

radiation on the absorption should become negligible, as it is in the case of heat conduction, because U processes are becoming much more frequent than collisions with damage centers. In all these points there seems to be good qualitative agreement with our experiments. (See Figs. 6, 7.) A more quantitative comparison between Berman's results and our data appears difficult, because the mean free path limiting the heat conduction in an irradiated crystal and the mean free path responsible for sound absorption are not necessarily the same.

We would like to emphasize that the relaxation process which we have sketched here has been applied only in the absorption of *longitudinal* hypersonic waves

in quartz. Whether it also causes the absorption of transverse waves cannot be answered until data about the effect of shear strain on the elastic constants in quartz are available.

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Nonlocal Current-Field Relationship in Metals*

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It is found that Chambers' formula for the response of the conduction electrons of a metal to an internal transverse electric field must be supplemented by a diffusion current when it is used for longitudinal fields. A new derivation of Chambers' formula is given by means of the Boltzmann transport equation, and the additional diffusion term in the current is exhibited explicitly.

THE purpose of this note is to point out that the well-known and widely accepted formula due to Chambers¹ for the response of a metal to an internal transverse electrical field is actually not immediately applicable to a longitudinal field, but can be used provided that it is supplemented by an additional term. No error exists in the applications of this relationship by Chambers and Pippard² to the determination of the anomalous skin depth because of the restriction to transverse fields. But with the increasing interest in the propagation and attenuation of ultrasound in metals,³

as well as in other phenomena in which longitudinal fields are generated (e.g., screening and slowing down of moving point charges), it seems desirable to improve Chambers' formula so that it is also correct for such fields. Such a diffusion current term is even mentioned by Chambers,¹ but it does not seem to have appeared in the published literature. Chambers' formula, including the additional term, will be exhibited explicitly below, but for the purpose of illustrating the basic idea in the present paper, it will be useful first to consider the special case of a static electric field. If, in addition, the electric field varies only slowly in space (with the scale taken as the mean free path of the conduction electrons in the metal), then Chambers' formula reduces simply to Ohm's law

$$\mathcal{J}_C(\mathbf{x}) = \sigma_0 \mathcal{E}(\mathbf{x}). \quad (1)$$

This is a completely local relationship between the electric field \mathcal{E} and the current density \mathcal{J}_C at the point \mathbf{x} . σ_0 is the dc conductivity of the metal. It is immediately clear, however, that this contribution to the current cannot represent a steady state solution for the response of the electrons to a longitudinal electric

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¹ R. G. Chambers, Proc. Phys. Soc. (London) **A65**, 458 (1952).

² A. B. Pippard, *Advances in Electronics and Electron Physics* (Academic Press, Inc., New York, 1954), Vol. VI. This article gives a brief derivation of Chambers' relationship from kinetic theory and except for the omission of the diffusion current, presents it in the same form as in the present paper.

³ W. P. Mason and H. E. Bömmel, J. Acoust. Soc. Am. **28**, 930 (1956). In the study of the ultrasonics of metals it is essential also to take into account not only the diffusion current, but also the additional current resulting from the collision drag effect. We make no attempt to treat this additional modification here. For detailed treatments of this effect see the work of Steinberg (reference 7), the recent paper of Holstein [T. Holstein, Phys.

Rev. **113**, 479 (1959)], and the thesis of J. L. Warren, University of Maryland, 1959 (unpublished). In the last named reference the attenuation and dispersion of phonons is worked out using the dielectric constant approach. The results are identical to those of Steinberg (reference 7).

field. The currents which flow will, in such a case, cause charge to concentrate in certain regions of the metal and thereby produce an excess in the electron density, $n(\mathbf{x})$, over and above the equilibrium value n_0 . The resulting concentration gradients will give rise to the diffusion current density

$$\mathcal{J}_D(\mathbf{x}) = eD \text{grad} n(\mathbf{x}), \quad (2)$$

where the diffusion constant is given in terms of the mean free path l and the Fermi velocity v_0 by the usual formula

$$D = lv_0/3. \quad (3)$$

Consequently the total current density is the sum

$$\mathcal{J} = \mathcal{J}_C + \mathcal{J}_D. \quad (4)$$

This equation contains the essential point of this paper; to give a complete description of the current in a metal, one must add to the Chambers' term the additional diffusion term.

