The first of these is readily integrated using Eq. (11), and the second term equals $\pi(p_c/b)^2$ where p_c is the classical impact parameter corresponding to θ_c as tabulated in reference I. The result is

$$\sigma/b^2=(\pi a^2/\bar{\lambda}^2)L_c^2a^2/(L_c^2a^2+1)+\pi(p_c/b)^2. \tag{16}$$

For $b/\bar{\lambda}\leq 1$ the total cross section is given entirely by extending I_1 from 0 to π, and the well-known result is

$$\sigma/b^2=(4\pi a^4/\bar{\lambda}^4)/(1+4a^2/\bar{\lambda}^2). \tag{17}$$

The total cross sections are shown in Fig. 3. The solid lines are those in which θ_c was chosen as the transition angle, and the dotted lines have θ_b as the transition angle. At the top and the left edge of the figure all lines have slopes -2 on log log paper and may be extended without limit.

5. LIMITED TOTAL CROSS SECTION

The limited total cross section σ_α is defined here as the cross section for scattering to angles in excess of a specified angle α. Experimental measurements of scattering cross sections must have a lower angular limit because of the finite angular resolution of the apparatus and the impossibility of differentiating between an unscattered particle and one which has been scattered through a negligible angle.

A study of Table I shows that θ_c and θ_b are very small angles, much less than one degree for large values of $b/\bar{\lambda}$. Thus the classical solution is often valid for angular regions which can be measured experimentally.

The limited total cross section is found very simply from

$$\sigma_\alpha=\pi p_\alpha^2, \tag{18}$$

where p_α is the classical impact parameter corresponding to scattering through an angle α as found in reference I. For convenience in comparison with experiments Fig. 4 plots values of this cross section as it depends on b/a and the specified angle α.

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Efficiency of Field Ionization at a Metal Surface

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In this paper the expression for the transmission coefficient, which was derived formerly by Müller for field emission, is integrated by making certain approximations. The formula for the efficiency of field ionization could then be integrated. Furthermore the supply function is calculated by regarding the molecule as moving in a central field of force. The main object of the paper is that of deriving analytical formulas which give a better picture of the dependence of the field ion current on the various parameters.

1. INTRODUCTION

MÜLLER[1] has shown that, in a field up to 500×10^6 volts/cm, the mechanism by which field emission of positive ions takes place in the field ion microscope depends upon the supply of molecules and their field ionization probability.

As the molecule approaches the tip from the low-field region where the ionization is negligible, to the high-field region near the tip, the ionization will increase due to a reduction in height and breadth of the potential wall which binds the electron and will reach a sharp limit when the ground level of the molecule sinks below the Fermi level of the emitter.

Müller[1,2] has calculated the ionization probability for a molecule approaching the tip from a distance L A up to $(L-1)$ A with a speed depending on the field F and the molecular polarizability. He has taken the effective potential of the escaping electron to be

$$V(x)=\frac{-e}{L-x}+Fx-\frac{e}{4x}+\frac{e}{L+x}, \tag{1}$$

in a one-dimensional approximation of the problem. The The terms on the right are, respectively, associated with the Coulomb attraction between ion and electron, the applied field, the image of the electron in the metal (tip) surface and the image of the ion. A wave mechanical calculation leads to the formula

$$D(L)=\exp\left\{-\left(\frac{8em}{\hbar^2}\right)^{\frac{1}{2}}\int_{x_1}^{x_2}\left[V(x)-FL+V_I-\frac{e}{4L}\right]^{\frac{1}{2}}dx\right\} \tag{2}$$

for the penetration probability of the electrons where V_I is the ionization potential.

In a rough approximation for the three dimensional

* For this investigation, H. Fiedeldey received a bursary from the Council for Scientific Industrial Research, Pretoria.

