where c is a constant. As Sartori and Rubinow have already observed, the two coupled differential equations satisfied by ψ' and ψ'' [Eqs. (14) and (15) of reference 4] may be combined to yield the Schrödinger equation for the system, in which the spin and isotopic spin variables have been removed [see Eq. (10) of reference 7]. Verde defines an integral J, involving ψ' and ψ'', which vanishes if the correct functions are used and whose first variation is zero. With the aid of Eq. (A.4) and the symmetry properties of ψ' and ψ'' it may be shown that

$$J=\text{const}\int \Psi\Lambda\Psi d\tau.$$

The normalization used by Troesch and Verde in their application of the Verde principle is equivalent to the choice $\theta=\pi/2$ [see Eq. (2.2)], i.e., it corresponds to that proper to the Kohn principle. The variational parameters were evaluated according to the prescription of Hulthén, i.e.,

$$J=0,$$
$$\partial J/\partial a_i=0, \quad i=1, 2, \cdots, N,$$

where the a_i are the N variational parameters in the trial function, excluding the trial scattering length. As discussed in I, the Kohn method for evaluating the parameters, in a problem in which it provides a bound, is superior to that of Hulthén but the results in the latter case still give an upper bound on the true scattering length.

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Electron Capture Decay of Tm^{168} and Tm^{166}

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The electron capture of Tm^{168} (87 day) to levels in Er^{168} was investigated with permanent magnet spectrographs and coincidence scintillation spectrometers. The following levels in Er^{168} have been established: 79.8(2+), 264.3(4+), 548.9(6+), 822.4(2+), 897.0(3+), 996.2(4+), 1095.1(3− and $T_{\frac{1}{2}}=1.2\times10^{-7}$ sec), and 1543.1(3)−) kev. The internal conversion data for Tm^{166} (7.7 hour) suggest levels in Er^{166} at 80.6(2+), 265.1(4+), 545.3(6+), 787.1(2+), 860.6(3+), and 957.2(4+) kev. with many more high lying levels. Energy level schemes are proposed for both Er^{168} and Er^{166}. The levels at 822, 897, and 996 kev in Er^{168} and 787, 861, and 957 kev in Er^{166} may possibly be associated with electric quadrupole (gamma) vibrations. Some general features regarding these vibrational levels are discussed and compared with available data on other even-even nuclei in the rare earth region.

I. INTRODUCTION

PRELIMINARY results of a survey of the radioactivities of neutron deficient rare earth isotopes have been reported previously.[1–5] This paper is concerned with a more detailed study of the energy levels of two even-even nuclei of this region, Er^{168} and Er^{166} which are reached by electron capture of the neutron deficient isotopes Tm^{168} and Tm^{166}, respectively. These nuclei are in the region where the nuclei are strongly deformed and have spheroidal equilibrium shape. According to the unified model[6–8] these are expected to exhibit energy levels which are collective in nature. This study was undertaken to obtain more data on such collective levels.

II. RELEVANT PREDICTIONS OF THE UNIFIED MODEL

The lowest modes of collective excitation of deformed nuclei correspond to rotations in which the nuclear shape remains unchanged. The resulting energy spectrum for a deformed even-even nucleus is given by

$$E_I=(\hbar^2/2\mathcal{I})I(I+1)+BI^2(I+1)^2, \tag{1}$$

where $I=0$, 2, $4\cdots$. The quantity $\mathcal{I}$ represents an effective moment of inertia. The second term in (1) is a correction term due to rotation-vibration interaction.[7] The quantity B is related to the vibrational quanta $\hbar\omega_\beta$ and $\hbar\omega_\gamma$ associated with the so called beta and gamma

* Supported by the U. S. Atomic Energy Commission.
† Operated for the U. S. Atomic Energy Commission by the Union Carbide Nuclear Company.
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8 Alder, Bohr, Huus, Mottelson, and Winther, Revs. Modern Phys. **28**, 432 (1956).

vibrations and is given by

$$B=\left(\frac{12}{(\hbar\omega_\beta)^2}+\frac{4}{(\hbar\omega_\gamma)^2}\right)\left(\frac{\hbar^2}{2\mathcal{J}}\right)^3. \qquad (2)$$

The ground-state rotational band of an even-even nucleus, given by (1), is characterized by $K=0$ (K is the projection of total angular momentum I, on the nuclear symmetry axis) and even parity.

Nuclei can also be excited by collective vibrations about their equilibrium shapes. The lowest order shape vibrations are of quadrupole type. In even-even nuclei of spheroidal shape, these vibrations give rise to sequences of rotational states with $I=0, 2, 4\cdots$ (beta vibrations) and $I=2, 3, 4\cdots$ (gamma vibrations). For the beta vibrational spectrum $K=0$, while $K=2$ for the gamma vibrations. $M1$ radiation is forbidden in the decay of vibrational states even when $\Delta I=0$ or 1.

Deformed even-even nuclei may also give rise to higher order shape vibrations, such as octupole vibrations. Octupole vibrational states are expected to have odd parity and $K=0$, 1, 2, or 3. The associated rotational levels will have $I=K$, $K+1$, $K+2\cdots$ with the exception of the $K=0$ case, where the sequence is expected to be 1, 3, $5\cdots$.

The gamma branching ratios in the emission of various multipole order (henceforth abbreviated as M.O.) radiations which are not K forbidden, from a given state to different members of a rotational band, have been treated theoretically by Alaga *et al.*[9] and it has been shown that the ratios of the reduced transition probabilities (henceforth abbreviated as R.T.P.) can be expressed in terms of vector addition coefficients involving only the M.O. and the spins and K values of the states in question.

TABLE I. Conversion electron data for decay of Tm^{168} to Er^{168}.

Transition energy (kev)	K	$L_{\rm I}$	$L_{\rm II}$	$L_{\rm III}$	M	N	Remarks[b]
	Intensity (N_e)[a]						
74.6	*w*	*w*	~4.5	~4.5	d	*w*	$E2+(M1)$?
79.8	>300	c	1000c	960	470	130	$E2(2+\rightarrow 0+)$
99.0	~3c	d	d	6	<4d	<1d	$E2+(M1)$?
99.3	40	<10d	⩽2	2	d	d	$(E1)$?
173.7	*w*						
184.5	200	c	52c	42	25	8	$E2(4+\rightarrow 2+)$
198.4	140	20	⩽3.5c	3.5	5	*w*	$(E1)$
273.0	4.0	0.65		*w*	*w*		$(E1)$?
284.6	0.4						$(6+\rightarrow 4+)$?
348.8	0.6	*w*					
421.9	0.8	*w*			*w*		
447.7	58	9			2.2		
546.7	~0.7						
557.7	*w*						
632.6	5.7	0.8			0.2		
646.2	~0.4						
720.7	2.0	0.3					
731.9	2.1	~0.4					
742.7	4.5	0.7			~0.2		
817.2	14	2.2			~0.5		
822.4	3.5	~0.45			d		
831.2	0.7	d					
916.6	0.7	~0.1					
1015.7	~0.05						
1279.5	~0.12						
1463.9	*w*						

[a] Intensity data are arbitrarily normalized to 1000 units for the most prominent line. "*w*" indicates a weak line.
[b] Multipole assignments were made on the basis of K/L or L ratios.
[c] Conversion line is not completely resolved.
[d] Conversion line is a composite of two lines.