To complete this illustration, it is of interest to make use of the above equations to obtain a relationship between the electron density and the electrostatic potential $\varphi(\mathbf{x})$. Assume that the electric field contains no divergenceless parts. Then it is clear that \mathcal{J} is also an irrotational field which must vanish by virtue of the continuity equation

$$\text{div} \mathcal{J} = e \partial n / \partial t = 0. \quad (5)$$

(We continue to limit ourselves for the moment to the steady state case, with no time dependence.) By introducing the standard formula for the DC conductivity

$$\sigma_0 = n_0 e^2 l / m v_0 \quad (6)$$

we find, after making use of Eqs. (1)–(6) and carrying out an integration,

$$n(\mathbf{x}) = (3n_0 e / m v_0^2) \varphi(\mathbf{x}). \quad (7)$$

This equation will be recognized as the linearized form of the basic equation of the Thomas-Fermi statistical model.⁴ The most common application of this equation is to the screening of a unit positive point charge by the electron gas.⁵ From Poisson's equation and Eq. (7) one eliminates the electron density and obtains an equation of the Yukawa type for the screened potential φ , the solution of which is

$$\varphi(\mathbf{x}) = e e^{-q|\mathbf{x}|} / |\mathbf{x}|, \quad (8)$$

where the screening constant is given by

$$q = (12\pi n_0 e^2 / m v_0^2)^{1/2}. \quad (9)$$

We hasten to add that the above derivation is valid only for $|\mathbf{x}| \gg l$. This shortcoming is remedied by the nonlocal formulation below. In any case, the essential

point is that it is only by including the diffusion portion of the current that one can obtain the familiar result expressed by Eq. (8) for the static screening of a point charge impurity. If only Chambers' contribution to the current is used, then it is not possible according to Eq. (1) for a field to exist in the metal without having a current flow. Consequently for a static situation for which there can in principle exist no current, it would follow from Chambers' relationship that there can be no field present inside the metal, or in other words, that the screening is complete. This result is, of course, not correct and raises the question of what in general is the correct current-field relationship inside a metal. With this brief orientation, we now proceed to derive Chambers' formula and the expression for the additional diffusion current for the general case of an electric field with arbitrary spatial and temporal variation.

We use transport theory and carry out calculations similar to those done by Lindhard⁶ in studying the general dielectric properties of an electron gas and by Steinberg⁷ in studying the attenuation of ultrasound in metals. There are, however, some essential differences which will be brought out as they arise. But because the method is well-known, we describe the calculation only very briefly.⁸ To begin with, we introduce the distribution function $f(\mathbf{p}, \mathbf{x}, t)$, where \mathbf{p} is the electron momentum measured at the point \mathbf{x} . Already we violate the uncertainty principle, but this is a familiar trouble with the semiclassical transport theory and is known not to lead to serious error as long as the results are applied only to disturbances which are relatively smooth in space. In other words, we actually only study the mean values of space-dependent quantities averaged over relatively large distances Δx , and require only limited resolution Δp in momentum space, so that Heisenberg's inequality $\Delta x \Delta p > \hbar$ can be satisfied.

Once the distribution function has been obtained then we can obtain the current density at any point \mathbf{x} and at any time t by means of the integral

$$\mathcal{J}(\mathbf{x}, t) = -(2e/\hbar^3 m) \int \mathbf{p} f(\mathbf{p}, \mathbf{x}, t) d^3 p, \quad (10)$$

where $-e$ and m are the electron charge and mass and \hbar is Planck's constant. (Generally we use lower case or script letters for space and time dependent quantities and capital letters for their Fourier transforms.) The distribution function must satisfy the Boltzmann transport equation

$$\frac{\partial f}{\partial t} + \frac{d\mathbf{p}}{dt} \cdot \nabla_{\mathbf{p}} f + \mathbf{v} \cdot \nabla_{\mathbf{x}} f = \left[\frac{df}{dt} \right]_{\text{coll}}, \quad (11)$$

⁶ J. Lindhard, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **28**, No. 8 (1954).

⁷ M. S. Steinberg, Phys. Rev. **111**, 425 (1958).

⁸ A derivation of Chambers' formula, evidently without the diffusion term, has been given using the Boltzmann transport equation by V. Heine, Phys. Rev. **107**, 431 (1957).

⁴ L. I. Schiff, *Quantum Mechanics* (McGraw-Hill Book Company, Inc., New York, 1955), second edition, pp. 281–282.

