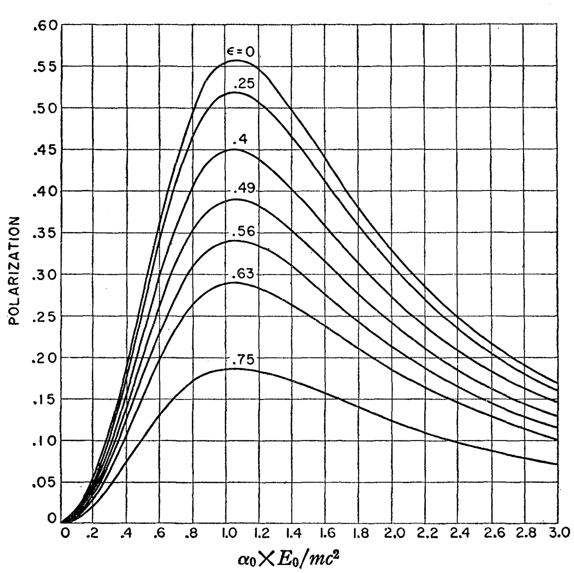

FIG. 2. May's calculated values of polarization (no multiple scattering correction).

The argument above suggests that a method of taking into account the effects of beam size, angular divergence, and multiple scattering is to refrain from deflecting the electrons after they have passed through the radiator and to measure their distribution when they arrive in the region where the polarized bremsstrahlung will be observed. This measured distribution can then be folded into the polarization formula as if it were due to the multiple scattering of electrons from an infinitely narrow beam of zero angular divergence. The major approximation involved is in obtaining the correct value of the multiple scattering of the electrons at the time the radiation is produced. In this measurement we chose to use a radiator half as thick as the radiator used in the production of the bremsstrahlung. This radiator gives the correct value for the rms multiple scattering, but the electrons radiate uniformly as they pass through the foil and those which radiate as they enter the foil have a narrower angular distribution than those radiating just before they leave the foil. The distribution which we fold in is of the correct rms width but is approximately Gaussian in shape, while the real distribution should be the integral of a Gaussian which becomes wider as the electrons move through the foil. In a similar analysis of the production of polarized

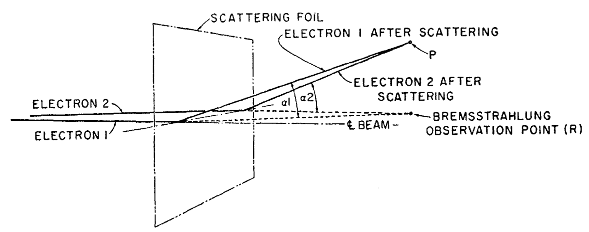

FIG. 3. Angular relations determining the effects of beam size, angular divergence, and multiple scattering.

bremsstrahlung at lower energies, Miller[6] analyzed the multiple scattering in this manner, pointing out that this integral is the logarithmic integral tabulated in Jahnke and Emde.[7] Such an approach would be necessary for bremsstrahlung from a very narrow beam of negligible angular divergence. The beam we used was of appreciable size before it entered the radiating foil, and this finite size reduces the difference in the distribution of the electrons as they enter and leave the foil, making insignificant the difference between the real angular distribution and that which we determine from a foil of one-half the real thickness.

We have measured the effects of multiple scattering, beam size, and angular divergence in the manner described, arriving at an approximately Gaussian angular distribution with a half-width at $1/e$ of 0.546 $\times(E_0/mc^2)$ radians. An aluminum foil of 13.8 mg/cm^2 (0.002 in.) would produce multiple scattering of this amount, and hence with the approximation described

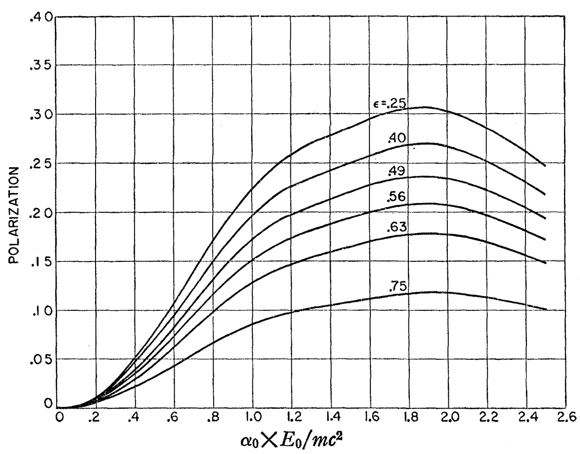

FIG. 4. Calculated values of polarization with multiple scattering of 27.6 mg/cm^2 aluminum radiator included.

above, this is the multiple scattering correction which should be folded into the polarization for a foil of twice this thickness if the electron beam had no size or angular divergence.

The resulting modification of the polarization angle curves is shown in Fig. 4.

EXPERIMENTAL ARRANGEMENT

The Stanford linear accelerator is ideally suited to the production of a polarized bremsstrahlung beam since it produces an intense ($\cong$0.3 μa) external electron beam of small angular divergence and size. The high intensity is necessary since the radiator must be very thin to avoid too much multiple scattering. Since a thin radiator is to be used, any radiation from collimator

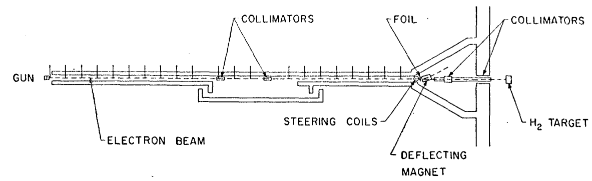

FIG. 5. Linear accelerator and equipment for the production of polarized bremsstrahlung.

edges could easily be larger than that from the radiating foil. To avoid background from the collimator we placed our radiator at the end of the accelerator (see Fig. 5) while the beam was collimated 85 and 125 ft earlier. The radiation from the collimator was farther from the experimental equipment and at a lower energy than that from the radiator. The lower-energy radiation has larger bremsstrahlung and multiple-scattering angles which increase its inverse square attenuation, and in addition the lower-energy bremsstrahlung is less capable of producing pions than the 600-Mev peak-energy bremsstrahlung from the radiator. After passing through the collimators the electron beam was held together by two strong-focusing quadrupoles mounted over the accelerator.