[1] E. W. Müller and K. Bahadur, Phys. Rev. **102**, 624 (1956).
[2] R. H. Good and E. W. Müller, *Encyclopedia of Physics* (Springer-Verlag, Berlin, 1956), Vol. 21.

penetration probability $\Delta(L)$, Müller finds

$$\Delta(L) = -D(L)/\ln D(L). \tag{3}$$

Finally Müller obtains the formula

$$P(L) = \frac{\nu \times 10^{-8}}{F(\alpha/M)^{\frac{1}{2}}}\Delta(L) \text{ in A}^{-1}, \tag{4}$$

for the ionization probability where ν is the frequency of the bound electron and $1/F(M/\alpha)^{\frac{1}{2}}$ the time taken by the ion to move through 1 A with a radial velocity depending only on dipole attraction.

The formula (4) was integrated graphically by Müller.[2]

2. THE INTEGRATION OF $D(L)$

The contribution of the image potentials in formula (1) can be neglected for large values of L. By keeping the term $-e/4L$ in formula (2) the effect of the image potentials is not entirely neglected. With this approximation Eq. (2) reduces to

$$-\ln D = \left(\frac{8m}{\hbar^2}\right)^{\frac{1}{2}} \int_{x_1}^{x_2} \left(eFx - \frac{e^2}{L-x} - eFL + eV_I - \frac{e^2}{4L}\right)^{\frac{1}{2}} dx. \tag{5}$$

On substituting

$$W = \tfrac{1}{4}(eV_I - e^2/4L), \quad z = L - x \quad \text{and} \quad F_0 = \tfrac{1}{4}F$$

Eq. (5) becomes

$$-\ln D = 2\left(\frac{8m}{\hbar^2}\right)^{\frac{1}{2}} \int_{z_1}^{z_2} \left(-eF_0 z - \frac{e^2}{4z} + W\right)^{\frac{1}{2}} dz. \tag{6}$$

This integral differs from the corresponding expression for the field electron transmission coefficient[2] by a

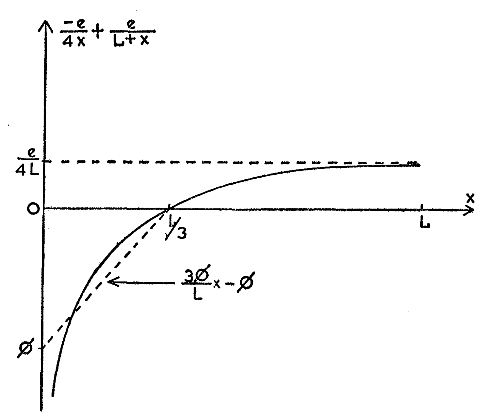

FIG. 1. The image potentials.

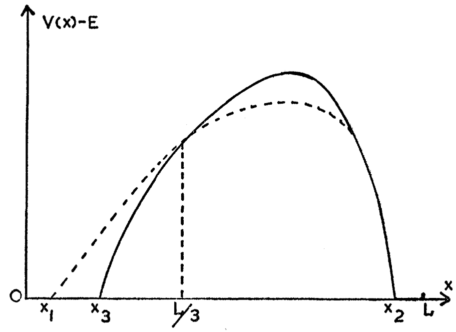

FIG. 2. The potential barrier.

factor 2 only and can be written down by comparison[3,4]:

$$D(L) = \exp\left[-\frac{4(2m)^{\frac{1}{2}}}{3\hbar eF}\left(eV_I - \frac{e^2}{4L}\right)^{\frac{3}{2}} v(y)\right], \tag{7}$$

where

$$y = \frac{2(e^3F)^{\frac{1}{2}}}{eV_I - e^2/4L}, \quad \begin{matrix} z_1 \\ z_2 \end{matrix} = \frac{eV_I - e^2/4L}{2eF}\left[1 \pm (1-y^2)\right]^{\frac{1}{2}},$$

and

$$v(y) = \frac{1}{\sqrt{2}}\left[1 + (1-y^2)^{\frac{1}{2}}\right]^{\frac{1}{2}}\left[E(k) - (1 - (1-y^2)^{\frac{1}{2}})K(k)\right],$$