⁵ N. F. Mott, Proc. Cambridge Phil. Soc. **32**, 281 (1936). See also J. Friedel, Phil. Mag. **43**, 153; 1115 (1952).

where $\mathbf{v} = \mathbf{p}/m$ is the electron velocity. The collision term in the right-hand member represents the effect of collisions at the point \mathbf{x} . These collisions carry particles both into and out of any given elementary volume of momentum space. The collision term can be represented by the sum of two relaxation terms:

$$\left[\frac{df}{dt} \right]_{\text{coll}} = -\frac{f - \langle f \rangle_{\Omega}}{\tau} - \frac{\langle f \rangle_{\Omega} - f_F}{\tau'} \quad (12)$$

The relaxation time τ is the same as that of the theory of resistivity and corresponds to the scattering of electrons *elastically* by the impurities and lattice defects. The quantity $\langle f \rangle_{\Omega}$ is simply the distribution function averaged, or, better, "smeared out" over the directions of \mathbf{p} . The second term in the right-hand member represents the *inelastic* processes which bring about the relaxation of this angularly smeared out distribution to that of the degenerate gas at zero temperature. (We limit ourselves in this paper to nonsuperconducting metals at absolute zero.) This relaxation requires that heat be carried away by phonons. Consequently the corresponding mean relaxation time τ' can be expected to be very much greater than τ .^{9,10} The distribution f_F corresponds to complete Fermi degeneracy at the local density $n(\mathbf{x}, t)$:

$$\begin{aligned} f_F(\mathbf{p}, \mathbf{x}, t) &= 1, & |\mathbf{p}| < p_0(\mathbf{x}, t) \\ &= 0, & |\mathbf{p}| > p_0(\mathbf{x}, t). \end{aligned} \quad (13)$$

Here we have introduced the local Fermi momentum which in its linearized form for very small density changes, $n(\mathbf{x}, t) \ll n_0$, is given by

$$p_0(\mathbf{x}, t) = p_0 + (p_0/3n_0)n(\mathbf{x}, t). \quad (14)$$

Basing f_F on the local density rather than on the unperturbed density n_0 is essential, and is the feature which distinguishes our treatment from most previous ones. Without this specification for f_F , the requirement of local conservation of charge cannot be satisfied. The usual procedure of basing f_F on n_0 instead of on the actual local density corresponds to assuming a relaxation process capable of annihilating or creating electrons at the point \mathbf{x} , [depending upon the sign of $n(\mathbf{x}, t)$]. Such a process does not, of course, represent the true relaxation mechanisms operative in the metal, and cannot lead to completely correct results.

⁹ For a discussion of the difference in these relaxation processes see H. W. Lewis, *Solid State Physics*, edited by F. Seitz and D. Turnbull (Academic Press, Inc., New York, 1958), Vol. VII, p. 353. Actually, as emphasized by Steinberg (reference 7), this cooling process cannot be represented by a mean relaxation time, since the relaxation rate is strongly dependent upon the electron temperature. For this reason the term in Eq. (12) containing τ' should be regarded as only a schematic representation of the process by which the heat generated in the ohmic process is carried away. Since our calculation is not sensitive to the cooling, it is not necessary to formulate it more precisely in Eq. (12).

¹⁰ Steinberg (reference 7) has, however, taken into account the relaxation with respect to the local density in his calculation of ultrasonic attenuation, and our work is quite similar to his, in this respect.

Only the angular dependent part of the distribution

$$f_1 = f - \langle f \rangle_{\Omega} \quad (15)$$

contributes to the current in Eq. (10). In the absence of a force on the electrons, f_1 vanishes. Hence, treating the force $d\mathbf{p}/dt$ as a weak perturbation and working only to lowest order, we neglect $(d\mathbf{p}/dt) \cdot \nabla_{\mathbf{p}} f_1$ in Eq. (11). A further approximation is indicated by the fact that the distributions $\langle f \rangle_{\Omega}$ and f_F are very similar. The former is merely a slightly radially smeared out form of the latter, corresponding to the Fermi distribution at a finite but very small temperature. The difference will be important only in the problem of determining the thermal gradient in the metal and can be neglected here. With this approximation, a differential operator applied to f_F is simply proportional to the delta function at the Fermi surface. Thus, with $f_1 = f_S \delta(p - p_0)$, we obtain