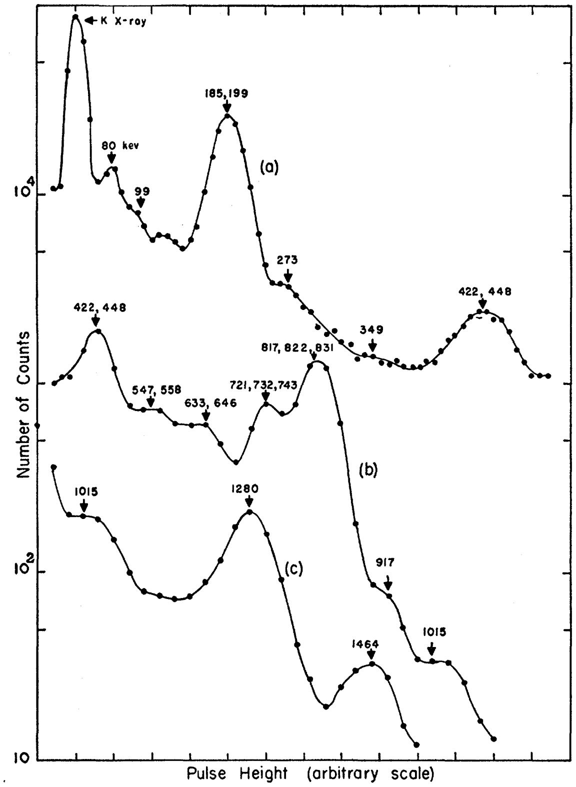

FIG. 1. Gamma-ray spectrum of Er^{168}. (a) Low-energy spectrum, obtained with 1 in.×1 in. NaI crystal. (b) and (c) High-energy spectrum, obtained with 2 in.×2 in. NaI crystal.

III. EXPERIMENTAL METHODS

The conversion electron data were obtained using 180° permanent magnet photographic recording spectrographs. The spectrographs of field strengths 64, 127, 180, and 360 gauss are capable of recording conversion lines of from 6 to 1600 kev. The electron intensities were determined photometrically. Gamma-ray measurements were made with NaI(Tl) scintillation spectrometers. The fast-slow coincidence apparatus used in this work has been discussed elsewhere.[10] The radioactive sources were made by proton irradiation in the Oak Ridge National Laboratory 86-inch cyclotron.[11] Ion exchange separations were performed and the spectrograph sources were made by electrolysis.

[9] Alaga, Alder, Bohr, and Mottelson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **29**, No. 9 (1955).
[10] Mihelich, Ward, and Jacob, Phys. Rev. **103**, 1285 (1956).
[11] Harmatz, Handley, and Mihelich, Phys. Rev. **114**, 1082 (1959).

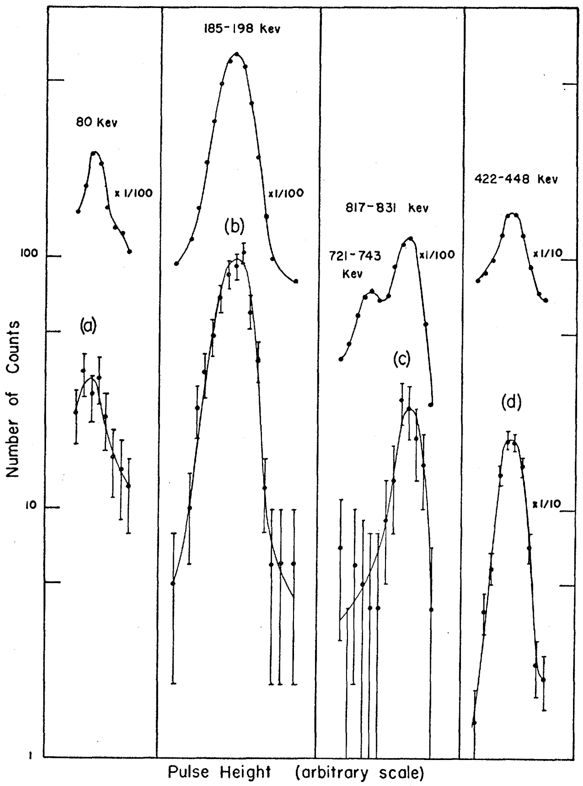

FIG. 2. Results of delayed coincidence measurements made with gamma rays of Er.[168]. (a), (b), and (c) show radiations succeeding the delayed state, while (d) shows those preceding the delayed state ($T_{\frac{1}{2}}=1.2\times10^{-7}$ sec).

IV. ERBIUM-168

It has been shown that Tm^{168} decays to levels in Er^{168} via electron capture with a half-life of 87 days.[12,13] In addition a metastable level in Er^{168} with a half-life of 1.2×10^{-7} sec has been observed.[2] Preliminary results on this decay scheme were reported previously.[3] The levels in Er^{168} have also been studied by resonant neutron capture[14,15] in Er^{167}.

The conversion electron data for Tm^{168} are presented in Table I. The uncertainties in the measured energies and intensities presented in this paper are the same as those discussed previously.[1] A typical gamma-ray spectrum of Er^{168} as obtained with a scintillation spectrometer is shown in Fig. 1. The relative gamma-ray intensities were obtained by analyzing a number of spectra in the conventional way starting with the highest energy gamma-ray peak and extrapolating the Compton distribution and finally correcting for the peak efficiency of the crystal.[16,17] The photon intensities are presented in Table IV together with other data. The photon intensities of transitions other than those listed in Table IV are considered to be too small to affect the scheme proposed on the basis of the more intense transitions. The photon intensities are good to within ±25%, except in the case of low energy and low intensity gamma rays, where the errors may be larger.

In view of the fact that there is an isomeric level for this nucleus, spectra of radiations both succeeding and preceding the delayed state were obtained using delayed coincidence techniques. The results are shown in Fig. 2. Radiations succeeding the delayed state are

TABLE II. Results of prompt coincidence measurements.