K and E are the complete elliptic integrals of the first and second kinds defined by

$$K(k) = \int_0^{\pi/2} \frac{d\varphi}{(1-k^2\sin^2\varphi)^{\frac{1}{2}}} \quad \text{and}$$

$$E(k) = \int_0^{\pi/2} (1 - k^2\sin^2\varphi)^{\frac{1}{2}} d\varphi,$$

$$k^2 = \frac{2(1-y^2)^{\frac{1}{2}}}{1 + (1-y^2)^{\frac{1}{2}}},$$

The function $v(y)$ has been evaluated for representative values of y by Burgess, Kroemer, and Houston.[4] For sufficiently large values of L, $e^2/4L$ becomes small compared to eV_I and can be neglected. In this region $D(L)$ will be denoted by $D_\infty(L)$.

We shall now discuss the case where the image potentials cause an appreciable lowering of the potential barrier. This is possible only when the width of the potential barrier is greater than $2/3L$ (see Fig. 1).

In Fig. 2 the dotted curve is a representation of $V(x) - E$ when the image potentials are not taken into account. The full curve represents the real potential barrier. eE denotes the energy of the tunnelling electron, where $E = FL - V_I + e/4L$.

The image potentials cause a displacement of the

[3] L. W. Nordheim, Proc. Roy. Soc. (London) **A121**, 626 (1928).
[4] Burgess, Kroemer, and Houston, Phys. Rev. **90**, 515 (1953).

classical turning point x_1 to x_3, but do not effect a notable change in the position of the other classical turning point x_2. Denoting the distance from the tip surface to the border of ionization zone by L_0, it is seen that $x_3=0$ when $L=L_0$. Thus the width of the barrier, which changes slowly, is approximately equal to L_0. Hence a lowering of the potential barrier can only be expected when $L<3/2L_0$. The lowering will be appreciable when $L<5/4L_0$. The evaluation of $D(L)$ in the interval $(L_0,5/4L_0)$ requires a known value of $z_3=L-x_3$. In order to calculate z_3 we approximate the image potentials in the region $(0,L/3)$ by $(3\phi/L)x-\phi$, where $e\phi$ is the work function of the metal (see Fig. 1). This approximation is justifiable because the image potentials are not well known in a region of 2–3 A from the metal surface.

Hence $V(z)-E\approx-(F+3\phi/L)z+V_I+2\phi$, and

$$z_3\approx(V_I+2\phi)/(F+3\phi/L). \tag{8}$$

When $L=L_0$ the ground level of the atom, which is given by $FL_0-V_I-e/4L$, becomes equal to $-V$, so that $L_0\approx(V_I-\phi)/F$. Substitution of this value of L_0 into (8) yields $z_3=L_0$, as one should expect. This confirms that the approximation (8) is a good one in the interval $(L_0,5/4L_0)$.

An approximation for $D(L)$ in this interval can now be obtained by neglecting the image potentials in the expression for $V(x)-E$, but integrating between the limits x_3 and x_2 shown in Fig. 2, this becomes in terms of z:

$$\int_{z_2}^{z_3}[eV(z)-eE]^{\frac{1}{2}}dz=\int_{z_2}^{z_1}[eV(z)-eE]^{\frac{1}{2}}dz-\int_{z_3}^{z_1}[eV(z)-eE]^{\frac{1}{2}}dz.$$

