Elastic Scattering of Heavy Nuclei*

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A program is presented for determining the differential cross section for the elastic scattering of heavy nuclei. It utilizes the unitary property of the S matrix and a less drastic L dependence of the absorption than the sharp cutoff model of Blair. It is shown that experimental data can be fitted quite well.

ANGULAR distribution data of the elastic scattering of heavy ions are becoming more prevalent, and it is interesting to try to analyze them from as simple considerations as possible. Among the first data published are those of Reynolds and Zucker[1] of N^{14}-N^{14} scattering at bombarding energies ranging from 15 to 21.7 Mev, analyzed by them in terms of a model proposed by Blair for α-particle scattering. In this model one assumes that all particles of angular momentum L which would correspond to classical particles actually touching are completely absorbed from the Coulomb wave of the incident beam, thus producing shadow scattering. The critical value of L determines a sort of nuclear radius, in Reynolds' and Zucker's analysis, amounting to $R_{\text{nucleus}}=1.66A^{\frac{1}{3}}10^{-13}$ cm, somewhat higher than nuclear radii defined by other means. Porter[2] has also analyzed the same data with an optical model and has been just as successful in fitting the data with this quite different approach.

The interpretation of $N^{14}+N^{14}$ elastic scattering has been under consideration from a more general viewpoint for some time.[3] This makes use of general properties of the scattering matrix combined with plausible assumptions regarding the variation of the absorption cross section with the orbital angular momentum quantum number L. It has been proposed to the writers by Professor G. Breit and some of the relevant effects have already been discussed by him.[4] Although the calculations have been completed so far only in a specialized form it appears worth while to report them since they demonstrate the possibilities of obtaining fits to experiment without the physical inconsistency of abnormally large nuclear radii involved in the application of the Blair procedure and also without the unnatural assumption of the existence of a static potential in the description of the interaction of two complex quantum mechanical systems. To simplify the considerations it was assumed that L, the orbital angular momentum in units of $\hbar$, is a good quantum number. The further simplification is then made that for each L wave, there are only two channels, with two corresponding matrix elements, one for the elastic scattering, S_{LSc}, and the other into which are lumped all inelastic scattering and reaction processes, S_{LR}. The S-matrix elements are then estimated as follows: a "nuclear radius," or distance beyond which nuclear forces become negligible, is assumed, e.g., $R_{\text{nucleus}}=1.45A^{\frac{1}{3}}10^{-13}$ cm. At twice this radius, one determines $F_L^{(c)}(\rho)$ where $F_L^{(c)}$ is the Coulomb function regular at the origin and satisfying the equation $d^2F_L^{(c)}(\rho)/d\rho^2+\{1-(2\eta/\rho)-[L(L+1)/\rho^2]\}F_L^{(c)}(\rho)=0$ with the following meaning of the symbols: $\eta=Z_1Z_2e^2/\hbar v$ where v is incident velocity, Z_1,Z_2 are, respectively, charges on incident and target particles, $k/(2\pi)$ is the wave number, μ the reduced mass, while $\rho=kr$. The $F_L^{(c)}(2kR_{\text{nucleus}})$ so calculated may then be used to estimate the reaction matrix element from the following reasoning. Since so many inelastic and reaction pro-