Gating with	Coincides with
185–198 kev	80, 185–198, 721–743, 817–831, and possibly with 633–646-kev peaks
422–448 kev	No coincidences with 421–448-kev peak
721–743 kev	80, 185–198, 721–743, and 817–831-kev peaks
817–831 kev	80, 185–198, and 721–743-kev peaks. No coincidences with the 817–831-kev peak

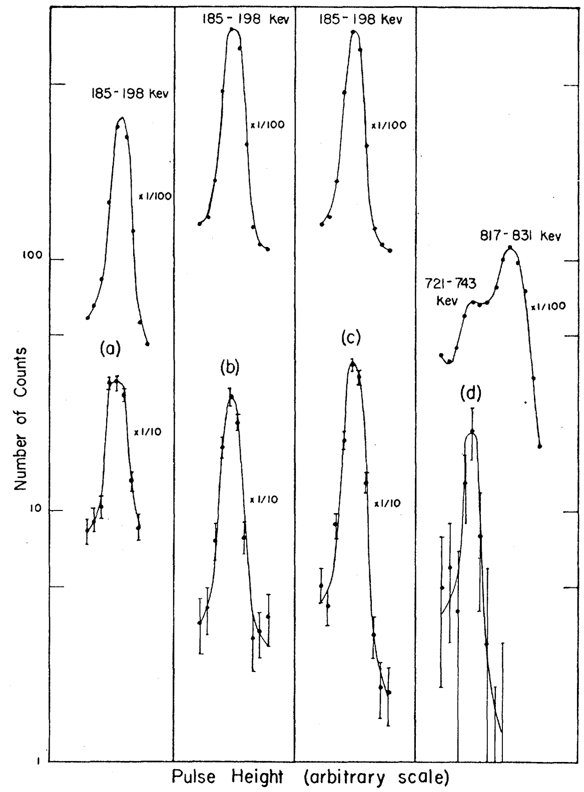

FIG. 3. Results of prompt coincidence measurements made with gamma rays of Er^{168}. (a) Coincidences with 185–198 kev. (b) Coincidences with 721–743 kev. (c) and (d) Coincidences with 817–831 kev.

[12] T. H. Handley and E. L. Olson, Phys. Rev. **90**, 500 (1953).
[13] M. C. Michael and D. H. Templeton, Phys. Rev. **93**, 1422 (1954).
[14] Fenstermacher, Draper, and Bockelman, Nuclear Phys. **10**, 386 (1959).
[15] Skliarevskii, Stepanov, and Obiniakov, Atomnaya Energ. **4**, 22 (1958).
[16] P. R. Bell, *Beta- and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North-Holland Publishing Company, Amsterdam, 1955), Chap. V.
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TABLE III. Theoretical I.C.C. for transitions in Er^{168}.

Transition (kev)	K shell (α_K)				L shell (α_L)			
	E1	E2	M1	M2	E1	E2	M1	M2
80	5.0(−1)[a]	1.7(0)	4.5(0)	4.6(1)	7.8(−2)	4.0(0)	6.6(−1)	1.5(1)
185	5.5(−2)	2.0(−1)	4.2(−1)	2.4(0)	7.5(−3)	1.0(−1)	5.7(−2)	5.1(−1)
198	4.6(−2)	1.6(−1)	3.5(−1)	1.9(0)	6.2(−3)	7.1(−2)	4.7(−2)	3.6(−1)
273	2.0(−2)	6.6(−2)	1.5(−1)	6.3(−1)	2.8(−3)	2.1(−2)	2.0(−2)	1.2(−1)
349	1.1(−2)	3.5(−2)	7.9(−2)	2.8(−1)				
448	6.1(−3)	1.8(−2)	4.1(−2)	1.2(−1)	8.4(−4)	3.7(−3)	5.4(−3)	2.3(−2)
547	3.9(−3)	1.1(−2)	2.5(−2)	7.0(−2)				
558	3.8(−3)	1.0(−2)	2.4(−2)	6.6(−2)				
633	3.0(−3)	7.8(−3)	1.7(−2)	4.6(−2)	3.8(−4)	1.4(−3)	2.3(−3)	7.7(−3)
646	2.8(−3)	7.4(−3)	1.6(−2)	4.3(−2)				
721	2.2(−3)	5.8(−3)	1.2(−2)	3.2(−2)	2.9(−4)	9.5(−4)	1.6(−3)	5.2(−3)
732	2.1(−3)	5.6(−3)	1.2(−2)	3.0(−2)	2.9(−4)	9.2(−4)	1.6(−3)	5.1(−3)
743	2.0(−3)	5.4(−3)	1.1(−2)	2.9(−2)	2.8(−4)	8.9(−4)	1.5(−3)	4.9(−3)
817	1.7(−3)	4.4(−3)	8.6(−3)	2.2(−2)	2.3(−4)	6.9(−4)	1.2(−3)	3.6(−3)
822	1.7(−3)	4.3(−3)	8.4(−3)	2.2(−2)	2.2(−4)	6.8(−4)	1.2(−3)	3.6(−3)
831	1.6(−3)	4.2(−3)	8.2(−3)	2.1(−2)				
917	1.4(−3)	3.4(−3)	6.5(−3)	1.6(−2)	1.8(−4)	5.2(−4)	9.8(−4)	2.6(−3)

[a] All I.C.C. obtained using Rose's tables of I.C.C. (reference 18). The numbers in parenthesis indicate the power of 10 by which the preceding figures should be multiplied.

those of 80, 185+198, 817+822, 831 kev, while photons of 422 and/or 448 kev precede the delayed state. Prompt coincidence measurements were made gating with 185–198, 421–448, 721–742, and 817–831-kev composite peaks. Some of the results are shown in Fig. 3. The results are summarized in Table II.

The theoretical and experimental internal conversion coefficients (henceforth abbreviated as I.C.C.) for the various gamma rays are presented in Tables III and IV, respectively. All theoretical I.C.C. were obtained using Rose's[18] tables of I.C.C. In obtaining the I.C.C. for the various gamma rays from the relative photon and electron intensities, the 80-kev transition is taken to be pure $E2$ and all experimental I.C.C. are normalized to the theoretical value, α_L (80 kev) = 4.0. There are many cases where two or more photon peaks are not resolved with the scintillation spectrometer. However, by assuming various trial M.O.'s for the transitions in

TABLE IV. Transition data for the decay of Tm^{168}.

