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Four-Fermion Interactions with Spin 3/2 Neutrinos*

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Beta decay involving a spin $\frac{3}{2}$ neutrino in weak competition with the spin $\frac{1}{2}$ neutrino has been investigated. This process could explain the deviations from allowed shapes in the Gamow-Teller interactions recently reported by Langer *et al.*, but it would then be in disagreement with μ-decay observations.

I. INTRODUCTION

THE possibility of a spin $\frac{3}{2}$ value for the neutrino emitted in beta decay was considered by Kusaka[1] and Ono[2] using a parity conserving interaction. This idea was discarded because the theory permits only Gamow-Teller transitions and because it predicts relatively more electrons at low energy than are observed.

Recently Langer *et al.*[3,4] observed deviations from the theoretical shape in the Fermi-Kurie plots of Na^{22}, In^{114}, Y^{90}, and P^{32}, all of which decay by pure Gamow-Teller transitions. They find that a factor of $(1+b/W)$, where $0.2<b<0.4$, must be applied to the theoretical curve to produce agreement with the experiments. This is a larger discrepancy than has been previously observed.[5] Also Langer finds that these deviations cannot be attributed to the effects of finite de Broglie wavelength, screening, contributions from second forbidden matrix elements, or Fierz interference because the deviations are so large and of the same sign for the positron emitter Na^{22} as for the electron emitters.

The facts that the deviations are for Gamow-Teller transitions and increase the low-energy end of the spectrum suggest that the idea of beta decay with a spin $\frac{3}{2}$ neutrino should be re-examined as a less probable mode of the decay. As is shown below, there are two possible interactions as long as parity need not be conserved and, with the proper choice of the coupling constants, one can fit Langer's observations with either of them. However, if the same branching interaction is applied to the decay of the μ meson, a Michel parameter in disagreement with experiment is obtained.

II. NOTATION

The simplest way to form scalars from the four fermion wave functions is to use spinor products. The notation used is that the spinor[6] $\Omega_{\dot\rho\dot\sigma\ldots}$ transforms with respect to continuous Lorentz transformations,

$$x_k'=a_{kl}x_l,$$

according to the rule

$$\Omega_{\dot\rho\dot\sigma\ldots}'(x')=\Lambda_{\rho\mu}\Lambda_{\sigma\nu}\cdots\Omega_{\dot\mu\dot\nu\ldots}(x), \tag{1}$$

where Λ is to be found from

$$\Lambda^H\sigma_k\Lambda=a_{kl}\sigma_l.$$

Spinors with undotted indices transform like the complex conjugate of $\Omega_{\dot\rho\dot\sigma\ldots}$ and spinors with upper indices are defined by $\Omega^{\dot1}=\Omega_{\dot2}$, $\Omega^{\dot2}=-\Omega_{\dot1}$.

In this notation, writing the Dirac equation as

$$(-\boldsymbol{\alpha}\cdot\mathbf{p}-\beta)\psi=i\partial\psi/\partial t,$$

and using the matrices

$$\boldsymbol{\alpha}=\begin{pmatrix}-\boldsymbol{\sigma} & 0\\ 0 & \boldsymbol{\sigma}\end{pmatrix},\quad \beta=\begin{pmatrix}0 & -1\\ -1 & 0\end{pmatrix},$$

one can write the wave function as two spinors:

$$\psi=\begin{bmatrix}\Omega_{\dot1}\\ \Omega_{\dot2}\\ \chi^1\\ \chi^2\end{bmatrix}. \tag{2}$$

For the spin $\frac{3}{2}$ neutrino a specialization of the arbitrary spin theory given earlier[7,8] is used. The particle is right-handed and the equations

$$\psi_{\frac{3}{2}}=\Omega_{\dot1\dot1\dot1},\quad \psi_{\frac{1}{2}}=\sqrt{3}\Omega_{\dot1\dot1\dot2},\quad \psi_{-\frac{1}{2}}=\sqrt{3}\Omega_{\dot1\dot2\dot2},\quad \psi_{-\frac{3}{2}}=\Omega_{\dot2\dot2\dot2}, \tag{3}$$

give the relationship between the components ψ of the earlier paper and a third rank spinor which is symmetric between all pairs of indices.

The conventional left-handed spin $\frac{1}{2}$ neutrino is used in the μ-decay interaction. The wave function is a spinor of the type χ^ρ.

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[2] K. Ono, Progr. Theoret. Phys. (Kyoto) **6**, 238 (1951).
[3] Hamilton, Langer, and Smith, Phys. Rev. **112**, 2010 (1958).
[4] Johnson, Johnson, and Langer, Phys. Rev. **112**, 2004 (1958).
[5] See, for example: Pohm, Waddell, and Jensen, Phys. Rev. **101**, 1315 (1956) and G. B. Henton and B. C. Carlson, ISC 1006 Ames Laboratory of the Atomic Energy Commission; Porter, Wagner, and Freedman, Phys. Rev. **107**, 135 (1957); H. Daniel, Nuclear Phys. **8**, 191 (1958).

[6] Greek indices run from 1 to 2 and Latin indices from 1 to 4. The coordinate x_4 is ict and the σ_k are the Pauli matrices with $(\sigma_4)_{\rho\sigma}=i\delta_{\rho\sigma}$. Relativistic units ($m$, c, $\hbar=1$) are used.
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[8] C. L. Hammer and R. H. Good, Jr., Phys. Rev. **111**, 342 (1958).