The position of the radiator was defined without the help of a nearby collimator by making it from a $\frac{1}{4}$-in. diameter 0.003-in. thick aluminum foil which was suspended in the center of the electron beam tube on thin wires. The whole assembly was on a motor-driven support which could place in the beam a zinc sulfide screen, the foil, or a blank holding only the equivalent of the suspension wires. By steering with Helmholtz coils just after the nearer collimator 85 ft away, the beam could be centered on the zinc sulfide screen which could subsequently be replaced by the foil. The blank with wires was inserted for background runs.

After passing through the foil the electron beam was deflected by a magnet while the bremsstrahlung went straight ahead. A collimator $\frac{1}{4}$ in. in diameter was placed in the γ-ray beam 40 ft from the radiator to select the polarization region (see Fig. 6), while an additional collimator was placed about 15 ft from the radiator. The two collimators defined a solid angle which eliminated many possible radiation sources other than the

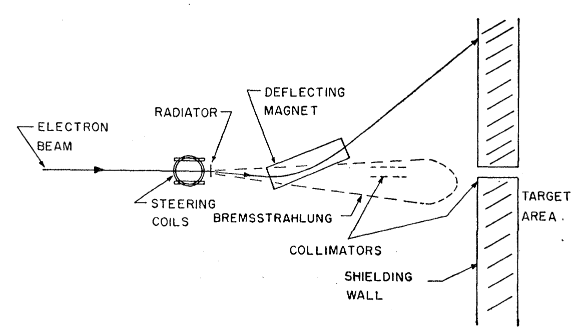

FIG. 6. Equipment for the production of polarized bremsstrahlung.

[6] J. Miller, Rapport C.E.A. No. 655, Centre d'Etudes Nucleaires de Saclay, 1957 (unpublished).
[7] E. Jahnke and F. Emde, *Table of Functions* (Dover Publications, New York, 1948), p. 1.

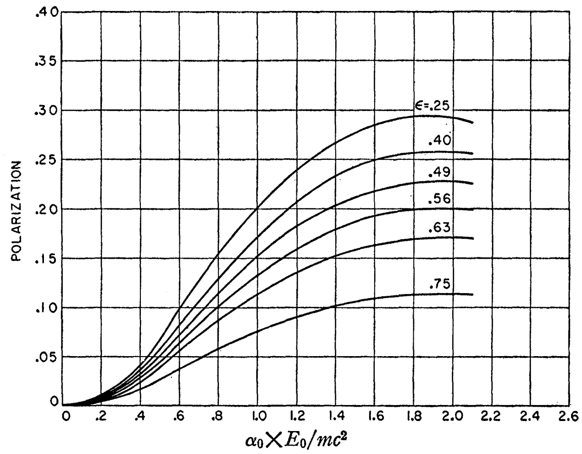

FIG. 7. Calculated values of polarization with multiple scattering and $0.6\times mc^2/E_0$ diameter aperture included.

radiator foil. The farther collimator was in a 5-ft steel and concrete wall separating the beam-deflecting area from the experimental area.

The region of polarization could in principle be selected by moving the collimator to the appropriate portion of the bremsstrahlung, but it was easier to move the bremsstrahlung angular distribution by deflecting the electron beam slightly by Helmholtz coils just before it hit the radiator. These coils were so close to the radiator that they changed the beam angle but not its position. Polarization of either alignment could be selected by the proper choice of current in the coils. The $\frac{1}{4}$-in. diameter collimator aperture is then folded into the polarization curves (Fig. 4) to obtain the polarization values used in these measurements (see Fig. 7). After passing through the foil the beam was energy-analyzed by a deflecting magnet and then stopped inside of the well-shielded beam-deflecting area.

To obtain data on the beam size, angular divergence, and multiple scattering, as mentioned above, the beam was allowed to hit an aluminum foil half as thick as that used for the radiator during the experimental runs. Instead of being deflected, the scattered beam was allowed to move straight ahead past the positions of both collimators, which had been removed, and allowed to hit a glass plate. The darkening of the glass was measured with a densitometer, giving a combined measurement of the beam size, angular divergence, and rms multiple scattering.[8] The precaution taken to ensure that the darkening was linear was to make three plates with differing densities. The two less dense plates showed the same shape of electron distribution, indicating that they were in a linear region. In addition, the actual exposure was in a region (10^{13} electrons/cm^2) crudely determined to be linear by making a series of exposures with different total numbers of electrons as determined by a secondary emission monitor.

[8] Mozley, Smith, and Taylor, Phys. Rev. **111**, 647 (1958).

DETECTION OF POLARIZATION

It would be desirable to determine the percentage polarization experimentally using a well-understood reaction such as pair production, since the calculation of the asymmetry of pair emission is almost identical to the bremsstrahlung calculation.[3] The difficulty is that the angle of emission of the pair members is again mc^2/E, and even a completely polarized bremsstrahlung beam would produce only a partial pair asymmetry. Since one must resolve small angles and measure a total asymmetry of about 5%, we decided that this method of calibration was not feasible. Compton scattering could be used but offers even greater difficulties.

We felt that a nuclear interaction was needed with reaction products appearing at reasonably large angles. Since the ultimate purpose of the experiment was to use the polarized bremsstrahlung for meson production, it was necessary to make the polarization measurement at an energy of approximately 300 Mev. All nuclear reactions such as photodisintegration of the deuteron are poorly understood in this energy region due to virtual meson processes which take part in the interaction. The photodisintegration process, however, is an excellent method of calibration for very low-energy photons near the peak of the electric-dipole interaction.

We decided to use positive pion production as our method of studying polarization. Although it is a poorly understood detection process, it is the reaction we eventually wished to study. Moreover, the 2-μsec μ-decay is an easily observed reaction when using the Mark III linear accelerator. The duty cycle of the accelerator is about 3×10^{-5} since the beam is on for at most 0.6 μsec and off for 1/60 sec. In order to detect pions one counts the electrons in the decay chain $\pi\rightarrow\mu\rightarrow e$. The half-life of the μ meson is long enough that most μ decays take place after the beam of the accelerator is no longer present, thus allowing one to avoid the problem of trying to identify a meson during the period when the background from the accelerator is highest.