Transition (kev)	Photon intensity		ϵK	ϵL	M.O.	Transition intensity
K-x-ray						6000
80	490	490	≥6.1(−1)	{4.0(0)}[a]	E2	3880[b]
185 }	3330	1000	{2.0(−1)}	9.4(−2)	E2	1330
198 }		2330	6.0(−2)	1.2(−2)	E1	2500
273	≤480	≤480	≥8.4(−3)	≥1.4(−3)	E1?	≤485
422 }	1390	≤110	{7.0(−3)}		E1	≤110
448 }		1280	4.5(−2)	7.0(−3)	M1	1345
547 }	330	≤330	≥2.0(−3)		E1	≤330
558 }					E2?	
633 }	710	570 [700][c]	1.0(−2) [7.9(−3)]	1.4(−3) [1.1(−3)]	E2+M1 [E2]	575 [710]
646 }		140 [10]	{2.8(−3)} [{4.3(−2)}]		E1 [M2]	140 [10]
721 }	780	60 [200]	{3.2(−2)} [9.9(−3)]	4.8(−3) [1.5(−3)]	M2 [E1+M2]	60 [200]
732 }		230 [170]	9.1(−3) [{1.2(−2)}]	1.7(−3) [2.4(−3)]	E2+M1 [M1]	235 [175]
743 }		490 [410]	9.1(−3) [{1.1(−2)}]	1.4(−3) [1.7(−3)]	E2+M1 [M1]	495 [415]
817 }	2940	1685 [2090]	8.3(−3) [6.7(−3)]	1.3(−3) [1.0(−3)]	M1+E2 [M1+E2]	1700 [2105]
822 }		815	{4.3(−3)}	5.5(−4)	E2	820
831 }		440 [35]	1.6(−3) [2.1(−2)]		E1 [M2]	440 [35]
917	185	185	3.8(−3)	5.4(−4)	E2	185
1016	~100	~100	5.0(−4)		E1?	~100
1280	~210	~210	5.7(−4)		E1	~210
1464	~ 80	~ 80			E1?	~ 80

[a] All experimental I.C.C.'s are normalized to the theoretical value, α_L(80 kev) = 4.0. The numbers in { } are theoretical values of I.C.C.'s used to resolve composite gamma-ray peaks (see text).
[b] Corrected for the K-conversion line intensity, (K/L taken as 0.42).
[c] The numbers in [ ] show alternate or limiting choices which are possible.

[18] M. E. Rose, *Internal Conversion Coefficients* (North Holland Publishing Company, Amsterdam, 1958).

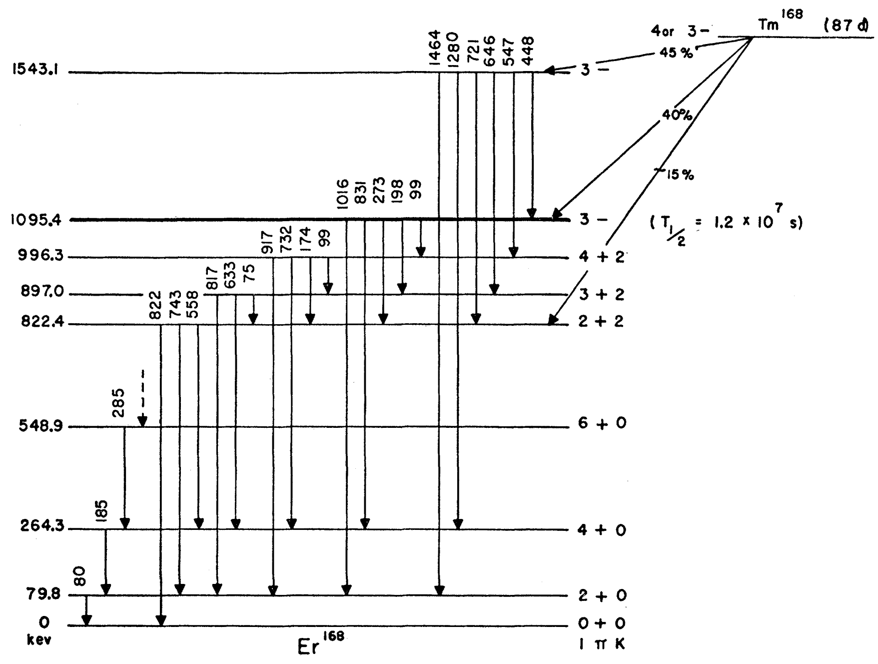

FIG. 4. Energy level scheme for Er^{168}.

question and using the conversion line intensities one can obtain reasonable and consistent experimental values (extreme values in certain cases) for the I.C.C. of the individual transitions. We shall discuss the various cases in more detail. The results of the coincidence measurements and I.C.C. determinations are consistent with the proposed level scheme as shown in Fig. 4. We present this scheme now in order to facilitate the discussion of the analysis of the data.

When one makes the photon and electron intensity normalization for the 80-kev transition, one may find the conversion coefficient for the 198-kev transition, by assuming that the 184-kev transition is pure $E2$ and subtracting the appropriate intensity from the composite 184–198-kev photon peak. We were not able to obtain reliable photon intensity data for the 99, 273, and 285-kev transitions. The 422–448-kev composite photon peak is almost entirely due to the 448-kev transition. The K-conversion line intensities are well measured, and regardless of the M.O. assumed for the weaker transition, the experimental I.C.C. for the 448-kev transition is definitely consistent with an $M1$ assignment.

As regards the composite photon peak of 547 and 558 kev, since no conversion line intensity for the 558-kev transition was obtained, one may obtain an estimate of the I.C.C. of the 547-kev transition, considering all of the photon intensity to be due to it. The value obtained is consistent with an $E1$ assignment. We have measured the intensities of the K-conversion lines of the transitions of 633 and 646 kev and also have obtained their composite photon peak intensity. These data are most consistent with an assignment of $E2+M1$ and $E1+M2$ for the transitions of 633 and 646 kev, respectively.

The next case to consider is that of the three transitions of 721, 732, and 743 kev, for which we obtain discrete K-conversion lines but a single photo peak with the scintillation counter. If one obtains the "composite" I.C.C. by dividing the sum of the three K-line intensities by the composite photon intensity, a value of $1.1(-2)$ consistent with an $M1$ assignment for all three transitions is obtained. However, since the 721-kev transition proceeds between levels of different parity, it must be $E1$ and/or $M2$. If it is taken to be pure M2, then the transitions of 732 and 743 kev are $M1+E2$ ($\sim$40%), this possibility representing the maximum amount of $E2$ admixture. On the other hand if the transitions of 732 and 743 are pure $M1$, then the 721-kev transition is $E1+M2(25\%)$. These should be the two extreme possibilities. This situation will be discussed further in our presentation of the level scheme.

