section. Below 41.9 Mev, however, the size of this background correction became considerably larger, and, consequently, at the lower energies the error associated with this background correction accounts for most of the tabulated relative error on each cross section.

At the low pressures of helium gas in the chamber, the possibility of the loss of some beam from the Faraday cup by multiple scattering in the gas was negligible, and was calculated to be less than 0.1%.

The errors listed in Table I are relative errors. The absolute errors were compounded separately and result in a ±3.8% error which should be applied to the ordinates of each bombarding energy.

RESULTS

The measured differential cross sections are tabulated with their relative errors in Table I and plotted in Figs. 2 and 3. Angles and cross sections are in the c.m. system. The single prominent minimum seen at 36.85 and 38.83 Mev, gives way to two minima at the higher energies. This transition from one to two minima with increasing energy is also present in the 12- to 23-Mev alpha-alpha data, where resonance scattering from a virtual excited state (4+) around 11 Mev in Be^8 is observed.[11]

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We are grateful to Professor A. C. Helmholz for helpful discussions and support during the course of this work and to Dr. D. J. Prowse and Dr. G. W. Farwell for sending us their results prior to publication. Two of us (G. I. and R. S.) wish to also thank the other staff members of the Lawrence Radiation Laboratory who made it possible for us to participate in this work.

We wish to express our appreciation to the crew of the 60-in. cyclotron under the direction of William B. Jones for their assistance during the cyclotron runs.

[11] R. Nilson, W. K. Jentschke, G. R. Briggs, R. O. Kerman, and J. S. Snyder, Phys. Rev. **109**, 850 (1958).

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Optical Model Analysis of the Scattering of Alpha Particles from Helium*

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An optical model analysis using a complex potential

$$\frac{V+iW}{1+\exp[(r-r_0)/d]}$$

has been made of the elastic scattering of alpha particles from helium. In the data which are analyzed the bombarding energy ranges from 23.1 Mev to 47.1 Mev. The best agreement with the angular distributions taken at eight different bombarding energies was obtained when the parameters V, W, r_0, and d were -112 Mev, -1 Mev (for bombarding energies near 40 Mev), 1.8×10^{-13} cm, and $0.6\pm0.1\times10^{-13}$ cm, respectively. The value -112 Mev for V is an average value; V decreases by 15% when the bombarding energy is increased from 23 Mev to 47.1 Mev. Since W is small, the central depth of the real part of the potential V has significance. This is in contrast to the scattering of alpha particles from heavier elements where the absorption is so large that the central part of the potential is not easily determined. No lower limit was placed on r_0, however, r_0 must be less than 2.7×10^{-13} cm. The phase shifts obtained from this analysis are in good agreement with the preliminary results of Snyder below 42 Mev. Above 42 Mev they continue to vary slowly with no new states of Be^8 appearing up to 47.1 Mev.

INTRODUCTION

THE optical model has been quite successful in describing the scattering of complex particles from nuclei. So far, optical model analyses of the scattering of alpha particles,[1–3] deuterons,[4] and nitrogen ions[5] have been reported. The parameters r_0 and d which enter into the Woods-Saxon potential[6]

$$(V+iW)/\{1+\exp[(r-r_0)/d]\},$$

are expected to be increased due to the finite size of the projectile. By analyzing the scattering of alpha particles from helium, the contribution to r_0 due to the finite size of the alpha particle may be estimated.

The scattering of alpha particles from nuclei[3] (and the scattering of nitrogen ions from nitrogen[5]) determine the real part of the potential in the surface region, and

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† Fulbright Fellow 1958–1959.

[1] G. Igo and R. M. Thaler, Phys. Rev. **106**, 126 (1957).

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[3] G. Igo, Phys. Rev. Letters **1**, 72 (1958), and Phys. Rev. **115**, 1665 (1959).

[4] M. A. Melkanoff (private communication).

[5] C. E. Porter, Phys. Rev. **112**, 1722 (1958).

[6] R. D. Woods and D. S. Saxon, Phys. Rev. **95**, 1617 (1954).

