Analysis of Alpha-Particle Elastic Scattering Experiments*

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By a simple modification of the sharp cutoff (Blair) approximation, a phase-shift analysis has been found to reproduce the experimental alpha-particle elastic scattering data from silver. Only two adjustable parameters were required to fit the experimental data for 22-Mev scattering; four parameters were used for the 40-Mev data. The uniqueness of the fits has not been determined.

The purpose of this note is to show that good fits to alpha particle elastic scattering angular distribution experiments can be obtained by using a phase shift analysis of the partial waves representing the scattered alpha particles. Two examples are shown to illustrate the method. For 22-Mev alpha particles scattered by silver,[1] two adjustable parameters are found to be sufficient to fit the experimental data; however, for 40-Mev alpha particles scattered by silver,[2] four adjustable parameters are required. It will be seen that the adjustable parameters are related to conditions of the nuclear surface where the scattering process takes place.

The method is based on the sharp cutoff model for the elastic scattering of alpha particles[3] as a first approximation. The idea for modifying the sharp cutoff model is not new as several calculations have already been made incorporating such modifications. Calculations by Wall, Rees, and Ford[1] and by Ellis and Schecter[4] using modified sharp cutoff models did not significantly improve on the sharp cutoff model results however. Substantial improvement was obtained though by Turner, McIntosh, and Park[5] for nitrogen-nitrogen scattering. The method used here is similar in approach to that in references 1 and 4; however, much better fits have been obtained to the experimental data than were obtained in these papers. A check on the results of references 1 and 4 with the computer code used in the following work indicates that an error has been made in the calculations in each of these references.[6]

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[2] Igo, Wegner, and Eisberg, Phys. Rev. **101**, 1508 (1956).

[3] J. S. Blair, Phys. Rev. **95**, 1218 (1954).

[4] R. E. Ellis and L. Schecter, Phys. Rev. **101**, 636 (1956).

[5] Turner, McIntosh, and Park, Bull. Am. Phys. Soc. **3**, 223 (1958). J. S. McIntosh, Proceedings of the Conference on Reactions of Complex Nuclei [Oak Ridge National Laboratory Report ORNL-2606 (1958), p. 181]. McIntosh, Park, and Turner, Phys. Rev. (to be published).

[6] We have recalculated several of the curves in references 1 and 4 using our IBM 650 computer code and have checked our computer calculations with hand calculations. Our calculations do not agree with the results of references 1 and 4. Professor Ford (reference 1) has kindly rechecked the results of reference 1 and has discovered an error in the calculations. Dr. Ellis (reference 4) has examined our hand calculations and has informed us that our procedure is the same as that in reference 4. Presumably, then, the modified sharp cutoff curves in reference 4 are also incorrect. The sharp cutoff curves in references 1 and 4 have been spot checked by us also and these seem to be correct. Professor Ford has also verified that the sharp cutoff calculation in reference 1 is correct.

The modifications to the sharp cutoff model in the procedure to be reported here are as follows:

(1) The amplitudes of the scattered partial waves, A_l, in the neighborhood of l_A', are varied smoothly from 0 to 1 over a range Δl_A through the relation

$$A_l=\{1+\exp[-(l-l_A')/\Delta l_A]\}^{-1}. \tag{1}$$

In the sharp cutoff model, $(\Delta l_A=0)$, $A_l=0$ for $l\leq l_A'$ and $A_l=1$ for $l>l_A'$. While it is plausible that A_l should vary smoothly from 0 for small l to 1 for large l, there are not theoretical considerations involved in selecting the particular function used in Eq. (1).

(2) A nuclear phase shift δ_l is introduced in addition to the Coulomb phase shift for partial waves with l values near l_δ' through the relation:

$$\delta_l=\delta\{1+\exp[(l-l_\delta')/\Delta l_\delta]\}^{-1}. \tag{2}$$

Again, there is no theoretical basis for this particular function. The scattering differential cross section, σ, is then related to the Rutherford scattering cross section, σ_R, by the equation

$$\sigma/\sigma_R=|-i\exp[-i\eta\ln\sin^2(\phi/2)]-\sin^2(\phi/2)/\eta \times\sum_{l=0}^{\infty}(1-A_l\exp 2i\delta_l)(2l+1)P_l(\cos\phi) \times\exp 2i(\sigma_l-\sigma_0)|^2. \tag{3}$$

Here $\eta=ZZ'e^2/\hbar v$ while the other notation is that of reference 3.

From Eqs. (1)–(3) it is seen that there are five adjustable parameters available for fitting the data: l_A', Δl_A, l_δ', Δl_δ, and δ. The two l' parameters are related semiclassically to the interaction radius of the colliding nuclei while the two Δl parameters are related to the thickness of the region in which the nuclear interaction between the colliding nuclei takes place without destruction of the identity of either of the nuclei. The remaining parameter, δ, is required to introduce the strength of the nuclear phase shift.