$$\left(\mathbf{v} \cdot \nabla_{\mathbf{x}} + \frac{\partial}{\partial t} + \frac{1}{\tau} \right) f_S = \frac{\mathbf{p} \cdot (d\mathbf{p}/dt)}{p_0} - \frac{p_0}{3n_0} \left(\mathbf{v} \cdot \nabla_{\mathbf{x}} + \frac{\partial}{\partial t} \right) n, \quad (16)$$

where f_S depends upon \mathbf{p} only through its direction. If \mathcal{E} and \mathcal{H} are the electric and magnetic fields in the metal, the acceleration term in Eq. (16) becomes

$$\begin{aligned} \mathbf{p} \cdot (d\mathbf{p}/dt) &= -e\mathbf{p} \cdot [\mathcal{E} + (1/c)\mathbf{v} \times \mathcal{H}] \\ &= -e\mathbf{p} \cdot \mathcal{E}. \end{aligned} \quad (17)$$

Note that the Lorentz force disappears, so that a weak magnetic field produces no current in the present calculation. Thus the Landau diamagnetism, which is a weak quantum effect, is not accounted for by the semiclassical transport theory. This establishes the order of magnitude of the error to be expected by the violation of the uncertainty principle, as discussed above. By comparison with Lindhard's quantum mechanical treatment¹¹ (for the special case of infinite mean free path) we see that the fractional error due to the neglect of the Landau diamagnetism is of the order of $[\hbar^2 k^2 / (2m\hbar\omega)]^2$. The error is therefore quite small for long wavelengths, and not-too-small frequencies. Even for static fields, however, the actual magnitude of the error is generally negligible, corresponding to a susceptibility of the order of magnitude of only 10^{-4} .

To solve Eq. (16), it is convenient to Fourier analyze the spatially dependent quantities f_S , n , and \mathcal{E} . These can be taken to be superpositions of plane waves of the form $\exp(i(\mathbf{k} \cdot \mathbf{x} - \omega t))$. Consequently we find for the Fourier transforms the relationship

$$\begin{aligned} F_S(\mathbf{p}, \mathbf{k}, \omega) &= -\frac{(e/p_0)\mathbf{p} \cdot \mathbf{E}(\mathbf{k}, \omega) + (ip_0/3n_0)(\mathbf{k} \cdot \mathbf{v} - \omega)N(\mathbf{k}, \omega)}{i(\mathbf{k} \cdot \mathbf{v} - \omega) + \tau^{-1}}. \end{aligned} \quad (18)$$

¹¹ See reference 6, Eq. (3.18).

When this equation is substituted into the Fourier transform of Eq. (10) and the integrations carried out, it is convenient to consider the transverse and longitudinal cases separately. One finds the following expression for the transverse part of the current:

$$\mathbf{J}^t(\mathbf{k}, \omega) = \sigma^t(k, \omega) \mathbf{E}^t(\mathbf{k}, \omega), \quad (19)$$

where the expression for the conductivity is

$$\sigma^t(k, \omega) = \frac{3in_0e^2}{4mkv_0} \left(2u' + (1-u'^2) \ln \frac{u'+1}{u'-1} \right). \quad (20)$$

Here we have introduced the abbreviation $u' = u + ia$, where $u = \omega/(kv_0)$ and $a = 1/(kv_0\tau) = 1/(kl)$. It should be noted that Eq. (20) is completely equivalent to Lindhard's expression for the transverse dielectric constant. This is so even though Lindhard claims no validity for his calculation for the case of finite mean free path; he regards τ^{-1} as infinitely small and as only introduced for convenience in the mathematical manipulations. This restriction does, however, apply to his longitudinal dielectric constant.

The integrations for the longitudinal case are also readily carried out and yield

$$J^l(\mathbf{k}, \omega) = \left(1 + \frac{u'}{2} \ln \frac{u'-1}{u'+1} \right) [-3i(n_0e^2u'/mkv_0)E^l(\mathbf{k}, \omega) + i(p_0ea/m)N(\mathbf{k}, \omega)]. \quad (21)$$