Although meson production is not well understood, all calculations predict a reasonable amount of asymmetry in the production. Since the production of pions is fundamentally magnetic dipole in the region of the $(\frac{3}{2},\frac{3}{2})$ resonance, enhancement of production at right angles to the electric field vector of the photons is expected.

The experimental work was divided into two periods. During the first, the equipment was designed entirely to maximize the counting rate, sacrificing any ability to determine accurately the energy and angle of the mesons produced. The purpose of the experiment at this time was to demonstrate that a polarized γ-ray beam had been produced. The second part sacrificed counting rate in order to obtain energy and angular resolution in studying positive pion production.

PRELIMINARY EXPERIMENT

The polarized photon beam was passed through a liquid hydrogen target and positive pions produced in the hydrogen were detected by two plastic scintillation counters placed at 90° to the γ-ray beam. They were also placed 90° apart in azimuth so that if one detected mesons produced along the electric field vector the other detected mesons produced at right angles to it. (See Fig. 8.)

The target was two ft long and made of Styrofoam surrounded by a liquid nitrogen cooled radiation shield, while the plastic scintillators were 6 in. in diameter, 5 in. thick, and located $2\frac{1}{2}$ ft from the target. To determine the meson energy, iron absorbers were placed between the counters and the target. The energy range in the detectors was ± 17 Mev, the polar angle subtended $\pm 25°$ and the azimuthal angle $\pm 9°$. All particles were detected which lost sufficient energy in the plastic during a period of about 6.6 μsec immediately after the beam pulse. That they were due to pions was determined by establishing that they had the characteristic 2.2-μsec $\pi-\mu-e$ decay. In addition, it could be shown that the threshold for their production agreed with that for π-mesons and that their production cross section ratios in hydrogen and carbon were characteristic of mesons rather than of neutrons. There was no way of ensuring that the pions had stopped in the plastic, but the chance that they had stopped elsewhere was reduced by keeping the counters as far from any other material as possible. They were 1 ft from the absorbers and $2\frac{1}{2}$ ft from the target, and no shielding was used around the counters. A telescope arrangement would have been necessary to remove this problem and this in turn would have greatly reduced the counting rate. Although a magnetic analysis system would also have solved the problem none was available with sufficient solid angle to make the experiment feasible. Moreover, in this preliminary experiment it was felt that the exact location of the decaying mesons and even their identification as pions were not essential to determining that polarization was present.

The procedure for measuring the asymmetry of meson production was as follows. The ratio of mesons detected in counters 1 and 2 was measured with the polarization of the bremsstrahlung chosen so that the majority of the photons had their electric field vector directed toward counter 1. The polarization was then rotated 90° by changing the currents in the Helmholtz coils immediately before the radiator, and the ratio was again measured. Dividing the first of these ratios by the second gives a value of the square of the meson production asymmetry, R, which is independent of counter characteristics.

In describing meson production we label the number of photons, N, by subscripts, $_{\parallel}$ and $_{\perp}$, referring to the plane of photon emission, thus making the polarization, $P=(N_{\perp}-N_{\parallel})/(N_{\perp}+N_{\parallel})$, greater than 0. On the other

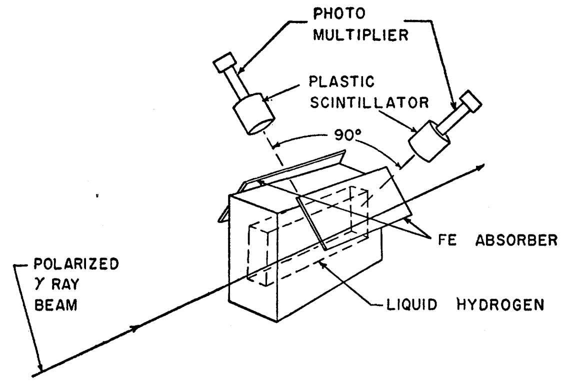

FIG. 8. Hydrogen target and meson detection scintillators (preliminary experiment).

hand, the subscripts of the meson production cross sections, $d\sigma_{\parallel}$ and $d\sigma_{\perp}$, refer to the relation of the plane of polarization of the meson producing photon to the plane of meson production. If we make the plane of photon emission coincide with the plane of meson emission defined by counter 2, the counting rate of counter 2 will be proportional to $(N_{\parallel}d\sigma_{\parallel}+N_{\perp}d\sigma_{\perp})C_2$, where C_2 contains the counter characteristics. Correspondingly the rate of counter 1 for the same bremsstrahlung intensity will be $(N_{\perp}d\sigma_{\parallel}+N_{\parallel}d\sigma_{\perp})C_1$, since those photons which are polarized parallel to the plane of meson emission of counter 2 are perpendicular to that of counter 1.

We measure the ratio of the counting rates of counters 1 and 2 with the plane of photon emission first directed toward counter 1 and then toward counter 2 and obtain the following value of the square of the meson production asymmetry, R.

$$R^2=\frac{(N_{\perp}d\sigma_{\parallel}+N_{\parallel}d\sigma_{\perp})C_1}{(N_{\parallel}d\sigma_{\parallel}+N_{\perp}d\sigma_{\perp})C_2}\bigg/\frac{(N_{\parallel}d\sigma_{\parallel}+N_{\perp}d\sigma_{\perp})C_1}{(N_{\perp}d\sigma_{\parallel}+N_{\parallel}d\sigma_{\perp})C_2}. \quad (3)$$

Using this measurement and the calculated value of polarization P, we obtain the meson cross-section asymmetry:

$$\frac{d\sigma_{\parallel}}{d\sigma_{\perp}}=\frac{P(1+R)-(1-R)}{P(1+R)+(1-R)}. \quad (4)$$

It can be seen that if one is studying the polarization variations caused by changing the angle of bremsstrahlung or other parameters while holding $d\sigma_{\parallel}/d\sigma_{\perp}$ constant

$$P\propto(1-R)/(1+R).$$

For values of R not much different from unity, $(1-R)$ will be approximately proportional to the polarization.

In the experiment we are now describing we are concerned only with indicating qualitatively that polarized bremsstrahlung was present.

Errors associated with the drifts in the counter and electronics were reduced by taking the ratios in many short runs so that a negligible drift could take place

in the time required for making a complete ratio measurement. Problems of beam centering were handled by using the polarization in four quadrants around the beam (see Fig. 9). This allows the meson counting rate to be used as a check on the alignment and, in addition, causes a partial cancellation of the errors since too high a polarization in one quadrant is compensated for by too low a polarization in the opposite quadrant unless one is working at the polarization maximum where the error would be negligible.