The next group of transitions to be discussed are those of 817, 822, and 831 kev, for which we have discrete K-conversion lines and a composite photon peak. The "composite" I.C.C. indicates that these are $E2+M1$. However, one may analyze the situation as follows: since the 822-kev transition is a $2+\rightarrow 0+$ transition it must be pure $E2$. The designation of the 822-kev level as $2+$ is consistent with the observed branchings of gamma rays proceeding from this level to the $0+$, $2+$, and $4+$ levels. Therefore, one may reduce the photon peak to a doubly composite peak by subtracting the contribution from the 822-kev transition. Since the 831-kev transition proceeds from the odd parity level at 1095 kev (this placement of the "831" being based on energy fit only) it should be

$E1+(M2)$. Now, if one assumes the 831-kev transition to be pure $E1$, then the 817-kev transition will be $M1+E2$ (<20%). If the 831-kev transition is taken to be pure $M2$, the 817-kev transition becomes $M1+E2$ (~50%).

The 917-kev photon peak is single and we obtain an I.C.C. consistent with an $E2$ assignment. No reliable I.C.C. was obtained for the 1016-kev transition. The relatively intense photon peak of the 1280-kev transition allows a good measurement of its I.C.C. which is in agreement with that expected for $E1$. We have obtained only an upper limit for the K-conversion line intensity for the 1464-kev transition. Hence we have an upper limit for its K-I.C.C. which agrees with an $E1$ assignment.

The levels at 80, 264, and 549 kev can be interpreted as members of a ground-state rotational band with $I=2$, 4, and 6, respectively, $K=0$, and even parity. The level at 549 kev is suggested purely on the basis of energy. The experimental value of the postulated 6+ level is 548.9 kev as compared to the theoretical value of 549.4 kev.

The levels at 822, 897, and 996 kev may form a second rotational band with the 822-kev level as the ground state. These levels would have $I=2$, 3, and 4, respectively and even parity. The assignment of $K=2$ to these levels is made on the basis of ratios of R.T.P.'s for transitions from these to the members of the ground-state rotational band. The experimental ratios of R.T.P.'s for transitions from these levels, assuming pure $E2$ radiations, are presented in Table X, which are to be compared to the theoretical ratios given in Table XI.

For the 822 kev (2+) level, $K=2$ gives a satisfactory fit, whereas the data definitely is inconsistent with $K=1$ and $K=0$, even if one admits large $M1$ admixtures in the 742 and 557-kev transitions. The information on the radiations from the 897- and 996-kev levels is less clear, since the $E2+M1$ mixture ratios in them cannot be estimated from the measurements. Despite the uncertainties in the experimental data, it appears that the theoretical and experimental branching ratios of transitions from these two levels do not agree very well, if one assumes the transitions to be pure $E2$. Such a discrepancy has been noted in other cases also and will be discussed later (Sec. VII). With an assignment of $K=2$, the levels at 822, 897, and 996 kev may be interpreted as a rotational band associated with electric quadrupole (gamma) vibrations.

The 1095-kev level, which is the delayed state [$T_{1/2}=(1.2\pm0.2)\times10^{-7}$ sec] is assigned $I=3$ and odd parity. This assignment is consistent with the results of resonant neutron capture studies with Er^{167}, where gamma rays with energies of 80, 185, 199, 285, 750, and 820 kev have been observed.[14] In particular no transition of 448 kev was observed. Apparently all levels of 1095 kev or less are populated in that case. Er^{167} has a ground-state spin[19] of $\frac{7}{2}$ and thus one would expect the compound nucleus, $(Er^{167}+n)^*$, to have a spin of 4 or 3 $(7/2\pm1/2)$. Since the compound nucleus would decay predominantly by dipole radiation, a spin of 2, 3, or 4 for the 1095-kev level would be reasonable.

The 1543-kev level is assigned a spin of 3 and odd parity, since the transitions of 1464 and 1280 kev leading to the 2+ and 4+ levels are probably of $E1$ M.O. The assignment of 3- to the 1543-kev level is consistent with the $M1$ character of the 448-kev transition. The 1543-kev level decays to the three members of the possible gamma vibrational band which would have spins of 2+, 3+, and 4+ and these decays should proceed via $E1$ radiation. However, the conversion coefficient of the 721-kev transition is sufficiently large as to require a considerable amount of $M2$ admixture.

The comparative lifetimes[20] of transitions depopulating the levels of 1095 kev ($T_{1/2}=1.2\times10^{-7}$ sec) and 1543 kev (assuming $T_{1/2}=10^{-9}$ sec) have been computed and are presented in Table V. The experimental half-life of the 1543-kev level is less than 5×10^{-8} sec. In the case of transitions depopulating both these levels, it appears that the $E1$ transitions are considerably retarded in comparison with the single particle estimates.[21] The $M2$ comparative lifetimes were obtained from the intensity data and the M.O.'s stated in column 2 of Table V and are shown only for comparison.

Electron capture branchings to the various levels have been found from the total K x-rays due to electron capture and the intensities of transitions into and out of the various levels. These branchings are shown in the level scheme. The data show that there is little or no decay to the even parity levels, except to the 822 kev (2+) level. A scintillation spectrometer using an an-

TABLE V. Comparative lifetimes of transitions depopulating levels of 1095 and 1543 kev in Er^{168}.

Transition (kev)	M.O.	Comparative lifetimes $E1$ $(-14.2)^a$	$M1$ $(-13.4)^a$	$M2$ $(-8.1)^a$
	1095-kev level ($T_{1/2}=1.2\times10^{-7}$ sec)			
99	$E1$	-5.2		
198	$E1$	-7.3		
273	$E1$	-6.2		
831	$E1$	-4.6		
1016	$E1$	-3.8		
	1543-kev level ($T_{1/2}=10^{-9}$ sec)^b			
448	$M1$		-9.6	
547	$E1$	-7.3		
646	$E1$ or $M2$	-6.7		-6.0
721	$E1(75\%)+M2$	-6.7		-7.0
1280	$E1$	-6.0		
1464	$E1$	-5.4		

a Theoretical values. (See reference 21.)
b Assumed half-life.

[19] B. Bleany and H. E. D. Scovil, Proc. Phys. Soc. (London) A64, 204 (1951.

[20] M. Goldhaber and A. W. Sunyar, *Beta- and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North Holland Publishing Company, Amsterdam, 1955), Chap. XVI.

[21] S. A. Moszkowski, *Beta- and Gamma-Ray Spectroscopy*, edited by K.Siegbahn (North Holland Publishing Company, Amsterdam, 1955), Chap. XIII.

TABLE VI. Conversion electron data for Er^{166}.