The first term on the right has already been calculated. To simplify the integration of $\int_{z_3}^{z_1}[eV(z)-eE]^{\frac{1}{2}}dz$ we also neglect the term $-e/z$ in $V(z)$ which is of the order $-e/L$ in the interval (z_3,z_1). This means that the correction for the displacement of the classical turning point from z_1 to z_3, which is calculated below, is somewhat too great. However the lowering of the potential barrier between z_3 and $z=2L/3$ has also been neglected. These two opposing errors will partly cancel each other. The correction applied to $-\ln D(L)$ in the interval $(L_0,5/4L_0)$ is given by:

$$-\frac{2(2m)^{\frac{1}{2}}}{\hbar}\int_{z_3}^{z_1}(-eFz+eV_I)^{\frac{1}{2}}dz=+\frac{4(2m)^{\frac{1}{2}}}{3heF}[(eV_I-eFz_3)^{\frac{3}{2}}-(eV_I-eFz_1)^{\frac{3}{2}}].$$

Denoting $D(L)$ by $D_0(L)$ in the region $(L_0,5/4L_0)$ we get

$$D_0(L)=\exp\left\{-\frac{4(2m)^{\frac{1}{2}}(eV_I-e^2/4L)^{\frac{3}{2}}}{3\hbar eF}v(y)+\frac{4(2m)^{\frac{1}{2}}}{3\hbar eF}(eV_I)^{\frac{3}{2}}\left[\left(1-\frac{Fz_3}{V_I}\right)^{\frac{3}{2}}-\left(1-\frac{Fz_1}{V_I}\right)^{\frac{3}{2}}\right]\right\}$$

where

$$z_3=\frac{V_I+2\phi}{FL+3\phi}L,\quad z_1=\frac{eV_I-e^2/4L}{2eF}[1+(1-y^2)^{\frac{1}{2}}],\quad y=\frac{2e(eF)^{\frac{1}{2}}}{eV_I-e^2/4L}. \tag{9}$$

For values of L in the interval $(5/4L_0,3/2L_0)$, (7) is a good approximation, while for values of L greater than $3/2L_0$ the formula

$$D_\infty(L)=\exp\left[-\frac{4(2m)^{\frac{1}{2}}}{3\hbar eF}(eV_I)^{\frac{3}{2}}v(y)\right]\text{ where }y=\frac{2(e^3F)^{\frac{1}{2}}}{eV_I} \tag{10}$$

is a good one. The corresponding formulas for $P(L)$ can be obtained by using Eq. (4). The values of $P(L)$ obtained from those formulas are in good agreement with the graphically calculated values of Müller.

3. THE INTEGRATION OF $\int_{L_0}^{\infty}P(L)dL$

In the intervals $(L_0,5/4L_0)$, $(5/4L_0,3/2L_0)$ and $(5/2L_0,\infty)$ $P(L)$ is successively denoted by $P_0(L)$, $P_I(L)$, and $P_\infty(L)$. First we calculate $\int_a^\infty P_\infty(L)dL$. $P_\infty(L)$ can be written in the form

$$P_\infty(L)=A_0\exp[-(B_0/F)v(y)],\quad y=C_0F^{\frac{1}{2}}, \tag{11}$$

where

$$A_0=\frac{3\times10^{-6}\nu}{(\alpha/M)^{\frac{1}{2}}B_0v(C_0F_a^{\frac{1}{2}})},\quad B_0=6.83\times10^7(eV_I)^{\frac{3}{2}},\quad C_0=\frac{7.58\times10^{-4}}{eV_I}, \tag{12}$$

where $P_\infty(L)$ is expressed in A^{-1}, F in volts/cm and eV_I in electron volts.

Although A_0 is not a constant it changes so slowly in comparison with $\exp[-(B_0/F)v(y)]$ that it can be regarded a constant. We define the function

$$U(F)=(B_0/F)[1-v(y)]. \tag{13}$$

Because $\frac{1}{2}[t(y)+s(y)]\approx1$,[5] where $t(y)=v(y)-2/3ydv/dy$ and $s(y)=v(y)-\frac{1}{2}ydv/dy$ it follows that:

$$v(y)\approx1-y^{12/7}. \tag{14}$$

Substituting (14) into (13) we obtain

$$U(F)\sim D_0F^{-1/7},\quad D_0=B_0C_0^{12/7} \tag{15}$$

[5] See footnote 2, p. 187, Table I.

and

$$P_\infty(L)=A_0\exp[-(B_0/F)U(F)]$$
$$\approx A_0\exp[-(B_0/F)+D_0F^{-1/7}]. \quad (16)$$

It is clear that $U(F)$ changes very slowly in comparison with B_0/F, which shows that, to a first approximation, $U(F)$ can be treated as a constant.