An asymmetry was found and measured as a function of radiator thickness and angle of bremsstrahlung. Our major effort, however, was to establish that there were no poorly understood background effects which could cause a spurious asymmetry.

Background runs were made (1) with no radiator present and the hydrogen target filled, (2) with a radiator and no hydrogen in the target, and (3) with no radiator and no hydrogen. No statistically significant asymmetries were obtained in any of these runs. In addition, a run was made with the hydrogen target empty, and with a radiator much thicker than usual, and a test was made with more steering than usual to make certain that no steering effect could cause a background.

We concluded that the observed asymmetry could only be produced by polarized bremsstrahlung. Figure 10 shows asymmetry ratios measured as a function of bremsstrahlung angle and foil thickness. These are qualitatively in agreement with theoretical predictions. Although an identification of the particles counted as mesons was not necessary in order to establish that a polarized γ-ray beam had been obtained, such an identification is necessary if the polarization is to be established as transverse in nature. Although some conclusions concerning meson production could be obtained from this work, a new experimental arrangement was needed to allow a more accurate determination of the meson energy and angle.

FINAL EXPERIMENTAL ARRANGEMENT

The target size was reduced from 2 ft to 6 in; the detectors were of the same area as before but only 2 in. thick. They were located about $2\frac{1}{2}$ ft from the hydrogen and mounted on supports which allowed them to be moved in angle. The energy resolution was about ± 5 Mev and the angle $\pm 6°$.

We used no counter telescope and counted all particles losing over 5 Mev in the scintillator during a period 1–7.6 μsec after the beam pulse. A background gate delayed about 13 μsec was used to estimate and allow the subtraction of any slowly decaying activity. The particles were counted in two gates, one 2.2 μsec long starting 1 μsec after the beam and the other 4.4 μsec long starting immediately after the first. A partial determination of the half-life of the particles detected could be made since a particle decaying with a 2.2-μsec life would have twice as many counts in gate 1 as in gate 2. Just as in the preliminary experiment, there is no way of determining that the μ mesons which are counted actually stopped in the counters. Again no shielding was used and even more care was taken to keep all unnecessary mass away from the counters. The counters were mounted in very light-weight covers and suspended on light steel tubing supports. The target was made of Styrofoam and had no radiation shield. The presence of the copper absorber mass was unavoidable and although the effect was reduced by the distance of the absorber from the detectors it is estimated that about 11% of the mesons detected stopped in the absorber. The β rays from μ-decay penetrate about $\frac{1}{3}$ of a radiation length or about 0.5 cm of copper.

Similarly about 14% of the pions counted stopped in the Lucite light pipe between the plastic scintillator and the photomultiplier.

The effect of the Lucite and absorber was measured by a calibration run made with a magnetic analyzer for the mesons. The mean energy given by this measurement was used in the energy analysis rather than the slightly different value ($\Delta \simeq 5$ Mev) calculated from the absorbers.

The range-energy measurement also confirmed that the 2-μsec decay identified the decay particles as μ mesons. In addition a yield curve was obtained (without magnetic analysis) by varying the absorber thickness and comparing the yield with that calculated from the unpolarized bremsstrahlung spectrum and the known meson production cross section. The agreement was satisfactory at 90° c.m. angle and larger, but there were discrepancies in the forward angles which made

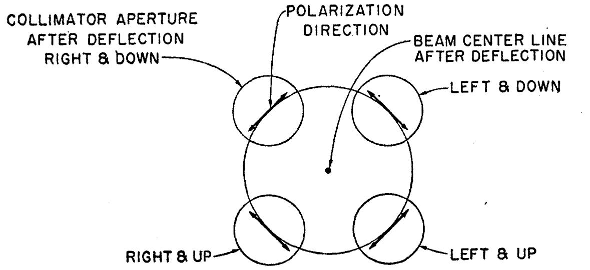

FIG. 9. Polarization direction cycle.

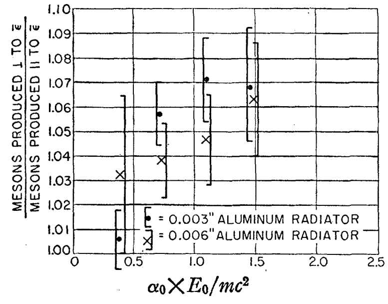

FIG. 10. Asymmetry ratios *vs* bremsstrahlung angle (preliminary experiment).

TABLE I. Data used in the evaluation of the meson cross-section asymmetry.[a]

Photon energy	Measured meson asymmetry	Photon polarization	Meson cross-section asymmetry $d\sigma_{\parallel}/d\sigma_{\perp}$	Correction due to π-pairs $\sigma_{\pi\text{-pair}}/\sigma_{\text{singles}}$	$d\sigma_{\parallel}/d\sigma_{\perp}$
242 Mev	0.950 ± 0.033	0.22	$0.792_{-0.12}^{+0.13}$	0.177 ± 0.02	$0.760_{-0.14}^{+0.15}$
296 Mev	0.914 ± 0.033	0.19	$0.618_{-0.12}^{+0.13}$	0.122 ± 0.04	$0.581_{-0.13}^{+0.14}$
337 Mev	0.834 ± 0.026	0.17	$0.304_{-0.07}^{+0.08}$	0.050 ± 0.04	$0.282_{-0.08}^{+0.08}$
376 Mev	0.909 ± 0.036	0.144	$0.503_{-0.14}^{+0.17}$	0.013 ± 0.01	$0.498_{-0.14}^{+0.17}$

[a] Errors shown are standard deviations.

more elaborate detection schemes desirable for measurements at small angles.

In the second experiment, in addition to modifying the target and the counters, the method of steering the electron beam to select the polarization was made automatic. The switching of the currents in the Helmholtz coils was synchronized with a switching of the storage scalers in which data was recorded, allowing the change of polarization to be triggered automatically by an integrator circuit measuring the bremsstrahlung. It was felt that by allowing this to cycle on the average once a minute during runs, the effects of drifts would be so reduced that they could be ignored as a source of systematic error.