Transition (kev)	K	L_I	Intensity (N_e)[a] L_{II}	L_{III}	M	N	M.O.
80.6	260		1000[b]	980	515	145	$E2$
84.1	1.9	w[c]					
147.2	1.6						
154.3	6.0	2.0					
170.0	1.3						
184.5	165		50[b]	30	24	7	$E2$
194.7	17	d					
215.1	11						
280.2	1.0			w[c]			($E2$)
345.5	3.4						
403.8	3.3	0.5					
459.3	8.0	1.3					
595.0	2.5	0.5					($E2$)
672.9	1.5						
675.4	0.65						
692.3	3.6	0.7			w[c]		($E2$)
706.2	5.6	1.0					($E2$)
758.9	2.3	d					
780.0	7.3	1.3			d		($E2$)
787.1	3.8	d					($E2$)
876.9	1.2						($E2$)
1081.6	w[c]						
1155.2	0.7						
1179.8	2.2	d					
1276.9	3.1	0.45					
1304.3	0.22						
1350.4	0.23						
1377.6	0.94	0.18					
1451.2	0.25						
1508.5	0.16						

[a] Intensity data are arbitrarily normalized to 1000 units for the most prominent line.
[b] L_I and L_{II} lines are not resolved.
[c] "w" indicates a weak line.
[d] Conversion line is composite of two lines.

thracene crystal indicated the presence of electrons attributable to internal conversion of the 448, ~700 and ~800-kev transitions. Any continuous β spectrum must be of much less intensity. A search for annihilation radiation gave negative results.

Tm^{168} is an odd-odd nucleus with 99 neutrons and 69 protons. In agreement with the predictions made on the basis of the Nilsson[22] specification of single paticle states in deformed nuclei, the ground state of Er^{167} is $(7/2+)$[19] which has 99 neutrons and 68 protons, while Tm^{169} which has 100 neutrons and 69 protons has a ground state of $(1/2+)$.[22] According to the coupling scheme of angular momenta for odd-odd nuclei,[6,22,23] one would expect Tm^{168} to have a ground state of $(4+)$ or $(3+)$. If one assumes the ground state of Tm^{168} to be $(4+)$, then the electron capture to the 822 kev $(2+)$ level will be of type $\Delta I=2$, no (second forbidden) and those to the 1095 kev (3) and 1543 kev $(3-)$ levels of type $\Delta I=1$, yes (first forbidden). With this choice of $(4+)$ for the ground state of Tm^{168} some difficulty arises with regard to the second forbidden transition to the 822-kev level, since the implied energy available for electron capture would then be quite large. A more reasonable assignment to the Tm^{168} ground state may be $(3-)$ or $(4-)$, with an energy difference, $Tm^{168}-Er^{168}$, of very roughly 1.6 Mev. The absence of electron capture to the ground-state rotational band may be due to the operation of the K-selection rule.

[22] B. R. Mottelson and S. G. Nilsson, Phys. Rev. **99**, 1615 (1955).
[23] C. J. Gallagher and S. A. Moszkowski, Phys. Rev. **111**, 1282 (1958).

V. ERBIUM-166

The electron capture of Tm^{166} (7.7 hr)[13] leads to levels in stable Er^{166}, some of which are also reached by the beta decay of the two isomers[24] of Ho^{166}. In each case high lying levels are populated. However many of the transitions observed in the decay of Tm^{166} are not present in the decay of Ho^{166} activities. Also electron capture in Tm^{166} populates levels in Er^{166} higher than those populated by beta decay. The beta decay of both the long-lived (30 yr)[24–26] and short-lived (23, 3 hr)[27–30] Ho^{166} activities have been studied and level schemes proposed for Er^{166}.

The conversion electron data for Er^{166} are presented in Table VI. The M.O. assignments are made on the basis of K/L and (or) L ratios and are only tentative (except for the cases of 81 and 185-kev transitions). It is rather difficult to make a unique M.O. assignment to high-energy gamma rays purely on the basis of K/L ratios. The M.O. assignments made in such cases are those which give the maximum internal consistency to the proposed level scheme. No claim as to the uniqueness of these assignments may be made.

The gamma-ray spectrum of Er^{166}, obtained with a scintillation spectrometer showed composite peaks with energies 59, 90, 192, 710, 1240, 1360, 1640, 1860, and 2100 kev.

A level scheme for Er^{166} is proposed on the basis of the available data. The level scheme is presented in

TABLE VII. Energies of ground-state rotational levels.

Nucleus	Level K, I	Energy (kev) (1)[a]	(2)[b]	(3)[c]
Er^{168}	0,2+	79.8		
	0,4+	264.3	266.0	
	0,6+	548.9	558.6	549.4
Er^{166}	0,2+	80.6		
	0,4+	265.1	268.7	
	0,6+	545.3	564.2	545.3

[a] Experimental values.
[b] Calculated values, using Eq. (3).
[c] Calculated values, using Eq. (1).

[24] F. D. S. Butement, Proc. Phys. Soc. (London) **A65**, 254 (1952).
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Fig. 5. Some of the very weak transitions are not included in the proposed scheme. The transitions seen only with the scintillation spectrometer are shown originating from the levels shown as dotted lines. The levels at 81, 265, and 545 kev can be interpreted as members of a ground-state rotational band with $I=2$, 4, and 6, respectively, $K=0$ and even parity. In analogy with Er^{168} levels, the levels at 787, 861, and 957 kev may form a second rotational band with $I=2$, 3, and 4, respectively, and may be interpreted as members of a rotational band associated with gamma vibrations ($K=2$ and even parity). The experimental ratios of R.T.P.'s for transitions from these levels to the members of the ground-state rotational band, assuming pure $E2$ radiations and $K=2$, are given in Table X. These ratios are to be compared with the theoretical values of Table XI. As in the case of Er^{168}, the experimental ratios of R.T.P.'s for transitions from the (3+) and (4+) levels are considerably smaller than the theoretical values.

Nothing much can be said about the levels above 958 kev. A comparison with the energy levels of Er^{166} reached by the beta decay of two Ho^{166} isomers suggest that some of the levels observed in the decay of Tm^{166} are common to both modes of decay. However, with our data it is not possible to say definitely which are really common levels, except the 81-kev level which is populated in the decay of both the Ho^{166} isomers[25–30] and the levels at 265 and 546 kev which are populated in the decay of 30 yr Ho^{166}.[25,26]