We assume that the electric field is given by the semiempirical formula

$$F=\frac{7.75Vr_0^{\frac{2}{3}}}{r^{\frac{4}{3}}}\equiv\frac{K}{r^{\frac{4}{3}}}, \quad (17)$$

hence

$$\int_a^\infty P_\infty dL=A_0e^{U(F_a)}\int_{r_a}^\infty \exp\left(-\frac{B_0}{K}r^{\frac{4}{3}}\right)dr, \quad (18)$$

where $r_a=r_0+a$. P_∞ is expressed in A^{-1} and A_0 has the same dimensions as P therefore r_a is expressed in A. Putting $t=(B_0/K)r^{4/3}$ and $x=B_0/F_a$ Eq. (18) becomes:

$$\int_a^\infty P_\infty dL=\tfrac{3}{4}A_0B_0^{-\frac{3}{4}}r_aF_a^{\frac{3}{4}}e^{U(F_a)}\Gamma(\tfrac{3}{4},x),$$

where $\Gamma(\alpha,x)=\int_x^\infty t^{\alpha-1}e^{-t}dt$ is the incomplete gamma function.[6] The asymptotic expansion of $\Gamma(\alpha,x)$ for large values of x is given by

$$\Gamma(\alpha,x)=x^{\alpha-1}e^{-x}\left[\sum_{n=0}^{N-1}\frac{(1-\alpha)_n}{(-x)^n}+O\{|x|^{-N}\}\right], \quad (19)$$

where $(\alpha)_n=\Gamma(\alpha+n)/\Gamma(\alpha)$, $(\alpha)_0=1$ and the terms which are neglected are of the order $1/|x|^N$. $\Gamma(\alpha)$ is the gamma function.

Since the values of x are relatively large, *viz.*, from 20–50, we may use the asymptotic expansion leading to:

$$\int_a^\infty P_\infty dL=\tfrac{3}{4}r_a(F_a/B_0)P_\infty(a)(1-\tfrac{1}{4}F_a/B_0). \quad (20)$$

If the numerical values of the constants, for r_a in cm, are inserted, the term $\frac{1}{4}F_a/B_0$ is seen to be negligible and Eq. (20) reduces to:

$$\int_a^\infty P_\infty dL=1.1r_aF_a(eV_I)^{-\frac{3}{2}}P_\infty(a). \quad (21)$$

Next we take the variation of $U(F)$ with F into account. Because $U(F)$ changes slowly one may put $e^{U(F)-U(F_a)}\approx 1+U(F)-U(F_a)$ and also $U(F)\approx DF^{-1/7}$ in the integral which then becomes:

$$\int_a^\infty P_\infty dL=\tfrac{3}{4}r_a\frac{F_a}{B_0}P_\infty(a)\left\{1+\left[\frac{U(F_a)}{7}-\tfrac{1}{4}\right]\frac{F_a}{B_0}\right\}. \quad (22)$$

The formula (22) can be approximated by (21) because $U(F_a)$ is of the order of 10.