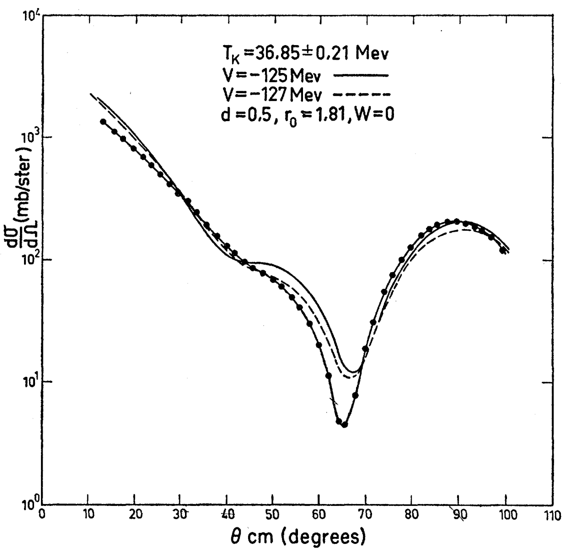

FIG. 1. Angular distribution of alpha particles elastically scattered from helium at T_k=36.85±0.21 Mev. The points through which a solid line have been drawn are from reference 13 and the curves are the optical model calculations. The quantities d and r_0 are in units of 10^{-13} cm.

not the central depth. However this statement does not apply to a helium scatterer when the bombarding energy T_k is less than 50 Mev since it is found in this analysis that the imaginary part of the potential must be close to zero. Consequently the central value of the real part of the potential has significance.

Levels of even spin and parity in Be^8 may be investigated by studying the angular distribution of alpha particles scattered by helium at a series of different energies. An earlier phase-shift analysis[7] of the data up to 23-Mev bombarding energy[7–9] has suggested, in addition to the well-known 1+ state at 2.9 Mev, a further state of spin 4+ at about 11–12 Mev. Preliminary phase-shift analysis by Snyder[10] of the data[11,12] in the energy range up to 42 Mev clearly shows that there is a level of spin 4+ at an energy near 13.0 Mev in Be^8 with a width of 4 Mev. The real part of the phase shifts (the imaginary parts are close to zero) obtained from the potential used in this analysis are in good agreement with the preliminary values of Snyder[10] in the energy range 23 to 42 Mev. In addition, the phase shifts obtained from this analysis of the

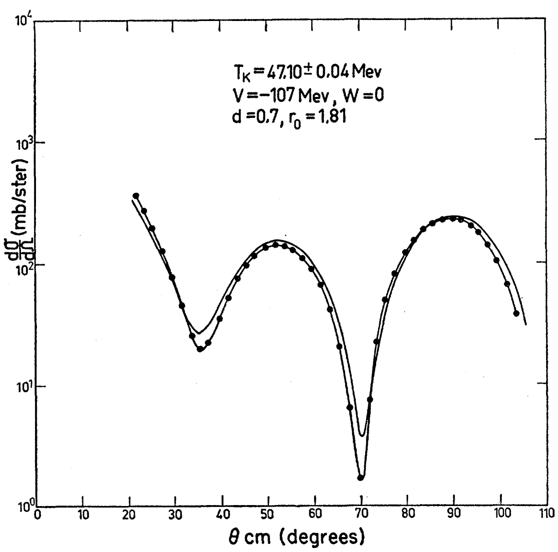

FIG. 2. Angular distribution of alpha particles elastically scattered from helium at T_k=47.10±0.04 Mev. The points through which a solid line have been drawn are from reference 13 and the curves are the optical model calculations. The quantities d and r_0 are in units of 10^{-13} cm.

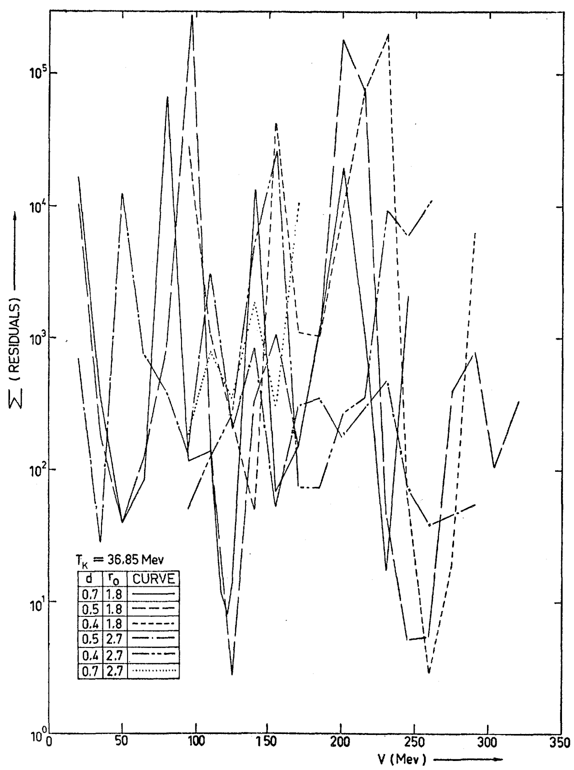

FIG. 3. The sum of the residuals of the least-square analyses at T_k=36.85 Mev *versus* V for various values of $d(\times 10^{-13}$ cm), $r_0(\times 10^{-13}$ cm), and for W=0.