The density can be eliminated by means of the equation of continuity:

$$N(\mathbf{k}, \omega) = -(k/e\omega)J^l(\mathbf{k}, \omega). \quad (22)$$

Thus we find

$$J^l(\mathbf{k}, \omega) = \sigma^l(k, \omega) E^l(\mathbf{k}, \omega), \quad (23)$$

where

$$\sigma^l(k, \omega) = -3i \frac{n_0e^2u'}{mkv_0} \left(1 + \frac{u'}{2} \ln \frac{u'-1}{u'+1} \right) / \left[1 + i \frac{a}{u} \left(1 + \frac{u'}{2} \ln \frac{u'-1}{u'+1} \right) \right]. \quad (24)$$

It may be noted at this point that Eqs. (22)–(24) establish a relationship between $N(\mathbf{k}, \omega)$ and the Fourier coefficient of the electrostatic potential, $\Phi(\mathbf{k}, \omega) = ik^{-1}E^l(\mathbf{k}, \omega)$. In the static limit ($\omega=0$) this reduces to the simple form

$$N(\mathbf{k}, 0) = (3n_0e^2/mv_0^2)\Phi(\mathbf{k}, 0), \quad (25)$$

thereby providing, as promised earlier, a proof of Eq. (7) not restricted to wavelengths longer than the mean free path.

The pairs of Eqs. (19) and (20), and (23) and (24) solve in principle the problem of calculating the response of the metal to an arbitrary electric field. It is only necessary to find the Fourier transform of the field, and then to substitute it into these equations in order

to obtain the corresponding Fourier coefficients of the current. The inverse Fourier transform will then give the space and time dependent current distribution in the metal. This procedure involves two integrations in four-dimensional space-time. It would be a useful simplification to make the standard interchange of order and to carry out once and for all the integration over \mathbf{k}, ω space. This would yield a Green's function, and would leave only one remaining integration to perform for any given field distribution. Unfortunately, because of the complexity of Eq. (24), it does not seem possible to obtain the Green's function in closed form. This is, however, possible for the separate terms of Eq. (21). Although the resulting expression is of limited practical value, since it contains the density which must still be eliminated by Eq. (22), it will nevertheless be of interest as a means of explicitly exhibiting Chambers' formula and the diffusion addition to it. It is most convenient to return to Eq. (18) and to introduce an auxiliary variable ξ , which enables us to write the denominator of the distribution function as

$$\begin{aligned} v_0^{-1} \left[i \left(\frac{\mathbf{k} \cdot \mathbf{v}}{v_0} - \frac{\omega}{v_0} \right) + \frac{1}{l} \right]^{-1} \\ = v_0^{-1} \int_0^\infty \exp \left\{ -\xi \left[i \left(\frac{\mathbf{k} \cdot \mathbf{v}}{v_0} - \frac{\omega}{v_0} \right) + \frac{1}{l} \right] \right\} d\xi \\ = v_0^{-1} \int_0^\infty \exp[-i(\mathbf{k} \cdot \boldsymbol{\xi} - \omega\xi/v_0)] e^{-\xi/l} d\xi. \end{aligned} \quad (26)$$

Here we have also introduced a vector $\boldsymbol{\xi}$, of magnitude ξ and direction \mathbf{v} . Denoting the portion of the current which depends explicitly on the field (i.e., excluding the implicit dependence through the density) by a subscript C , we find by substituting from Eqs. (26) and (18) into Eq. (10),

$$\begin{aligned} \mathbf{J}_C(\mathbf{k}, \omega) = \frac{3}{4\pi} \frac{n_0e^2}{mv_0} \int d^3\xi e^{-\xi/l} \\ \times \xi \frac{\mathbf{E}(\mathbf{k}, \omega)}{\xi^4} \exp[-i(\mathbf{k} \cdot \boldsymbol{\xi} - \omega\xi/v_0)]. \end{aligned} \quad (27)$$

Making the Fourier inversion, we obtain

$$\begin{aligned} \mathcal{J}_C(\mathbf{x}, t) = \int d^3k d\omega \mathbf{J}_C(\mathbf{k}, \omega) \\ = \frac{3}{4\pi} \frac{n_0e^2}{mv_0} \int d^3\xi e^{-\xi/l} \frac{\xi}{\xi^4} \boldsymbol{\xi} \cdot \mathcal{E}(\mathbf{x}', t'), \end{aligned} \quad (28)$$

precisely the Chambers' formula. Here we have introduced the variables $\mathbf{x}' = \mathbf{x} - \boldsymbol{\xi}$ and $t' = t - \xi/v_0$. In the special case that the field is static and does not vary appreciably over one mean free path, it may be taken

outside the integral. The latter then reduces to $4\pi l/3$ times the unit dyadic, and Eq. (28) becomes simply the local relation of Eq. (1).