In the interval $(5/4L_0,3/2L_0)$ we have

$$P_I(L)=A\exp[-(B/F)+U(F)]$$
$$\approx A\exp[-(B/F)+DF^{-1/7}]. \quad (23)$$

Since $3.6\times10^{-8}/L\ll eV_I$ for all values of L greater than L_0 (for He and fieldstrength of 4×10^8 volt/cm $(3.6\times10^{-8}/eV_I)(1/L_0)$ is equal to 0.03) we may put $A\sim A_0$,

$$B\sim B_0\left[1-\frac{3}{2}\frac{3.6\times10^{-8}}{eV_I}\frac{1}{L}\right],$$

$$D\sim D_0\left[1+\frac{3}{14}\frac{3.6\times10^{-8}}{eV_I}\frac{1}{L}\right], \quad (24)$$

where L is expressed in cm.

We are calculating $\int_{5/4L_0}^{3/2L_0}P_IdL$, so it is permissible to regard the fieldstrength F as a constant. Substituting

$$\lambda=\frac{3.6\times10^{-8}}{eV_I}\left[\frac{3}{2}\frac{B_0}{F_a}+\frac{3}{14}U(F_a)\right]\text{ cm}, \quad (25)$$

one obtain

$$\int_a^b P_IdL=10^8P_\infty(a)\int_a^b e^{\lambda/L}dL.$$

The factor 10^8 converts $P(a)$, which is expressed in A^{-1}, to cm^{-1}.

The integral can now be written in the form

$$\int_a^b P_I(L)dL=10^8P_\infty(a)\lambda\int_{-x}^{-y}\frac{1}{t^2}e^{-t}dt$$
$$=10^8P_\infty(a)\lambda\left(\frac{e^y}{y}-\frac{e^x}{x}-\int_{-x}^{-y}\frac{e^{-t}}{t}dt\right),$$

where $\lambda/L=-t$, $\lambda/a=x$, and $\lambda/b=y$.

By using the function[7] $E^*(x)=-\int_{-x}^\infty e^{-t}dt/t$ the integral reduces to

$$\int_a^b P_I(L)dL=10^8P_\infty(a)\lambda$$
$$\times[(e^y/y)-(e^x/x)+E^*(x)-E^*(y)], \quad (26)$$

where λ is expressed in cm and $P_\infty(a)$ is expressed in A^{-1}.

Finally we calculate $\int_{L_0}^{5/4L_0}P_0(L)dL$. It follows from (9) that

$$P_0(L)\sim P_\infty(L)\exp\left\{-\frac{\lambda}{L}+\frac{B_0}{F_0}\right.$$
$$\left.\times\left[\left(1-\frac{F_0z_3}{V_I}\right)^{\frac{3}{2}}-\left(1-\frac{F_0z_1}{V_I}\right)^{\frac{3}{2}}\right]\right\}, \quad (27)$$

[6] A. Erdelyi *et al.*, *Higher Transcendental Functions* (McGraw-Hill Book Company, New York, 1953), Vol. 2, p. 133.

[7] A. Erdelyi *et al.*, *Higher Transcendental Functions* (McGraw-Hill Book Company, New York, 1953), Vol. 2, p. 143. [See also E. Jahnke and F. Emde, *Tables of Functions* (Dover Publications, New York, 1945), p. 83.]

where F_0 represents the field strength at L_0. Because z_1 and $P_\infty(L)$ may be regarded as constants for purposes of the integration in the interval $(L_0, 5/4L_0)$, (27) becomes:

$$P_0(L) \approx M \exp\left[\frac{\lambda}{L}+\frac{B_0}{F_0}\left(1-\frac{F_0 z_3}{V_I}\right)^{\frac{3}{2}}\right]$$

where

$$M \equiv P_\infty(L_0) \exp\left[-\frac{B_0}{F_0}\left(1-\frac{F z_1}{V_I}\right)^{\frac{3}{2}}\right]. \quad (28)$$

It has already been shown that

$$z_3=\frac{V_I+2\phi}{F_0+3\phi/L}=\frac{V_I+2\phi}{F_0+3\phi/L_0-3\phi[(1/L_0)-(1/L)]}$$

$$\approx L_0\left[1+\frac{3\phi}{V_I+2\phi}\left(1-\frac{L_0}{L}\right)\right]. \quad (29)$$