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[7] R. Nilson, W. K. Jentschke, G. R. Briggs, R. O. Kerman, and J. N. Snyder, Phys. Rev. **109**, 850 (1958).

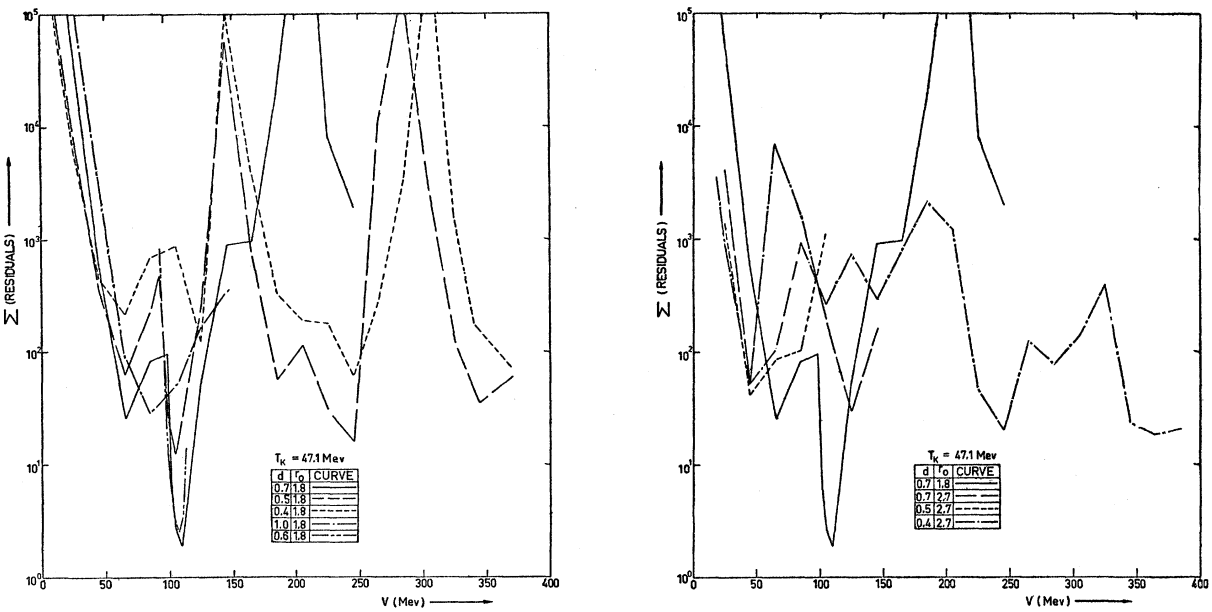
[8] J. L. Russell, Jr., G. C. Phillips, and C. W. Reich, Phys. Rev. **104**, 135 (1956).

[9] N. P. Heydenberg and G. M. Temmer, Phys. Rev. **104**, 123 (1956).

[10] W. E. Burcham, J. S. McKee, W. M. Gibson, D. Bredin, D. Evans, D. J. Prowse, and J. Rotblat, *Compt. rend. du Congres International de Physique Nucléaire Interactions Nucléaires aux Basses Energies et Structure des Noyaux, Paris, July*, 1958, edited by P. Guggenberger (Dunod, Paris, 1959), p. 366; and O. J. Prowse (private communication).

[11] W. M. Gibson, D. J. Prowse, and J. Rotblat, Nature **173**, 1180 (1954); Proc. Roy. Soc. (London) **A243**, 237 (1957).

[12] G. W. Farwell and A. I. Yavin (private communication).



(a) (b)

FIG. 4. (a) The sum of the residuals of the least-square analyses at $T_K=47.10$ Mev *versus* V for various values of $d(\times 10^{-13}$ cm), $r_0=1.8\times 10^{-13}$ cm, and for $W=0$. (b) The sum of the residuals of the least-square analyses at $T_k=47.10$ Mev *versus* V for various values of $d(\times 10^{-13}$ cm), $r_0(\times 10^{-13}$ cm), and for $W=0$.

data above 42 Mev[13] continue to vary slowly with energy with no new state of Be^8 appearing.