The portion of the current which depends explicitly on the density, denoted by subscript D , is calculated in parallel fashion. Note that in the numerator of Eq. (18) we can write

$$\mathbf{k} \cdot \mathbf{v} - \omega = (\mathbf{k} \cdot \mathbf{v} - \omega - i\tau^{-1}) + i\tau^{-1}, \quad (29)$$

where the quantity in parentheses cancels with the denominator and yields no current. Thus we find

$$\mathbf{J}_D = -\frac{ev_0}{4\pi l} \int d^3\xi e^{-\xi/l} \xi^{-3} \times \xi N(\mathbf{k}, \omega) \exp[-i(\mathbf{k} \cdot \xi - \omega\xi/v_0)] \quad (30)$$

so that the Fourier inversion gives

$$\mathcal{J}_D(\mathbf{x}, t) = -(ev_0/4\pi l) \int d^3\xi e^{-\xi/l} \xi^{-3} \xi n(\mathbf{x}', t'). \quad (31)$$

This is the diffusion current which must be added to Chambers' expression whenever longitudinal fields are acting in the metal. Here again it is of interest to check

the static limit for fields which vary slowly in space. Then we have, approximately

$$n(\mathbf{x}', t') \approx n(\mathbf{x}, t) - \xi \cdot \text{grad} n(\mathbf{x}, t). \quad (32)$$

The density gradient may now be taken outside the integral, so that the nonlocal, retarded diffusion formula of Eq. (31) reduces for this special case to Eq. (2), the simple uniform-gradient expression of kinetic theory.

To summarize, we began by emphasizing for the limiting case of time independence that it is necessary to include diffusion along with ohmic flow when describing longitudinal electric fields in metals. We then sketched a transport theory derivation of the relationship between the Fourier coefficients of the current and field. The result of this work is expressed by the conductivity functions of Eqs. (20) and (24). The diffusion is automatically included in the Boltzmann equation provided that the relaxation is based on the local density, rather than on the unperturbed density as is often done. Finally, we exhibited the explicit nonlocal, retarded relationship of current to field in the form of two integrals. The first is exactly Chambers' formula, while the second, given by Eq. (31), is the new diffusion term.

Strong-Coupling Limit in the Theory of Superconductivity*

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The Hamiltonian used by Bardeen, Cooper, and Schrieffer in their theory of superconductivity is studied in the strong-coupling limit. The complete set of energy levels can be found by using group theory, even for a finite system. An expression for the grand partition function can immediately be written down, and this expression can be evaluated in a simple manner for a large system. The results are in qualitative agreement with the weak-coupling theory, and in quantitative agreement with the strong-coupling limit of the expressions derived by Bardeen, Cooper, and Schrieffer. The second-order phase transition is a simple consequence of the form of the grand partition function. There is an energy gap independent of the total number of particles which goes to zero as the temperature approaches the critical temperature. The normal state is not metastable below the critical temperature.

1. INTRODUCTION

IN the theory of superconductivity developed by Bardeen, Cooper, and Schrieffer¹ (which we shall refer to as BCS) a system of interacting fermions with spin one-half is considered. The only interactions which are taken into account in the BCS theory are those between particles with opposite spin and momentum. Because of this, the single-particle states are paired off with each other. Each "pair state" consists of a single-particle state with a particular momentum and spin

direction, together with the state of opposite momentum and spin. The interaction then carries a pair of particles from one pair state to another pair state. The kinetic energy is, of course, diagonal in the pair states, since it is diagonal in the single-particle states. If a pair state is singly occupied at any time, it remains singly occupied, since there is no other particle with which the one particle can interact, and since no pair of particles can scatter into an already occupied pair state.

This problem was studied in BCS by a variational method. It was shown by Bogoliubov, Zubarev, and Tserkovnikov² that the perturbation series for the part

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¹ Bardeen, Cooper, and Schrieffer, *Phys. Rev.* **108**, 1175 (1957).

² Bogoliubov, Zubarev, and Tserkovnikov, *Doklady Akad. Nauk S.S.S.R.* **117**, 788 (1957) [translation: *Soviet Phys.-Doklady* **2**, 535 (1957)].