BACKGROUNDS AND CORRECTIONS

There were three major sources of background in the experiment. The largest of these (about 15%) from meson sources other than hydrogen could be measured by a run with the target empty. Another background due to radiation from sources other than the radiator producing mesons in the hydrogen could be estimated by measurements with the hydrogen target filled but no radiator present. (Wires duplicating those on which the radiator was suspended were present during this run.) The third and smallest type was due to radiation which produced counts in the scintillators with neither the radiator nor hydrogen present. All these backgrounds were measured and subtracted.

Mesons were counted which decayed during a 6.6-μsec interval following the beam pulse. In addition, a gate delayed 13 μsec measured the relatively constant background and this in turn was subtracted.

More difficult to handle were spurious counts due to mesons which did not reach our counters in the planned manner. The worst of these corrections was that due to π-pair production. We were using bremsstrahlung of about 300 Mev from 600-Mev electrons since it was necessary to use the low-energy bremsstrahlung from high-energy electrons in order to have appreciable polarization. When meson production is a two-body process, the energy of the meson identifies the energy of the bremsstrahlung producing it, but the very high-energy photons were able to pair-produce pions of the same energy as those produced singly by the 300-Mev photons. Measurements have been made by Friedman and Crowe[9] of the relative yield of positive pions from pair production and single production for 76-Mev pions. In addition, Bloch and Sands[10] have made similar measurements at 45 and 125 Mev. It is assumed that there is the same ratio of π-pair to single pion yield at our 73° laboratory angle as at 60° where the π-pair data have been obtained. From these relative yields we can make a correction for the presence of the pairs if we assume that (in the pion pair production) there is no basic asymmetry due to polarization. (See Table I.) Since the much higher-energy bremsstrahlung producing the π-pairs has a relatively smaller polarization than that producing the pions singly, this assumption is valid for the two higher-energy points but is increasingly poor at lower energies. It is impossible to estimate accurately how much asymmetry would be expected in π-pair production with completely polarized γ rays, but one would expect the asymmetry to be opposite that for single meson production since the presence of the second meson would change the parity rules and probably cause an increase of P-wave production along the electric field vector rather than at right angles to it. Since we are unable to estimate the asymmetry accurately we have not corrected for it. If it were taken into account it would probably increase slightly the corrected meson cross-section asymmetry (reduce the value of $d\sigma_{\parallel}/d\sigma_{\perp}$).

Another correction is due to pions which undergo μ-decay as they pass the absorbers separating the scintillators from the target. A few of the muons from very low-energy pions are given sufficient transverse momentum to allow them to penetrate the scintillator without passing through the absorber. In addition we detect electrons from the $\pi-\mu-e$ decay of low-energy pions which stop in the air near the scintillators. These two effects introduce a very low-energy component varying from 6% of the low-energy pions to 20% of the highest.

The third correction we have taken into account is that due to mesons which are incident on the absorber but stop in places other than the scintillator. This results in a slight shift of the mean energy and small tails on the resolution function. A similar small effect is due to pions which decay in flight into muons with a

[9] R. Friedman and K. Crowe, Phys. Rev. **105**, 1369 (1957).
[10] M. L. Bloch and M. Sands, Phys. Rev. **113**, 305 (1959).

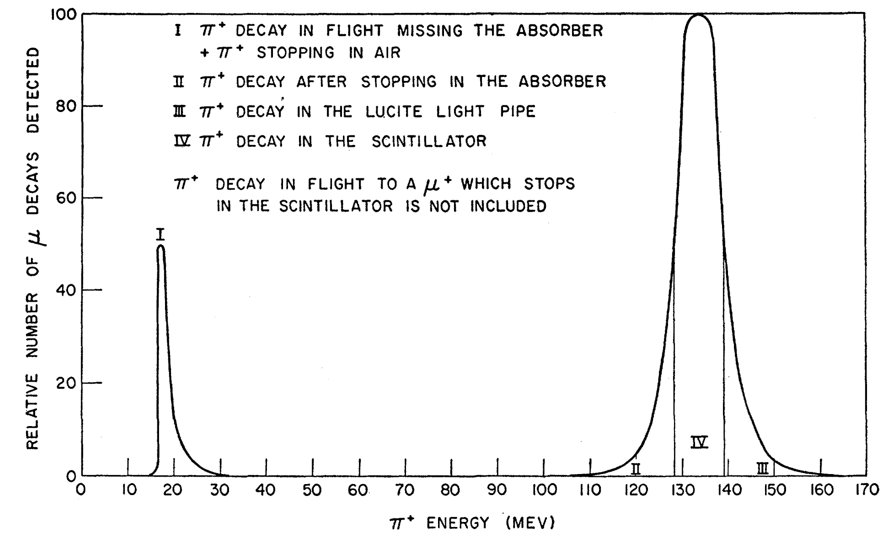

FIG. 11. Product of yield and energy resolution function.

different range. These spread out the energy resolution primarily by giving a high-energy tail. Approximately 5% of the mesons undergo such decays, but the total shift involved is small and is partially taken into account by the magnetic energy calibration. Figure 11 shows an estimate of the shape of the product of yield and energy resolution functions as compared with that from the idealized range energy relation in the scintillator. This function has been folded into the theoretical predictions (Fig. 14) to take into account the effects of the appreciable low-energy component.

ERRORS

The errors shown on the curves are entirely statistical counting errors. X^2 tests do not indicate the presence of systematic errors although the data fluctuations are somewhat larger than desirable. Errors due to the angle and energy resolution are trivial compared with the statistical errors.

The principal errors are due to the fundamental philosophy of the experiment. A primary source of error is our method of identification of the meson which is based almost entirely on the meson mean life. If there is an appreciable contamination of other particles such as neutrons which are not properly subtracted by the delayed gate measurement, this can cause an error, probably by reducing the asymmetry measured.

Another major source of error is in our estimate of the polarization. The measurement on which we based our multiple scattering calculation was made only once. Although the beam spot was of approximately the same size and location each time data were taken, there undoubtedly were fluctuations. Moreover, our fold of multiple scattering used approximations causing errors of about 3%. The polarization estimate is possibly accurate to about 10% of the quoted value if we assume that the theoretical predictions are correct. These errors could cause as much as 7% variation in the final $d\sigma_{\parallel}/d\sigma_{\perp}$ value but since they are by no means random they have not been combined with those quoted in Table I and Fig. 14.