NUMERICAL CALCULATIONS

The starting point of the analysis of the data is the optical model of the nucleus: the elastic scattering of the alpha particles from helium is calculated on the assumption that a point particle of charge two and with the mass of an alpha particle is scattered by a smooth nuclear potential which may have both real and imaginary parts plus the electric potential from a charge distribution characterized by a half-value parameter of 1.8×10^{-13} cm and an exponential dependence on radial distance.[14] This kind of analysis is familiar in nuclear physics, and it is not necessary to elaborate on the actual mechanics of the calculation beyond saying that it involves a partial wave analysis of the Schrödinger equation which makes essentially no approximation. Initially it is necessary to delineate the values of the four parameters characterizing the complex nuclear potential

$$\frac{V+iW}{1+\exp[(r-r_0)/d]}.$$

A preliminary four-parameter investigation was made to locate the regions in this four-dimensional space where the least-square residuals used as a criterion to

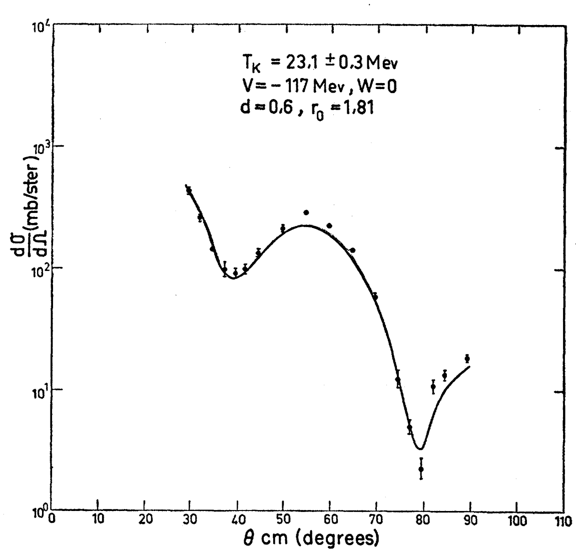


FIG. 5. Angular distribution of alpha particles elastically scattered from helium at $T_k=23.1\pm 0.3$ Mev. The quantities d and r_0 are in units of 10^{-13} cm. The points are from reference 10 and the curve is the optical model calculation.

[13] H. E. Conzett, G. Igo, H. C. Shaw, and R. J. Slobodrian, Bull. Am. Phys. Soc. 2, 305 (1957), and the preceding paper [Phys. Rev. **117**, 1075 (1960)].

[14] The half-value parameter is larger than 1.7×10^{-13} cm obtained in the Stanford electron scattering experiments [R. Hofstadter, *Comptes Rendue du Congres International de Physique Nucléaire Interactions Nucléaires aux Basses Energies et Structure des Noyaux, Paris, July*, 1958, edited by P. Guggenberger (Dunod, Paris, 1959), p. 47] to partially compensate for the finite charge distribution of the incident particle. However the electric potential is always overshadowed by the nuclear potential in the angular range where measurements exist. Consequently detailed assumptions about the charge distribution do not alter the angular distributions predicted in this analysis.

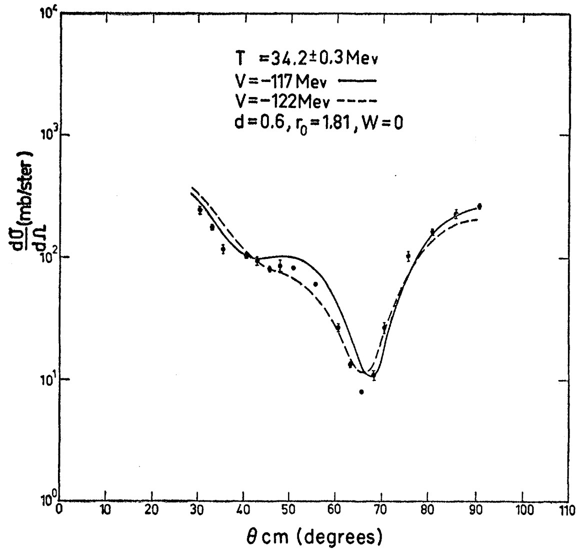


FIG. 6. Angular distribution of alpha particles elastically scattered from helium at $T_k=34.2\pm0.3$ Mev. The quantities d and r_0 are in units of 10^{-13} cm. The points are from reference 10 and the curves are the optical model calculations.

compare the calculation with the 41.9-Mev data would be small. It was soon obvious that the imaginary potential depth W would have to be small. The sharp minima in the 41.9-Mev angular distribution[13] (see the preceding paper) were always poorly fitted with $W\neq0$ since the effect of the imaginary part of the potential was to fill in the minima in the angular distribution.

However, it is known experimentally that the reaction cross section is not zero,[15] and consequently the imaginary part must be finite. A value of W of -1 Mev will give a reaction cross section of 155 mb at $T_k=41.9$ Mev. This is in reasonable agreement with the measured reaction cross section of 44 ± 3 mb at $T_k=38.5$ Mev since the reaction cross section will increase rapidly near threshold. Consequently, a small value of W (about 1% of V) will suffice to account for the absorption. The imaginary parts of the $l=0$, 2, and 4 phase shifts obtained with $W=-1$ Mev are about two degrees at $T_k=41.9$ Mev. The preliminary analysis also showed that the real values of the phase shifts are virtually unaffected by the presence of an imaginary potential with a depth of -1 Mev.