For helium, with $L=5/4L_0$, one finds that

$$\frac{3\phi}{V_I+2\phi}\left(1-\frac{L_0}{L}\right)=0.08$$

which shows that the approximation (29) was permissible. Substitution of (29) into $(1-F_0z_3/V_I)^{\frac{3}{2}}$ leads to

$$\left(1-\frac{F_0 z_3}{V_I}\right)^{\frac{3}{2}}=\left(\frac{\phi}{V_I}\right)^{\frac{3}{2}}\left[1-\frac{3(V_I-\phi)}{V_I+2\phi}\left(1-\frac{L_0}{L}\right)\right]^{\frac{3}{2}} \quad (30)$$

$$\approx\left(\frac{\phi}{V_I}\right)^{\frac{3}{2}}\left[1-\frac{9(V_I-\phi)}{2(V_I+2\phi)}\left(1-\frac{L_0}{L}\right)\right]. \quad (31)$$

By using this approximation one obtains:

$$\int_{L_0}^{5/4L_0} P_0(L)dL$$

$$=10^8 P'(L_0)\exp(-qB_0/F_0)\int_{L_0}^{5/4L_0} e^{\rho/L}dL$$

$$=10^8 P'(L_0)\rho\exp(-qB_0/F_0)\left[\frac{e^{4\rho/5L_0}}{4\rho/5L_0}-\frac{e^{\rho/L_0}}{\rho/L_0}\right.$$

$$\left.+E^*(\rho/L_0)-E^*(4\rho/5L_0)\right], \quad (32)$$

where

$$P'(L_0)=P_\infty(L_0)\exp\left\{\frac{B_0}{F_0}\left[\left(1-\frac{F_0L_0}{V_I}\right)^{\frac{3}{2}}-\left(1-\frac{F_0z_1}{V_I}\right)^{\frac{3}{2}}\right]\right\}$$

and z_1 is given by (9), while

$$\lambda=\frac{3.6\times10^{-8}}{eV_I}\left[\frac{3B_0}{2F_0}+\frac{3}{14}U(F_0)\right],$$

$$\rho\equiv\frac{B_0}{F_0}qL_0+\lambda, \quad\text{and}\quad q\equiv\frac{9}{2}\left(\frac{\phi}{V_I}\right)^{\frac{3}{2}}\frac{V_I-\phi}{V_I+2\phi}.$$

The final result may be written in the form:

$$\int_{L_0}^{\infty}P(L)dL=\int_{L_0}^{5/4L_0}P_0(L)dL$$

$$+\int_{5/4L_0}^{3/2L_0}P_I(L)dL+\int_{3/2L_0}^{\infty}P_\infty(L)dL, \quad (33)$$

where the terms on the right are given by Eqs. (32), (26), and (21), respectively.

In the case of large fields we can use the formula

$$\int_{L_0}^{\infty}P(L)dL=\int_{L_0}^{3/2L_0}P_I(L)dL+\int_{3/2L_0}^{\infty}P(L)dL, \quad (34)$$

while for small fields the last term in (33) may be omitted. It must however be pointed out that the choice of the intervals in which $P_0(L)$, $P_I(L)$, and $P_\infty(L)$ are applicable is somewhat arbitrary.

Müller has also shown that the field ion current is approximately given by

$$I^+=3eZ\int_{L_0}^{\infty}PdL, \quad (35)$$

and the total current I by $I=(\gamma+1)I^+$, where γ is the number of secondary electrons released per ion.

The supply function Z has been derived by Inghram and Gomer.[8] Values of I for helium calculated from the Eqs. (33) and (34) are in reasonable agreement with the experimental values given by Müller.[1]

4. ACKNOWLEDGMENTS

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[8] M. G. Inghram and R. Gomer, Z. Naturforsch. **10a**, 863 (1955).