By using Eq. (3) and an IBM 650 computer, alpha particle scattering data on silver at 22 Mev were fitted as shown in Fig. 1. The experimental points are indicated by the dots. In this figure, δ was set equal to zero for all of the calculated curves so that only two parameters were available for adjustment: l_A' and Δl_A. Curve 1 is the sharp cutoff or Blair model $(\Delta l_A=0)$,

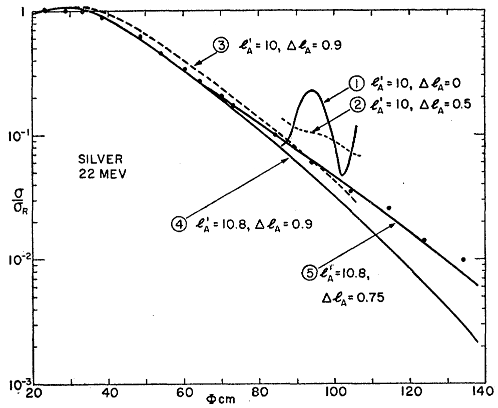

FIG. 1. The elastic scattering of 22-Mev alpha particles on silver. The dots are experimental points representing data taken in reference 1 (the points were actually taken from a curve in reference 7 which gave a logarithmic plot). The curves are calculated from Eq. 3. δ is zero in all curves. Thus, only two parameters, l_A' and Δl_A, are available for adjustment. The values of these parameters for each curve are indicated in the figure. Curve 1 ($\Delta l_A = 0$) is the sharp cutoff (Blair) model result.

for $l_A' = 10$. By following the values of the parameters from Curve 1 to Curve 5 it is seen that four reasonable changes in parameter values quickly leads to the good fit of Curve 5. An exhaustive search has not been made to determine whether other possible combinations of l_A' and Δl_A will fit the data. However, the fit was obtained by a simple succession of improved fits and seems likely to be unique. It is interesting to note here that an optical model analysis of the same experimental data yields essentially the same values of l_A' and Δl_A as have been obtained here.[7] Nevertheless, it must be emphasized that the fit with two parameters may not be the "correct" fit since δ may not be really zero. The fit with two parameters indicates, in that case, that the experimental data are not precise enough to determine δ.

At 40 Mev, the fitting procedure is more difficult. Figure 2 shows the sharp cutoff result with Curve 1. Curve 2 shows the effect of introducing the correct amount of Δl_A to give oscillations of the same amplitude as those of the experimental data. It is seen, however, that the curve lies far above the data at the large angles. An increase in Δl_A can be made to lower the curve to fit the data but then the oscillations are damped out. Thus, it was necessary to introduce additional parameters. The addition of δ, with $l_A' = l_\delta'$ and $\Delta l_A = \Delta l_\delta$ improved the fit considerably, but in order to obtain the fit shown by Curve 3, it was necessary to give different values to Δl_A and Δl_δ. Thus, four adjustable parameters were used ($l_A' = l_\delta'$ still) in order to obtain a satisfactory fit. The uniqueness of the values of the parameters to give the fit of Curve 3 has not been investigated.

While optical model calculations have been made[7–9] to give fits to alpha particle scattering data, the adjustable parameters determined by the fits are not unique and are not measureable directly by the experiment. Thus, for example, the real and imaginary nuclear potentials at the center of the nucleus are two of the parameters of the calculation. Yet, the optical model calculations have shown that only the nuclear potential at the surface affects the alpha particle scattering so that the central nuclear potentials are not physical parameters.[7,9] On the other hand, the calculations reported here introduce parameters associated with the nuclear surface. Further calculations are needed to show whether these parameters can be uniquely determined by the alpha particle scattering data.

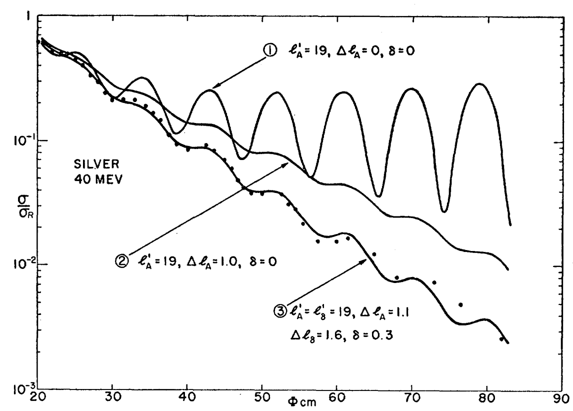

FIG. 2. The elastic scattering of 40-Mev alpha particles on silver. The dots are experimental points taken from a curve in reference 2. The curves are calculated from Eq. 3. For all curves, $l_A' = l_\delta'$. The values of the other four parameters for each curve are indicated in the figure. Curve 1 is the sharp cutoff (Blair) model result.

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