In order to fix the three remaining parameters (with W set to zero), the 36.85-Mev data[13] (Fig. 1) which exhibits one minimum in the angular range up to 90 degrees (c.m.) and the 47.1-Mev data[13] (Fig. 2) which shows two minima in this same range were subjected to analyses in which V, d, and r_0 were varied. Figures 3 and 4 summarize the results of the least-square analyses. The least-square residuals is plotted against V for three different values of d and two values of r_0 in Fig. 3. In Fig. 4 the least-square residuals are plotted for the 47.1-Mev data for five values of d and two values of r_0. At both energies the smaller radius 1.8×10^{-13} cm is favored over the larger radius 2.7×10^{-13} cm. At both energies values of d between 0.5 and 0.7×10^{-13} cm give the minimum least-square residuals.

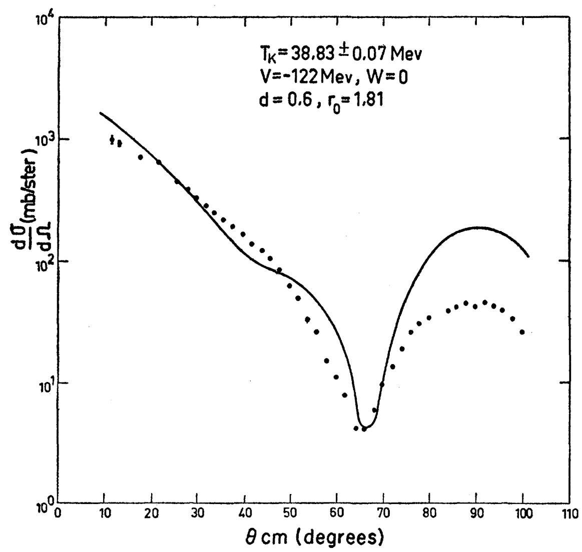


FIG. 7. Angular distribution of alpha particles elastically scattered from helium at $T_k=38.83\pm0.07$ Mev. The quantities d and r_0 are in units of 10^{-13} cm. The points are from reference 13 and the curve is the optical model calculation.

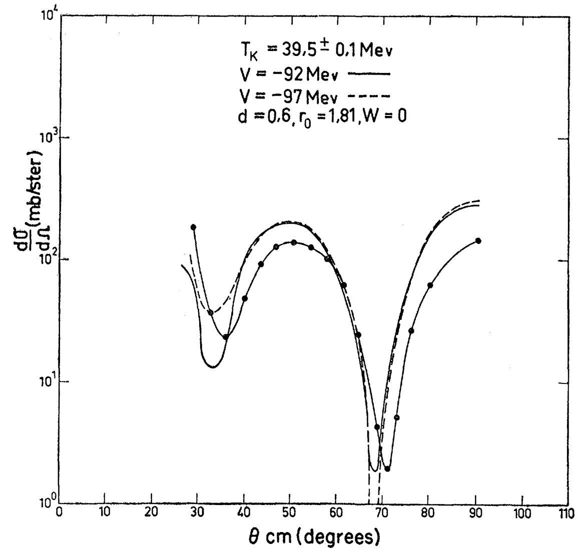


FIG. 8. Angular distribution of alpha particles elastically scattered from helium at $T_k=39.5+0.1$ Mev. The points through which a solid line have been drawn are from reference 13 and the other curves are the optical model calculations. The quantities d and r_0 are in units of 10^{-13} cm.

[15] W. E. Burcham, J. S. McKee, W. M. Gibson, D. Bredin, D. Evans, D. J. Prowse, and J. Rotblat, Nuclear Phys. **5**, 141 (1958).

At both energies the deepest minima in the least-square residuals occur at about −115 Mev, −250 Mev, and perhaps at larger values. The analyses were restricted, however, to values of $-V$ of less than 350 Mev. Figures 1 and 2 show the best fits obtained in the vicinity of the first minimum. Values of d, r_0, and W were fixed at 0.6×10^{-13} cm, 1.8×10^{-13} cm, and 0, respectively, and V was varied in small steps between −92 Mev and −127 Mev to compare with the data at 23.1+0.3 Mev,[11] 34.2+0.21 Mev,[11] 38.83

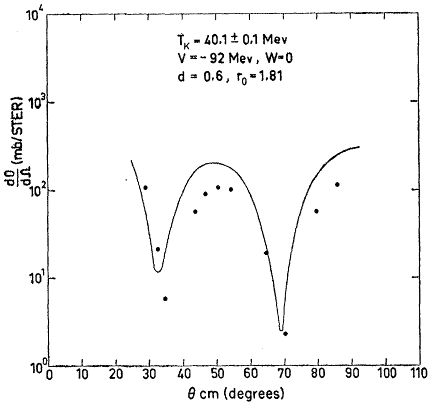


FIG. 9. Angular distribution of alpha particles elastically scattered from helium at $T_k=40.1\pm0.1$ Mev. The quantities d and r_0 are in units of 10^{-13} cm. The points are from reference 12 and the curve is the optical model calculation.