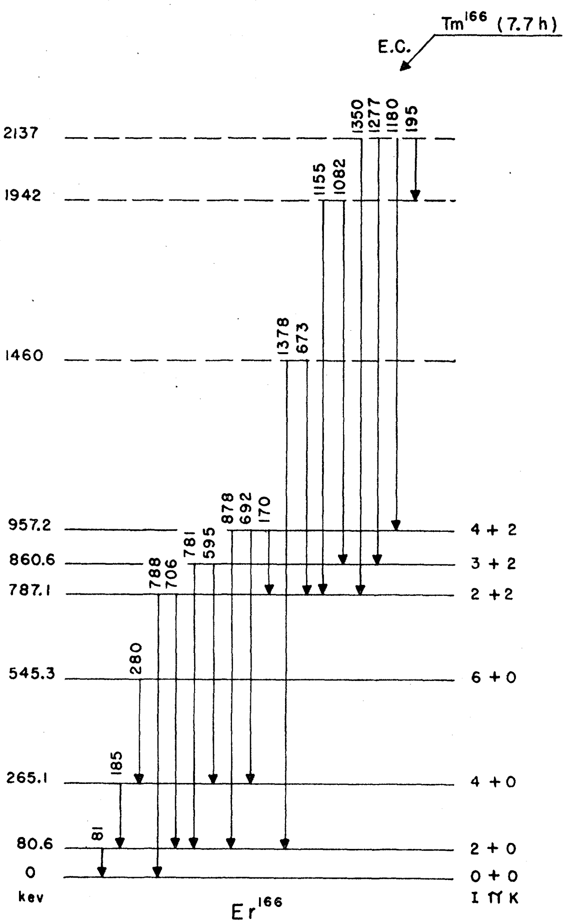

FIG. 5. Energy level scheme for Er^{166}.

TABLE VIII. Vibrational levels of even-even nuclei of the rare earth region.

Nucleus	$\hbar^2/2\mathscr{I}$ (kev)	B (kev)	Calc "$\hbar\omega$" (kev)	Possible vib. level (2, 2+)	$\hbar^2/2\mathscr{I}'$ (kev)	$\mathscr{I}^1/\mathscr{I}$	Reference
Sm^{152}	21.2	−0.145	1025	1082	24.0	0.883	31, 32
Gd^{154}	21.3	−0.138	1060	998	22.0	0.968	33
Gd^{156}	15.0	−0.027	1400	1134	15.8	0.949	34, 35, 43
Dy^{160}	14.6	−0.025	1440	964	13.8	1.058	36, 37
Er^{166}	13.5	−0.013	1750	787	12.3	1.098	This work
Er^{168}	13.3	−0.006	2500	822	12.4	1.075	This work
Yb^{172}	13.2	−0.008	2260	1107[a]	11.3	1.168	38
W^{182}	16.8	−0.015	2230	1222	18.3	0.928	40
W^{184}	18.7	−0.024	1540	904	17.0	1.100	39
Os^{186}	23.4	−0.083	1570	768			41, 42
Os^{188}	26.7	−0.138	1480	633			41, 42

[a] Energy of (2, 2+) level as calculated from the energies of (2, 3+) and (2, 4+) levels. No (2, 2+) was observed.

VI. GENERAL CONCLUSIONS

In both Er^{168} and Er^{166}, the first three levels can be interpreted as members of a ground-state rotational band with $I=2$, 4 and 6, $K=0$ and even parity. Their energies obey the predicted $I(I+1)$ law with a negative correction term proportional to $I^2(I+1)^2$, as given by (1). The results are given in Table VII.

If one assumes that the vibrational quanta associated with the two modes of quadrupole vibrations are equal, i.e., $\hbar\omega_\beta=\hbar\omega_\gamma$, then using the empirical values of the energies of the first two excited states, it is possible to estimate the energies of the vibrational levels using (1) and (2). The results of such calculations made for the nuclei investigated here and other even-even nuclei of the rare earth region are presented in Table VIII.[31–43] There is not much correlation between the experimental data and the calculated values. This is not too surprising for the details of the rotation-vibration correction term depend upon the assumptions of the incompressible

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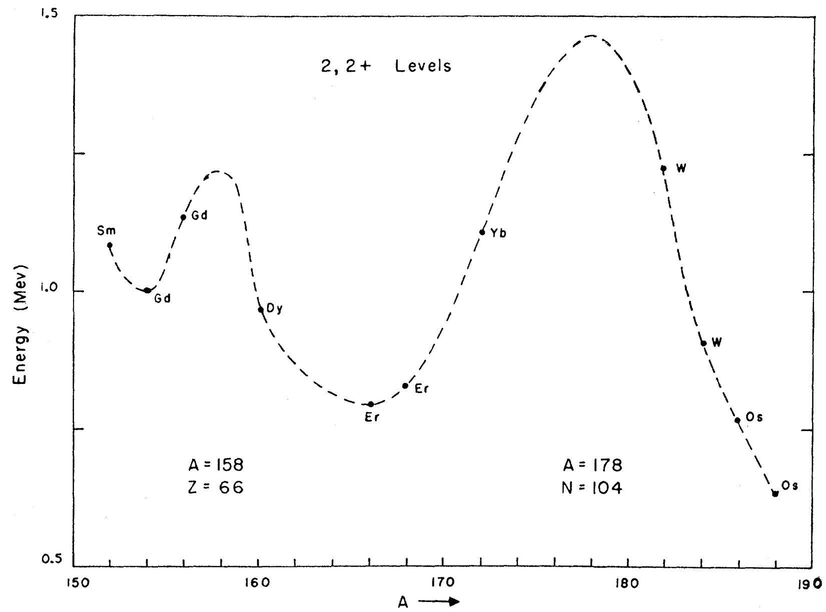

FIG. 6. Energy of possible (2, 2+) levels in the even-even nuclei $150<A<190$, *versus* the mass number.

model and as such are expected to give only an orientation in the magnitudes involved.[7] The more detailed analysis of the rotation-vibration effect will involve a knowledge of the proper modes of vibration of the deformed nucleus.[44]

In Fig. 6 the experimentally observed values of energies of the possible gamma vibrational levels (2, 2+) of the even-even nuclei of the rare earth region are plotted against the mass number. It is of interest to note here that the two peaks seen in the curve occur at $A=158$ (Z halfway between 50 and 82) and $A=178$ (N halfway between 82 and 126).

In both Er^{168} and Er^{166}, the rotational levels associated with gamma vibrations seem to obey the simple rotational energy formula given by

$$E_I=(\hbar^2/2\mathcal{J}')[I(I+1)-I_0(I_0+1)], \qquad (3)$$

where $\mathcal{J}'$ is an effective moment of inertia associated with the gamma band. The results are presented in Table IX. The data show that the calculated value for the energy of the (2, 4+) level in the case of Er^{166} is slightly higher than the experimental value, even though it is well within the possible uncertainties in the measured values. To say whether these rotational levels also need a correction similar to that of the ground-state rotational levels would require somewhat better energy measurements.