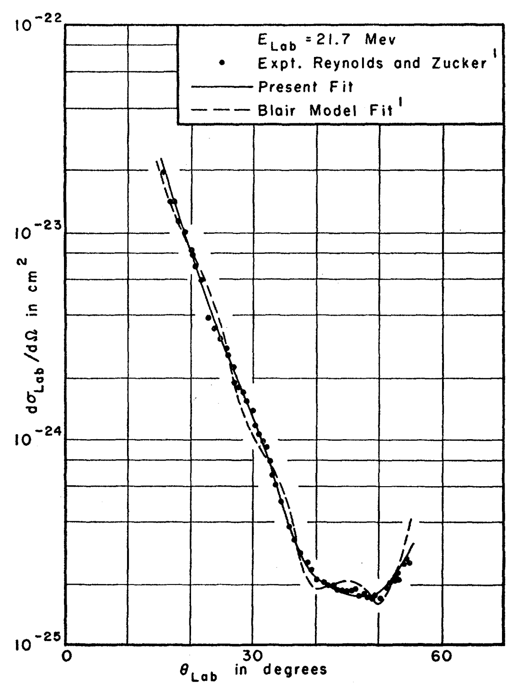

FIG. 1. Differential cross section for the scattering of 21.7-Mev nitrogen by nitrogen as a function of θ in the laboratory system. The vertical scale is logarithmic with the divisions 0.2, 0.4, 0.6, 0.8, 1.0 indicated. The Blair curve agrees with that of reference 1 except at 90° where the above curve is higher.

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1 H. L. Reynolds and A. Zucker, Phys. Rev. **102**, 1378 (1956).
2 Charles E. Porter, Phys. Rev. **112**, 1722 (1958).
3 J. E. Turner, J. S. McIntosh, and S. C. Park, Bull. Am. Phys. Soc. **3**, 223 (1958); J. S. McIntosh, Proceedings of the Conference on Heavy Nuclei, Gatlinburg, ONRL-2606 (1958), p. 181.
4 G. Breit, *Encyclopedia of Physics* (Springer-Verlag, Berlin, 1959), Vol. 41, Part 1, p. 403.

cesses can occur for complex nuclei, it may be assumed that if a particle from one nucleus gets inside the other nucleus, it causes the latter to be removed from the *elastic* scattering.

The actual procedure adopted for the calculation is as follows: (1) For "distant" collisions the true radial wave function times ρ will not differ markedly from $F_L^{(c)}(\rho)$, so that the "reaction" matrix element is taken roughly proportional to $|F_L^{(c)}(2kR_{\text{nucleus}})|^2$ for high L. The criterion chosen for considering L to be high enough for such a treatment was

$$|F_L^{(c)}(2kR_{\text{nucleus}})|^2<0.3.$$

Some adjustment of the right-hand side of this inequality has been made to obtain agreement with experiment but no claim is made that the best value of this quantity has been found. For the $N^{14}-N^{14}$ scattering at 21.7 Mev, L waves such that $L\geq 6$ are so treated. (2) At the opposite extreme, those L waves for which $F_L^{(c)}(2kR_{\text{nucleus}})$ has essentially reached a maximum at $r=2R_{\text{nucleus}}$ are assumed to be completely "absorbed" from the *elastic* scattering ($|S_{LR}|=1$, $S_{LSc}=0$). It is realized that $F_L^{(c)}(\rho)$ has little to do with the actual wave function for such L waves, but it is considered not unreasonable that if $F_L^{(c)}(\rho)$ is quite large, the chance of two nuclei overlapping and thus some reaction having taken place is quite good. For the $N^{14}-N^{14}$ scattering at 21.7 Mev the $L=0,1$ waves meet this criterion, and are considered completely absorbed. (3) The $|S_{LR}|$ for intermediate L are linearly interpolated between those of the other two regions of L. With the above chosen critical values of $F_L^{(c)}(2kR_{\text{nucleus}})$ separating each of the three regions of L it is possible to impose the condition that the total reaction cross section

$$\sigma_{\text{total }R}=\frac{\pi}{k^2}2\Big[\tfrac{2}{3}\sum_{L\text{ even}}(2L+1)|S_{LR}|^2+\tfrac{1}{3}\sum_{L\text{ odd}}(2L+1)|S_{LR}|^2\Big]$$

be approximately geometrical, thus fixing the $|S_{LR}|$. By geometrical cross section one here means $\pi\mathcal{P}_{\text{crit}}^2$ where $\mathcal{P}_{\text{crit}}$ is the impact parameter for two classical particles, each of radius $R_{\text{nucleus}}=1.45\times A^{\frac{1}{3}}$ fermis, at which the two particles repelled by the Coulomb field just graze one another in passing. The employment of the geometrical cross section in the above manner is not essential to the procedure. It was used as a first guess for the normalization of the $|S_{LR}|$ graph and since agreement with experiment was obtained the normalization was not changed.