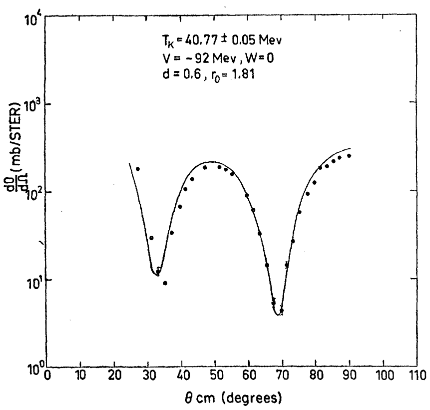


FIG. 10. Angular distribution of alpha particles elastically scattered from helium at $T_k=40.77\pm0.05$ Mev. The quantities d and r_0 are in units of 10^{-13} cm. The points are from reference 13 and the curve is the optical model calculation.

+0.07 Mev,[13] 39.5+0.1 Mev,[12] 40.1+0.1 Mev,[12] and 40.77+0.05 Mev.[13] The best fits obtained are shown in Figs. 5–10. No attempt was made to improve these latter fits by making small variations in other parameters besides V, since computing time was limited. However, the analysis shows that data ranging from 23 Mev to 47 Mev[11–13] can be fit reasonably well with values of the parameters near those which give the best fits at 36.85 Mev and at 47.1 Mev.

The reason that these markedly different angular

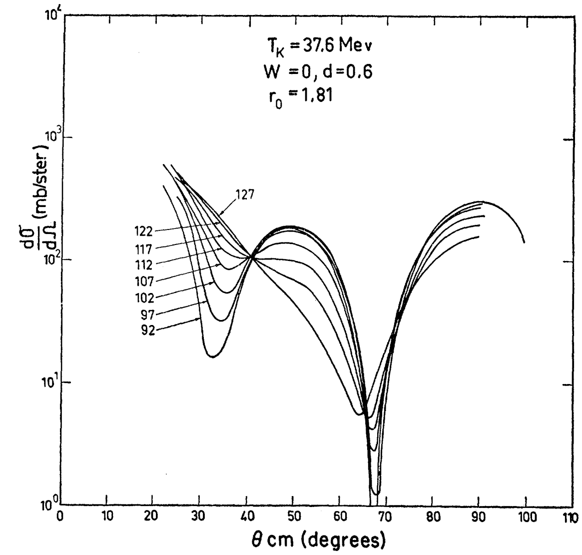


FIG. 11. The calculated angular distributions of 37.6-Mev alpha particles scattered from helium for several values of V. The quantities d and r_0 are expressed in units of 10^{-13} cm, and the curves are labeled by V in Mev.

distributions can be reproduced by making relatively small changes in the real part of the optical model potential while keeping all other parameters fixed can be seen by an examination of Fig. 11 where calculated distributions for different values of the potential in the range $V=-92$ Mev to -127 Mev are plotted. The distribution changes from a two minimum to a one minimum shape as the depth is increased. Figure 12 illustrates the slow variation with T_k of the angular distribution when the potential is kept constant, and with Fig. 11 illustrates that the potential depth must

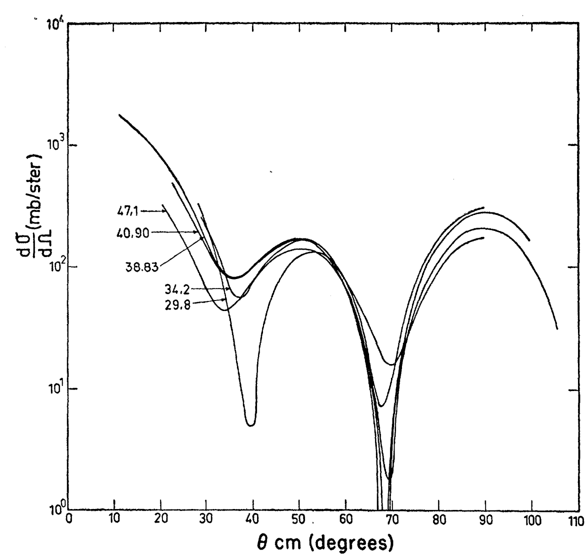


FIG. 12. The calculated angular distributions of alpha particles of several different bombarding energies scattered from helium. The quantities d and r_0 are expressed in units of 10^{-13} cm.