Another interesting feature is the behavior of the moments of inertia, $\mathcal{J}'$, associated with the gamma vibrations. The moments of inertia for the gamma band are very nearly equal to the ground-state moments of inertia, $\mathcal{J}$, in all cases (see Table VIII), with the exception of W^{184}.[39]

Theoretically, no $M1$ radiation is allowed in the decay of vibrational states in even-even nuclei even if $\Delta I=0$ or 1.[7,8] However, the experimental I.C.C.'s for transitions from the gamma vibrational levels in Er^{168}, indicate that some of them, in particular those from the (2, 3+) level, are $M1+E2$ mixtures. Similar results have been obtained in other cases also.[31,36,37] The above conclusion is further substantiated by the experimental data on gamma branching ratios for transitions from the different members of the gamma band. In Table X are presented the experimental ratios of R.T.P.'s, for

TABLE IX. Rotational energies associated with gamma vibrations.

Nucleus	Level K, I	Energy (kev) (1)[a]	(2)[b]
Er^{168}	2,2+	822.4	
	2,3+	897.0	
	2,4+	996.3	996.4
Er^{166}	2,2+	787.1	
	2,3+	860.6	
	2,4+	957.2	958.6

[a] Experimental values.
[b] Calculated values, using Eq. (3).

TABLE X. Experiment ratio of R.T.P.'s.

Nucleus	$\frac{B(E2;22\to00)}{B(E2;22\to02)}$	$\frac{B(E2;23\to02)}{B(E2;23\to04)}$	$\frac{B(E2;24\to02)}{B(E2;24\to04)}$	Reference
Sm^{152}	0.6	2.1		32
Gd^{154}	0.56	1.4		33
Gd^{156}	0.61	0.63		34
Dy^{160}	0.42	1.3		36
Er^{166}	0.52	1.3	0.17	This work
Er^{168}	0.58	1.2	0.17	This work
Yb^{172}		1.2		38
W^{182}	0.62	2.1		40
W^{184}	0.58	3.6		39

[44] A. Bohr, *Rotational States in Atomic Nuclei* (Ejnar Munksgaard, Copenhagen, 1954).

transitions depopulating the gamma vibrational levels, assuming pure $E2$ radiation. It appears that the ratio of R.T.P.'s for transitions depopulating the (2, 2+) levels agree with the theoretically predicted values (see Table XI). However if one takes into account the probable $M1$ admixture to the $(2, 2+) \rightarrow (0, 2+)$ transition, the experimental ratio, $B(E2;\, 22 \rightarrow 00)/B(E2;\, 22 \rightarrow 02)$ will increase and thus the agreement with theory might be only fortuitous. Similarly since the $(2, 4+) \rightarrow (0, 4+)$ transition is also probably $M1+E2$, the ratio $B(E2;\, 24 \rightarrow 02)/B(E2;\, 24 \rightarrow 04)$ will increase. In the case of transitions depopulating

TABLE XI. Theoretical ratio of R.T.P.'s.

K value	$\frac{B(E2;\,22\rightarrow 00)}{B(E2;\,22\rightarrow 02)}$	$\frac{B(E2;\,22\rightarrow 04)}{B(E2;\,22\rightarrow 02)}$	$\frac{B(E2;\,23\rightarrow 02)}{B(E2;\,23\rightarrow 04)}$	$\frac{B(E2;\,24\rightarrow 02)}{B(E2;\,24\rightarrow 04)}$
0	0.7	1.8		
1	2.8	2.3		
2	0.7	0.05	2.5	0.34

the (2, 3+) levels, since both $(2, 3+) \rightarrow (0, 2+)$ and $(2, 3+) \rightarrow (0, 4+)$ transitions are probably $M1+E2$ nothing much can be said until better experimental data are available.

Recently Davydov and Filipov[45] proposed a theory of the energy states and the electromagnetic transitions between them for nuclei which do not possess axial symmetry. A comparison of their predictions to experimental data is made in Table XII. The values of γ given in Table XII are those calculated from the ratio of the energy of the second (2+) state, $E'(2)$, to that of the first (2+) state, $E(2)$. The ratio $E'(2)/E(2)$ depends only on γ. According to their predictions $E'(2)+E(2)=E(3)$, where $E(3)$ is the energy of the (3+) state. In Table XII are given the experimental and theoretical ratio of R.T.P.'s for transitions from the second (2+) level. The theoretical values were calculated using the values of γ as determined by the ratio $E'(2)/E(2)$. It appears that almost in all cases the experimental values are higher than the theoretical values.

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TABLE XII. Comparison of experimental data with predictions of Davydov and Filipov.

Nucleus	$\frac{E'(2)}{E(2)}$	deg γ	Experimental (kev) $E'(2)+E(2)$	$E(3)$	$\frac{b(E2;\,2'\rightarrow 0)}{b(E2;\,2'\rightarrow 2)}$ Expt.	Theory	Reference
Sm^{152}	8.9	13.1	1204	1226	0.6	0.44	31, 32
Gd^{154}	8.1	13.9	1121	1130	0.56	0.41	33
Gd^{156}	12.7	11.2	1223	1229	0.61	0.50	34, 35, 43
Dy^{160}	11.1	11.9	1051	1047	0.42	0.48	36, 37
Er^{166}	9.8	12.7	868	861	0.50	0.45	This work
W^{182}	12.2	11.4	1322	1332	0.62	0.50	40
W^{184}	8.1	13.8	1015	1006	0.58	0.41	39
Os^{186}	5.6	16.5	905		0.45	0.32	41, 42
Os^{188}	4.1	19.2	788		0.41	0.31	41, 42

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$Cu^{65}(\gamma,3n)$ Reaction and Its Bearing on the Use of the $Cu^{63}(\gamma,n)Cu^{62}$ Reaction for Bremsstrahlung Monitoring

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The $Cu^{65}(\gamma,3n)$ reaction has been measured from threshold to 110 Mev. The integrated cross section at 110 Mev is 0.037 ± 0.004 Mev barn. The consequent error in bremsstrahlung monitoring through ignoring this contribution varies from 0.9% at 40 Mev to 1.4% at 110 Mev.

In developing a bremsstrahlung monitoring system based on the 9.7-minute Cu^{62} activity resulting from the $Cu^{63}(\gamma,n)$ reaction,[1] it was necessary to measure the $Cu^{65}(\gamma,3n)Cu^{62}$ cross section since this reaction will not only affect the absolute intensity calibration but may also introduce errors into the normalization of beams with different energies. This latter is of particular importance when using bremsstrahlung subtraction techniques[2] to obtain photon cross sections.

A target,[3] consisting of 196.6 mg of Cu^{65} (Cu^{63} contamination $0.6\pm0.08\%$) on a 0.001-in. platinum support, was irradiated at various beam energies and counted with the following schedule: (a) Irradiation 0–10 min. (b) First count 11–20.5 min. (c) Second

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