Once the $|S_{LR}|$ are determined, the magnitudes of the scattering matrix elements $|S_{LSc}|$ may be found from the unitarity property of S, viz.,

$$|S_{LSc}|^2=1-|S_{LR}|^2,$$

where $|S_{LSc}|=e^{-2\delta_{IL}}$, δ_{IL} representing the imaginary part of the phase shift. Incidentally the phases of the reaction matrix elements play no role since S_{LR} always comes in as $|S_{LR}|^2$. Therefore S_{LR} may be taken to be real without loss of generality.

TABLE I. Complex phase shifts vs L giving full curve in Fig. 1.

L	$2\delta_L$	L	$2\delta_L$
0	$i\,\infty$	6	$1.175+i\,0.026$
1	$i\,\infty$	7	$-1.159+i\,0.016$
2	$0.000+i\,0.350$	8	$-0.284+i\,0.004$
3	$0.274+i\,0.171$	9	$-0.175+i\,0.002$
4	$-2.471+i\,0.094$	10	$-0.087+i\,0.001$
5	$0.160+i\,0.062$	11	$0.000+i\,0.000$ etc.

The real parts of the phase shifts of S_{LSc}, of course, also affect the elastic scattering. Although successes with Blair's model demonstrate that in some cases the major contribution arises from the shadow scattering, this other contribution must also be estimated. In the low L region of complete absorption the real phases are, of course, irrelevant. In the high L region, the phases have been chosen to go to 0 as $L\rightarrow\infty$ and to vary with L as in an optical model calculation,[5] i.e., for high L, δ_L is taken roughly proportional to

$$\int_0^{2kR_{\text{nucleus}}}[F_L^{(c)}(\rho)]^2d\rho.$$

It has been found possible to choose the real parts of the phases of those L waves in the intermediate region so that the experimental angular distribution is reproduced.

Employment in the differential cross section

$$\sigma(\theta)=\frac{1}{k^2}\Bigg(\frac{2}{3}\Bigg|\frac{\eta}{2}\bigg(\frac{\exp(-i\eta\ln s^2)}{s^2}+\frac{\exp(-i\eta\ln c^2)}{c^2}\bigg)-\sum_{L\text{ even}}(2L+1)2P_L(\cos\theta)e^{2i(\sigma_L-\sigma_0)}\frac{S_{LSc}-1}{2i}\Bigg|^2+\frac{1}{3}\Bigg|\frac{\eta}{2}\bigg(\frac{\exp(-i\eta\ln s^2)}{s^2}-\frac{\exp(-i\eta\ln c^2)}{c^2}\bigg)-\sum_{L\text{ odd}}(2L+1)2P_L(\cos\theta)e^{2i(\sigma_L-\sigma_0)}\frac{S_{LSc}-1}{2i}\Bigg|^2\Bigg)$$

TABLE II. Center-of-mass cross sections.

	Experiment[a] (in 10^{-26} cm^2)	Model (in 10^{-26} cm^2)
$\sigma(50°)$	99.5	103.1
$\sigma(60°)$	36.5	35.1
$\sigma(70°)$	13.4	13.5
$\sigma(80°)$	7.3	7.2
$\sigma(90°)$	6.4	6.4

[a] See reference 1.

[5] Calculations with Dr. Rawitscher now in progress derive an optical model potential suitable for high L-waves, where the optical model would seem to be most sensible.

of the matrix elements determined by the above prescription gives agreement with the 21.7 Mev $N^{14}-N^{14}$ data with $S_L S_{Sc}=e^{2i\delta_L}(\delta_L$ complex) as shown in Fig. 1 and Tables I and II. Of course these results should not be taken too seriously in themselves as uniqueness is not claimed. The further calculations mentioned above should tighten the considerations, and analysis of the mounting heavy ion scattering data at different energies should provide much more rigorous tests for the method of analysis. The above considerations seem to justify the feasibility of this approach which is less extreme than that of the cutoff and optical models and emphasizes the role of the nuclear surface in the phenomena.

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Magnetic Moment of Fe^{57}

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An electron-nuclear double resonance study has been made on the spectrum of neutral iron atoms in silicon. These measurements lead to a value of $+0.0903\pm0.0007$ nm for the magnetic moment of Fe^{57}.