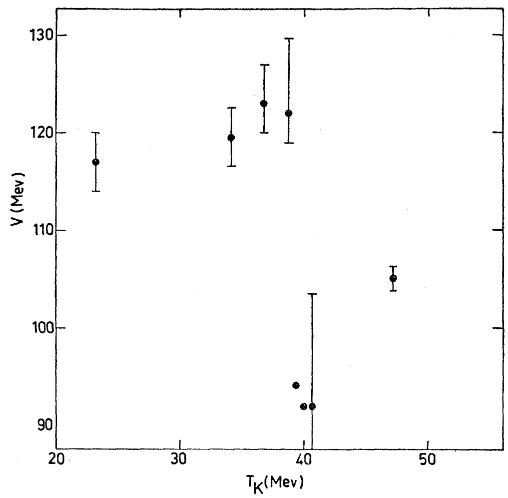


FIG. 13. The dependence of the depth of the real potential V on the bombarding energy T_k. The flags indicate the uncertainty in the determination of V in the analysis. The agreement with the $T_k=39.5$ and 40.1-Mev data was marginal, and no reliable estimate of the uncertainty in V can be made.

be energy dependent. The dependence of V on T_k is shown in Fig. 13 for $d=0.6$ and $r_0=1.8\times10^{-13}$ cm. As T_k increases above 39 Mev the potential depth drops by about 15% in the next 10 Mev. The flags represent the uncertainty in the best values of V in this analysis. At $T_k=39.5$ and 40.1 Mev, the agreement with experiment was not good enough to allow an estimate of the uncertainty (see Figs. 8 and 9). The rapid variation of the differential cross section just below 40 Mev seen clearly in the data of Farwell and Yavin[12] cannot be reproduced without making relatively rapid changes in the potential depth. This is not surprising since the optical model is expected to reproduce only gross structure, and therefore only after a suitable average over an energy interval in this region can a fit be expected.

It is interesting to note in passing that the phase shifts obtained from this analysis in the 23- to 42-Mev range are in good agreement with the preliminary values obtained by Snyder[10] except for the behavior of the $l=8$ phase shift near 40 Mev. In this region the $l=8$ phase shift is small in both analyses. In particular the existence of a 4+ state in Be^8 at 14 Mev is indicated by the behavior of the $l=4$ phase shift in agreement with Snyder's preliminary results.[10] The behavior of the phase shifts above 40 Mev is shown in Fig. 14 by the dashed line extensions. It is seen that the phase shifts vary smoothly up to 47 Mev with no new state in Be^8 appearing. The $l=6$ phase shift is rising above 40 Mev indicating a possible 6+ state at about 60 Mev. Since what is determined in this analysis is the tangent of the phase shift it is possible to determine the phase shift only within integer multiples of π. Thus in Fig. 14 δ_4, the phase shift for $l=4$ may continue to increase in the vicinity of 40 Mev where the phase shifts are poorly determined, pass through a maximum and drop to 145 degrees at $T_k=47.1$ Mev. It might also be argued that δ_0 should be increased by π since the total potential is mainly attractive.

In Table I, the Woods-Saxon potential[6] parameters and phase shifts obtained in this analysis are listed. In addition to the parameters listed in Table I, values obtained for r_0 and W are 1.81×10^{-13} cm and -1 Mev, respectively. The imaginary parts of the $l=0$, 2, and 4 phase shifts are of the order of 2 degrees when T_k is near 40 Mev.

CONCLUSIONS

The real parts of the phase shifts obtained in this analysis are in good agreement with the preliminary values obtained by Snyder below 42 Mev (with the exception of the $l=8$ phase-shift behavior near 40 Mev). Above 42 Mev the phase shift continues to vary smoothly and no further state in Be^8 is found up to 47.1 Mev. The imaginary parts of the $l=0$, 2, and 4 phase shifts are about two degrees near $T_k=40$ Mev and the $l=6$ and 8 imaginary parts are negligible. The effect of the introduction of a nonzero W of