These comments do not apply to our data on the polarization angular distribution, which are arbitrarily normalized; nor do they apply to our measurement of the bremsstrahlung intensity distribution, which has an error due primarily to an inadequate method of measuring the number of electrons hitting the foil. This type of normalization does not enter into the polarization measurements which are entirely ratio determinations using measurements of beam current merely to estimate the time required for background runs.

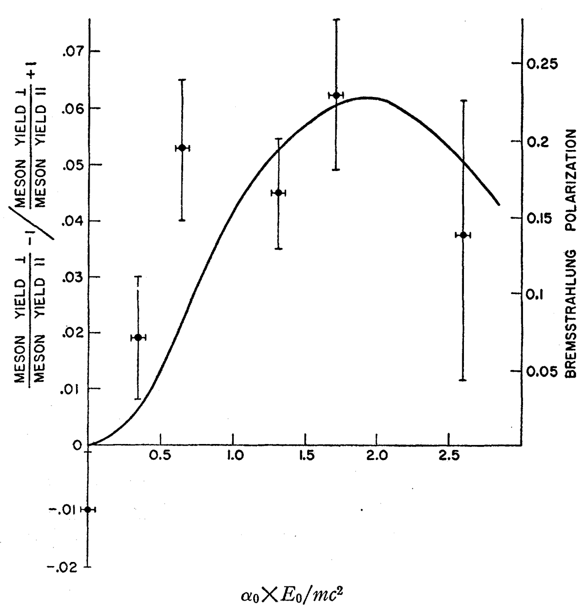

FIG. 12. Comparison of asymmetry ratios with calculated value.

RESULTS

Using the final experimental arrangement with reasonable energy and angle resolution, we obtained additional confirmation that the polarization varied with angle of bremsstrahlung qualitatively as predicted by theory. Figure 12 shows a plot of the measured ratio compared with the theoretically predicted value. The good fit is due to the arbitrary normalization since only the curve shapes are being compared. The data are poor statistically but cannot be said to disagree with the theory.

Figure 13 is a plot of meson production as a function of bremsstrahlung angle and compares the bremsstrahlung intensity with the angle variation predicted by theory. The data used are the same as those used above in studying the polarization angular distribution, and are obtained from the study of meson production by 300-Mev bremsstrahlung from 600-Mev electrons. In this case, however, instead of plotting ratios the total number of mesons produced was recorded and normalized against the number of electrons hitting the radiator. The electrons were measured in a secondary emitter just after the deflecting magnet which followed the radiating foil. The major error in the plot is due to the normalization since electrons which missed the radiator would still be counted by the secondary emitter. We feel that the fluctuations in the electron beam steering are so large that the data cannot be considered as being in substantial disagreement with theoretical predictions. The theory with which the data are compared is that of the May calculations which give the same result for unpolarized bremsstrahlung as the Schiff[11] approximate calculations.

We choose to regard the measurements as evidence that the polarization is as calculated, and we then use the calculated value of polarization and our measured

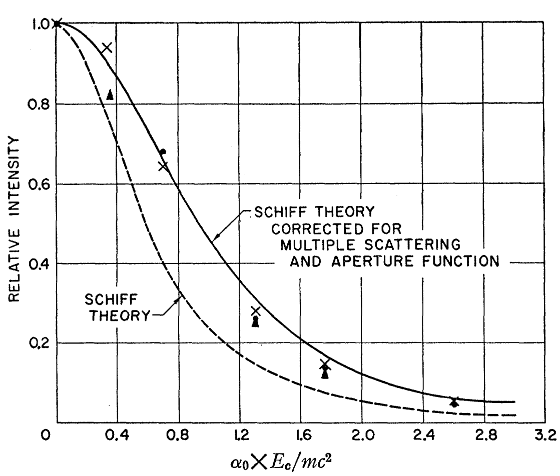

FIG. 13. Angular distribution of 300-Mev bremsstrahlung from 600-Mev electrons (3-mil aluminum radiator).

[11] L. Schiff, Phys. Rev. 83, 252 (1951).

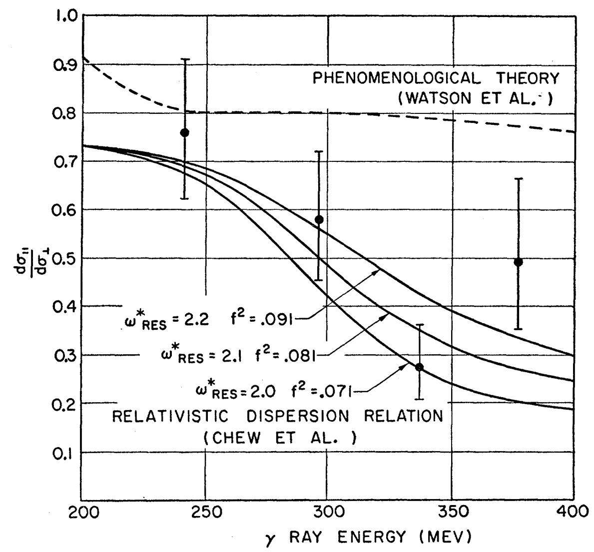

FIG. 14. Comparison of meson production asymmetry with theoretical predictions. The resolution function (Fig. 11) has been folded into the theoretical predictions causing a maximum change for the high-energy dispersion theory curves by raising them about 0.04–0.07.

meson production asymmetries to give the ratio of production of pions along and perpendicular to the electric field vector. The measurements which were made at 90° c.m. angle are listed in Table I along with the calculated polarization and corrections. In Fig. 14 they are compared with the predictions of the phenomenological theory[12,13] and the relativistic dispersion relations.[14]

The phenomenological analysis is based on the assumption of the presence of only S and P waves while the dispersion relation includes a retardation, meson current term. It can be seen that our data disagree with the phenomenological theory and are in good agreement with the dispersion theory. It is pointed out by Watson *et al.*[13] that additional terms are required to make a good fit to the experimental data, and our disagreement with the predictions of the theory is in part an additional confirmation of this.