III. INTERACTION TERMS

The processes

$$N \to P + e^- + \bar{\nu}_{\frac{3}{2}}, \tag{4a}$$

$$\mu^- \to \nu_{\frac{1}{2}} + e^- + \bar{\nu}_{\frac{3}{2}}, \tag{4b}$$

and

$$N \to P + e^- + \nu_{\frac{3}{2}}, \tag{4c}$$

$$\mu^- \to \bar{\nu}_{\frac{1}{2}} + e^- + \nu_{\frac{3}{2}}, \tag{4d}$$

which still allow universal four-fermion interactions, are considered. The corresponding interaction terms are

$$\mathcal{H}_a = G_{\frac{3}{2}} \int d\mathbf{x}\, \Omega_N{}^{\dot\lambda} \chi_P{}^{\dot\mu} \chi_e{}^{\dot\nu} \Omega_{n\dot\lambda\dot\mu\dot\nu} + \text{H.c.}, \tag{5a}$$

$$\mathcal{H}_b = G_{\frac{3}{2}} \int d\mathbf{x}\, \Omega_m{}^{\dot\lambda} \chi_n{}^{\dot\mu} \chi_e{}^{\dot\nu} \Omega_{n\dot\lambda\dot\mu\dot\nu} + \text{H.c.}, \tag{5b}$$

$$\mathcal{H}_c = G_{\frac{3}{2}} \int d\mathbf{x}\, \chi_N{}^{\lambda} \Omega_P{}^{\mu} \Omega_e{}^{\nu} \Omega_{n\lambda\mu\nu} + \text{H.c.}, \tag{5c}$$

$$\mathcal{H}_d = G_{\frac{3}{2}} \int d\mathbf{x}\, \chi_m{}^{\lambda} \chi_n{}^{\mu} \Omega_e{}^{\nu} \Omega_{n\lambda\mu\nu} + \text{H.c.}, \tag{5d}$$

where the subscripts P, N, e, m, n refer to the particles involved (m for the meson and n for the neutrinos). They are of the tensor type as can be seen by rewriting Eq. (5a), for example, as

$$\mathcal{H}_a = -i(G_{\frac{3}{2}}/16) \int d\mathbf{x}\, \psi_P{}^H \gamma_4 (\gamma_k\gamma_l - \gamma_l\gamma_k) \psi_N \times \chi_e{}^{\dot\lambda} (\sigma_2 \sigma_k{}^H \sigma_l)_{\mu\nu} \Omega_{n\dot\lambda\dot\mu\dot\nu} + \text{H.c.} \tag{6}$$

The interactions are parallel to the conventional ones, made with only two-component neutrinos, in the sense that they do not conserve parity and are invariant under combined space reflection and charge conjugation.

IV. RESULTS

The interaction (5a) leads to the beta spectrum[9]

$$N(W)dW = (G_{\frac{3}{2}}^2/96\pi^3)|M_{GT}|^2 F(Z,W) \times p(W_0 - W)^4 W dW, \tag{7}$$

and an electron polarization, including Coulomb corrections, of $-(v/c)$. The interaction (5c) leads to the same spectrum but an electron polarization of $+(v/c)$. If one of these reactions were in competition with the conventional one, the spectrum of an allowed Gamow-Teller transition, assuming no interference, would be

$$N(W)dW = (G_{\frac{1}{2}}^2/\pi^3)|M_{GT}|^2 F(Z,W) p(W_0 - W)^2 W dW \times [1 + (G_{\frac{3}{2}}^2/96 G_{\frac{1}{2}}^2)(W_0 - W)^2]. \tag{8}$$

The form factor in square brackets gives agreement with Langer's data for the allowed transitions in In^{114}, Na^{22}, and P^{32} with the choice

$$(G_{\frac{3}{2}}^2/96 G_{\frac{1}{2}}^2) \sim 0.008. \tag{9}$$

This admixture is too small to permit attributing the difference between the Fermi and the Gamow-Teller couplings to a spin $\frac{3}{2}$ neutrino mode of the decay.

The distribution in momentum and angle of the electrons emitted in the reactions (4b) and (4d) is

$$N^{-1} dn/dx = (5/2)x^2(1-x) \times [3 - x \pm (1+x)\cos\vartheta] \sin\vartheta d\vartheta, \tag{10}$$

where x is $p/p_{\max}$ and ϑ is the angle between the electron momentum and the initial spin direction. The upper sign applies to reaction (4b) and the lower to (4d). Assuming a normal decay mode of the μ meson in conjunction with either of the spin $\frac{3}{2}$ branches, one finds the energy spectrum to be

$$N^{-1}\frac{dn}{dx} = \frac{[2x^2(3-2x) + (\mu^2 G_{\frac{3}{2}}^2/128 G_{\frac{1}{2}}^2)x^2(1-x)(3-x)]}{[1 + (\mu^2 G_{\frac{3}{2}}^2/640 G_{\frac{1}{2}}^2)]}, \tag{11}$$

where μ is the μ-meson mass. The Michel parameter, defined by

$$\rho = (3/8N)(dn/dx)_{x=1}, \tag{12}$$

is seen to be

$$\rho = \tfrac{3}{4}[1 + (\mu^2 G_{\frac{3}{2}}^2/640 G_{\frac{1}{2}}^2)]^{-1}. \tag{13}$$

For the ratio of the coupling constants given in Eq. (9) ρ would be 0.01 in complete disagreement with the recent measurements of Plano and Lecourtois[10] which give ρ to be 0.79 ± 0.03.

One may conclude that, if there is a competing beta-decay process involving a spin $\frac{3}{2}$ neutrino, it is not universal in the sense that it does not also apply to μ-decay. Langer's observations are consistent with various types of form factors; further measurements especially at low energies might establish whether or not Eq. (8) applies.

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[9] The notation is that of M. E. Rose, in *Beta- and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (Interscience Publishers, Inc., New York, 1955), Chap. IX.

[10] R. J. Plano and A. Lecourtois, Bull. Am. Phys. Soc. **4**, 82 (1959).