A STUDY of the electron spin resonance spectrum of neutral iron atoms in silicon[1] has confirmed that the nuclear spin of Fe^{57} is $\frac{1}{2}$. The present note describes electron-nuclear double resonance measurements[2] on that spectrum which lead to a value of $+0.0903\pm0.0007$ nm for the magnetic moment of Fe^{57}.

The spin Hamiltonian appropriate to $(Fe^{57})^0$ in silicon is

$$\mathcal{H}=g\beta\mathbf{S}\cdot\mathbf{H}+A\mathbf{S}\cdot\mathbf{I}-g_I'\beta_N\mathbf{H}\cdot\mathbf{I}, \quad (1)$$

where the electronic g factor and the hyperfine interaction parameter A are isotropic. The parameter g_I' is an effective nuclear g factor which also is isotropic. As will be discussed later, the electronic g departs from the free electron value (2.0023) and g_I' departs from the nuclear g factor $(\mu/I\beta_N)$ if excited electronic states are present which must be taken into account.[3] To second order in the hyperfine interaction, the frequency f of the (M,m) to $(M, m-1)$ electron-nuclear double resonance transition is

$$hf=|AM-g_I'\beta_N H -[M(2m-1)+S(S+1)-M^2]A^2/2h\nu|, \quad (2)$$

where M and m are the quantum numbers specifying the orientation of the electron spin S and the nuclear spin I, respectively, and ν is the klystron frequency. Terms higher order in A are negligible for interpretation of the results.

Some resonance parameters[4] for $(Fe^{57})^0$ in silicon are given in Table I. Since $S=1$ and $I=\frac{1}{2}$, there are three possible electron-nuclear double resonance transitions. An average of several measurements gives the following values: $f=20.943\pm0.6925$ Mc/sec for $M=\pm1$; $f=0.7096$ Mc/sec for $M=0$ ($\nu=14$, 115.4 Mc/sec; $H=4868.6$ gauss). Using all three double resonance frequencies, it is possible to determine from Eq. (2) both the sign and the magnitude of g_I'. Taking $g_I'>0$, one calculates that $g_I'=0.1828\pm0.0002$ from the low-frequency transition and $g_I'=0.1824\pm0.0009$ from the two high-frequency transitions; the two determinations agree within the experimental error. If, however, g_I' is assumed less than zero, the two determinations differ by 0.009, showing that $g_I'>0$. Thus $g_I'=+0.1828\pm0.0002$ for $(Fe^{57})^0$ in silicon.

We believe that the ground-state wave function of $(Fe^{57})^0$ is an orbital singlet. The observations that the resonance lines are sharp at temperatures as high as 78°K and that the electronic g factor is close to the free

TABLE I. Some resonance parameters for several iron spectra in silicon. The hyperfine interaction parameter A is expressed in units of 10^{-4} cm^{-1}. g_I' is an effective nuclear g factor; its sign was determined only for $(Fe^{57})^0$.

Species	S	g	$\lvert A\rvert$	g_I'
$(Fe^{57})^0$	1	2.0699	6.984	$+0.1828\pm0.0002$
$(Fe^{57})^+$	$\frac{3}{2}$ or $\frac{1}{2}$	3.524	2.985	0.1976 ± 0.0018
$(Fe^{57}Ga^{71})^0$	$\frac{1}{2}$	5.089(‖); 2.530(⊥)	1.438(‖); 4.108(⊥)	0.2071 ± 0.0008

[1] Ludwig, Woodbury, and Carlson, Phys. Rev. Letters **1**, 295 (1958).
[2] G. Feher, Phys. Rev. **114**, 1219 (1959); G. Feher, C. S. Fuller, and E. A. Gere, Phys. Rev. **107**, 1462 (1957).
[3] See, for example, J. M. Baker and B. Bleaney, Proc. Roy. Soc. (London) **A245**, 156 (1958).
[4] A more complete discussion of the spectra of iron and other transition metals in silicon has been prepared. H. H. Woodbury and G. W. Ludwig, Phys. Rev. **117**, 102 (1960).