There is considerable latitude in one's choice of the S-wave and small P-wave phase shifts to put into the dispersion theory. It can be shown that the meson production asymmetry is relatively insensitive to the value of S-wave phase shift used.[15] The values used were $\delta_1 = 0.173q$ and $\delta_3 = -0.110q$.[16] Rather than

[12] M. Gell-Mann and K. Watson, *Annual Review of Nuclear Science* (Annual Reviews, Inc., Palo Alto, 1954), Vol. 4, p. 219.

[13] Watson, Keck, Tollestrup, and Walker, Phys. Rev. **101**, 1159 (1956).

[14] Chew, Goldberger, Low, and Nambu, Phys. Rev. **106**, 1337, 1345 (1957).

[15] A. Lazarus and F. Tangherlini (private communication).

[16] G. Puppi, *1958 Annual International Conference on High-Energy Physics at* CERN, edited by B. Ferretti (CERN Scientific Information Service, Geneva, 1958).

choose the four P-wave phase shifts, we reduced the number of parameters to two by using the dispersion relation connection between the phase shifts given by Chew *et al.*[14] The phase shifts δ_{11}, δ_{13}, δ_{31}, and δ_{33} are obtained from the value of the coupling constant f^2 and ω_r^*, the c.m. energy at resonance.

$$\left(\omega_r^* = \frac{\text{total energy at resonance (c.m.)} - \text{nucleon rest mass}}{\text{pion rest mass.}}; \text{ see Appendix.}\right)$$

Since both S-wave phase shifts appear together in a term $2\delta_1+\delta_3$, this reduces the 6 parameters to 3, *viz.* $2\delta_1+\delta_3$, f^2, and resonant energy. The values we quote are based on calculations performed by Lazarus and Tangherlini.[15] The value of S-wave phase shift used remained fixed and the value of f^2 and the resonant energy were varied; f^2 values of 0.071, 0.081, and 0.091 were used and ω_r^* values 2.0, 2.1, and 2.2 were used. Figure 14 shows three of the nine curves so obtained. A larger value of resonant energy or coupling constant produced a larger value of $d\sigma_{\parallel}/d\sigma_{\perp}$. The variation with resonant energy was about twice as large as that with the coupling constant. The data does seem to be in somewhat better accord with the larger values, but the experimental accuracy would have to be considerably improved to make any real conclusion possible. The approximations inherent in the use of the dispersion relations may lead to large inaccuracies in the higher energies and thus reduce the value of a more accurate investigation.

ACKNOWLEDGMENTS

We wish to thank Mr. Raymond Smith and Mr. Darrell Drickey for their assistance in all of the experimental work. Dr. Frank Tangherlini aided us considerably with detailed calculations and very helpful discussions. We have been helped materially by discussions with Dr. R. H. Dalitz, Dr. D. R. Yennie, and Dr. S. D. Drell, and we are grateful for the assistance we have received from Dr. W. K. H. Panofsky and Dr. K. M. Crowe. Mr. E. A. Wright and members of his shop were very helpful in the construction of equipment. Finally we want to thank Mr. Gordon Gilbert and the operating crew of the linear accelerator for many hours of trouble-free operation.

APPENDIX

The cross section for photomeson production is given by Chew *et al.*[14] as

$$\sigma = 2e^2 f^2 \frac{q}{k} |M|^2, \tag{5}$$

where

$$M = a\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon} + b\frac{2\boldsymbol{\sigma}\cdot(\mathbf{k}-\mathbf{q})\mathbf{q}\cdot\boldsymbol{\varepsilon}}{(\mathbf{k}-\mathbf{q})^2+1} + ui\mathbf{q}\cdot(\mathbf{k}\times\boldsymbol{\varepsilon}) + v\boldsymbol{\sigma}\cdot\mathbf{k}\mathbf{q}\cdot\boldsymbol{\varepsilon} + w\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}\mathbf{q}\cdot\mathbf{k}. \tag{6}$$

(The notation used is that of Moravcsik.)[17] We are informed by Chew[18] that two changes should be made in the formulas given. $1/[1+(\omega^*/M)]$ should precede each term which does not include a phase shift and ω^* should be substituted for ω. Then

$$\begin{aligned} a &= i\left[\frac{1}{1+(\omega^*/M)}\left(1+\omega^{*2}N^{(-)}-\frac{g_p+g_n}{2M}\omega^*\right) + \frac{i}{3}(2\delta_1+\delta_3)F_S\right], \\ b &= \frac{i}{1+(\omega^*/M)}, \\ u &= -i\lambda h^{(-+)} - (2/9)e^{i\delta_{33}}\sin\delta_{33}F_M, \\ v &= i\lambda h^{(--)} - \tfrac{1}{3}e^{i\delta_{33}}\sin\delta_{33}(F_Q+\tfrac{1}{3}F_M) - \frac{i}{1+(\omega^*/M)}\frac{g_p+g_n}{2M\omega^*}, \\ w &= -i\lambda h^{(--)} - \tfrac{1}{3}e^{i\delta_{33}}\sin\delta_{33}(F_Q-\tfrac{1}{3}F_M) + \frac{i}{1+(\omega^*/M)}\frac{g_p+g_n}{2M\omega^*}, \end{aligned} \tag{7}$$

where (all in c.m. system) $\hbar=c=\mu=1$, μ is the pion rest mass, and c is the velocity of light; $\boldsymbol{\sigma}$ is the nucleon spin, $\bar{\varepsilon}$ is the photon polarization, $\boldsymbol{k}$ is the photon momentum, $\bar{q}$ is the meson momentum, ω = meson total energy (including rest mass), γ = meson total energy/photon energy $=\omega/k$ $\beta=v/c=v$ = velocity of meson, $q=\beta\omega$, ω^* = total energy of the reaction minus the nucleon rest mass, M = nucleon rest mass, and

$$\begin{aligned} F &= 1+\frac{1-\beta^2}{2\beta}\ln\frac{1-\beta}{1+\beta}, \\ F_S &= 1-\tfrac{1}{2}F, \\ F_M &= \frac{3}{4q^2}F, \\ F_Q &= \frac{1}{\omega^{*2}}\left(1-\frac{3}{4\beta^2}F\right). \end{aligned} \tag{8}$$

$$\begin{aligned} h^{(--)} &= \tfrac{1}{3}(h_{11}-h_{31}-h_{13}+h_{33}), \\ h^{(-+)} &= \tfrac{1}{3}(h_{11}-h_{31}+2h_{13}-2h_{33}), \\ h_{ab} &= e^{i\delta_{ab}}(\sin\delta_{ab}/q^3). \end{aligned} \tag{9}$$

[17] M. Moravcsik, Phys. Rev. **104**, 1451 (1956).