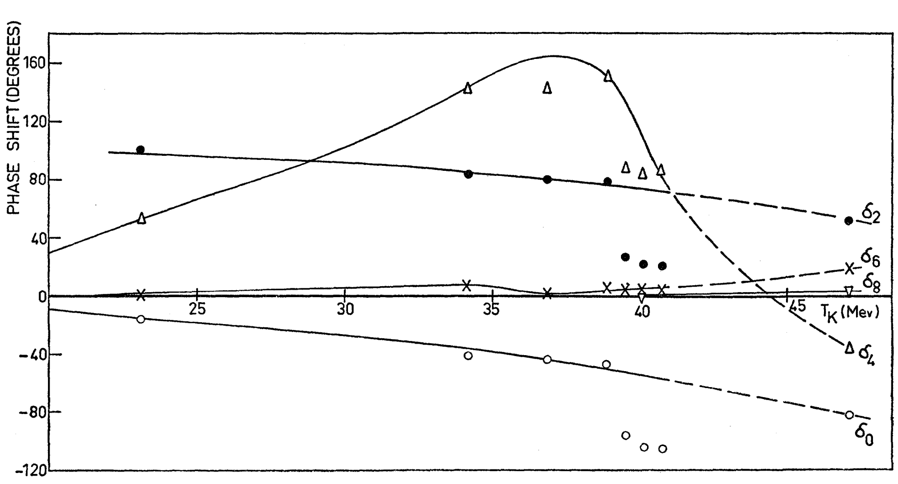


FIG. 14. The dependence of the phase shifts for $l=0$, 2, 4, 6, and 8 partial waves as a function of T_k. The dashed lines are discussed in the text. The symbols o, ●, Δ, x, and ∇ are, respectively, the values of the phase shifts for $l=0$, 2, 4, 6, and 8 obtained in this analysis.

TABLE I. Woods-Saxon potential[a] parameters and phase shifts.

T_k (Mev)	$-V$ (Mev)	d (10^{-13} cm)	tan δ_0	tan δ_2	tan δ_4	tan δ_6	tan δ_8 ($\times 10^{-3}$)	δ_0 (deg)	δ_2 (deg)	δ_4 (deg)	δ_8 (deg)	δ_8 (deg)
23.1 ±0.3	117	0.6	−0.287	−43.4	1.34	−0.0110	0.21	−16.0	10.1	53.3	0.6	0.012
24.2	117	0.6	−1.02	4.046	−0.924	0.062	2.8	−45.6	76.1	137	3.6	0.16
±0.3	122	0.6	−0.744	11.40	−0.588	0.065	2.9	−36.6	85	149	3.7	0.16
36.85	125	0.5	−1.00	4.74	−0.864	0.035	1.3	−45.0	78.7	139	2.0	0.07
±0.27	127	0.5	−0.882	6.68	−0.705	0.036	1.3	−41.4	81.5	145	2.0	0.07
38.83 +0.07	122	0.6	−1.06	5.15	−0.495	0.105	6.2	−46.6	78.8	154	6.0	0.35
39.5	92	0.6	4.23	0.422	7.41	0.083	5.5	−103	22.2	82.3	4.8	0.31
±0.1	97	0.6	16.1	0.641	−11.5	0.088	5.7	−93.6	32.6	95.0	5.0	0.32
40.1 ±0.1	92	0.6	3.84	0.409	10.6	0.089	6.1	−104.6	22.3	84.6	5.1	0.35
40.77 ±0.05	92	0.6	3.57	0.399	15.8	0.093	6.6	−105.6	21.8	86.0	5.4	0.38
47.10 ±0.04	107	0.7	−6.29	1.27	−0.687	0.338	49.1	−81	51.8	−34.5	18.7	2.8

[a] See reference 6.

magnitude necessary to reproduce the magnitude of the total reaction cross section[15] has a negligible effect on the real part of the phase shifts.

The depth of the real part of the nuclear potential V is found to be energy dependent in the range of T_k between 23 and 47 Mev with an average value of about −112 Mev. In the analysis of the elastic scattering of alpha particles from heavier targets, thecentral potential was ambiguous because of large absorption. In the present analysis W is less than 1% of V and the depth of the central potential, −112 Mev, becomes meaningful. A depth approximately twice as large is not ruled out; however, shallow real potentials are ruled out. This result is in agreement with the result of an analysis of alpha-particle decay systematics[16] and in disagreement with the results of an analysis of the inelastic scattering of alpha particles.[17]

The parameter r_0 represents some average over the potential distribution of the incident alpha particle and the helium scatterer. As a starting point, one might assume that the potential distribution of an alpha particle was characterized by the same radial parameter r_c found to characterize the charge distribution for the helium nucleus, 1.7×10^{-13} cm.[14] The quantity $(2r_c^2)^{\frac{1}{2}}$ is 2.4×10^{-13} cm which is compatible with the results of this analysis. The quantity $2r_c$ is however incompatible with the results of this analysis since it is larger than 2.7×10^{-13} cm, which does not reproduce the data. Since it is known from this analysis that $r_0<2.7\times10^{-13}$ cm, $r_0=(2r_c^2)^{\frac{1}{2}}$ and smaller values are not excluded. Further calculations at an intermediate value of r_0 are necessary to set a lower limit on the effective radius for the alpha particle.

[16] H. A. Tolhoek and P. J. Brussard, Physica **21**, 449 (1955).
[17] P. C. Gugelot and M. Rickey, Phys. Rev. **101**, 1613 (1956).

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