[18] G. Chew (private communication).

δ_{ab} are P-wave phase shifts of meson nucleon scattering, $a=2\times$isotopic spin, $b=2\times$angular momentum, δ_1 and δ_3 are the S-wave shifts of isotopic spin $\frac{1}{2}$ and $\frac{3}{2}$, $\lambda=(g_p-g_n)/4Mf^2$, $g_p=2.79$, the proton g-factor; $g_n=-1.91$, the neutron g-factor; f^2 is the coupling constant, and $N^{(-)}$ is an unknown real number associated with the electric dipole amplitude. It is considered small and set equal to zero.

If one squares the matrix element without averaging over polarization, one obtains

$$\begin{aligned}|M|^2=|a|^2+|b|^2&\frac{(1+\beta^2\gamma^2-2\beta\gamma\cos\theta)\beta^2\sin^2\theta\cos^2\phi}{(1-\beta\cos\theta)^2}\\
&+|u|^2\beta^2\gamma^2k^4\sin^2\theta\sin^2\phi\\
&+|v|^2\beta^2\gamma^2k^4\sin^2\theta\cos^2\phi\\
&+|w|^2\beta^2\gamma^2k^4\cos^2\theta\\
&-2\,\mathrm{Re}(a^*b)\beta^2\gamma\frac{\sin^2\theta\cos\phi}{1-\beta\cos\theta}\\
&+2\,\mathrm{Re}(a^*w)\beta\gamma k^2\cos\theta\\
&+2\,\mathrm{Re}(b^*v)\frac{(1-\beta\gamma\cos\theta)\beta^2\gamma k^2\sin^2\theta\cos^2\phi}{1-\beta\cos\theta}\\
&-2\,\mathrm{Re}(b^*w)\frac{\beta^3\gamma^2k^2\sin^2\theta\cos\theta\cos^2\phi}{1-\beta\cos\theta}.\end{aligned}\tag{10}$$

The relation connecting the phase shifts is given by Chew *et al.*[14] (We again change ω to ω^*.)

$$f_\alpha\cong\frac{\lambda_\alpha q^2/\omega^*}{1-r_\alpha\omega^*-\lambda_\alpha(q^3/\omega^*)},\tag{11}$$

where

$$\begin{aligned}&f_\alpha=\exp(i\delta_\alpha)\sin\delta_\alpha/q,\\
&\lambda_\alpha=\tfrac{2}{3}f^2\begin{pmatrix}-4\\-1\\-1\\+2\end{pmatrix}\quad\text{for}\quad\alpha=\begin{pmatrix}11\\13\\31\\33\end{pmatrix},\\
&\delta_{13}\cong\delta_{31}.\end{aligned}\tag{12}$$

We use r values given by Chew[19]; these values differ from those given earlier[14] since they contain $1/M$ corrections.

$$\begin{aligned}r_{13}\cong r_{31}&=-\frac{4}{5}\frac{1}{\omega^*_{\text{resonance}}}+\frac{9}{10M},\\
r_{11}&=-\frac{4}{5}\frac{1}{\omega^*_{\text{resonance}}}-\frac{9}{40M}.\end{aligned}\tag{13}$$

The expression for f_α reduces to the simpler

$$\frac{q^3}{\omega^*}\cot\delta_\alpha=(1-r_\alpha\omega^*)\Big/\tfrac{2}{3}f^2\begin{pmatrix}-4\\-1\\-1\\+2\end{pmatrix},\tag{14}$$

from which the small P-wave phase shifts may be calculated if f^2 and $\omega^*_{\text{resonance}}$ are known.

Phenomenological Theory

The matrix element is of the same form as that given in the dispersion relation above except that $b=0$. To reduce it to the form used by Gell-Mann and Watson,[12]

$$\begin{aligned}M/S=iE_1\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}&-M_1(\tfrac{1}{2})[k\times\boldsymbol{\varepsilon}\cdot\mathbf{q}-i\boldsymbol{\sigma}\cdot(\mathbf{k}\times\boldsymbol{\varepsilon})\times\mathbf{q}]k^{-1}q^{-1}\\
&-M_1(\tfrac{3}{2})[2\mathbf{k}\times\boldsymbol{\varepsilon}\cdot\mathbf{q}+i\boldsymbol{\sigma}\cdot(\mathbf{k}\times\boldsymbol{\varepsilon})\times\mathbf{q}]k^{-1}q^{-1}\\
&+\tfrac{1}{2}E_2[\boldsymbol{\sigma}\cdot\mathbf{k}\boldsymbol{\varepsilon}\cdot\mathbf{q}+\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}\mathbf{k}\cdot\mathbf{q}]k^{-1}q^{-1}.\end{aligned}\tag{15}$$

Let

$$\begin{aligned}&a=iE_1S,\\
&b=0,\\
&u=i[M_1(\tfrac{1}{2})+2M_1(\tfrac{3}{2})]Sk^{-1}q^{-1},\\
&v=i[M_1(\tfrac{3}{2})+\tfrac{1}{2}E_2-M_1(\tfrac{1}{2})]Sk^{-1}q^{-1},\\
&w=i[M_1(\tfrac{1}{2})-M_1(\tfrac{3}{2})+\tfrac{1}{2}E_2]Sk^{-1}q^{-1},\end{aligned}\tag{16}$$

where E_1 is the electric dipole moment, $M_1(\frac{1}{2})$ is the magnetic dipole moment with $j=\frac{1}{2}$, $M_1(\frac{3}{2})$ is the magnetic dipole moment with $j=\frac{3}{2}$, E_2 is the electric quadrupole moment, and

$$S=\frac{2\pi^2}{1+(k/Mc)}(2k\omega/e^2f^2)^{\frac{1}{2}}.$$

The multipole moment values used are those from the analysis of Watson *et al.*[18] In this $M_1(\frac{1}{2})=0$, $M_1(\frac{3}{2})$ is about the same size as E_2, and E_1 is approximately $2M_1(\frac{3}{2})$ at resonance.

[19] G. Chew, University of California Radiation Laboratory Report UCRL-Misc. 1957–45 (unpublished).