distribution, one cannot expect to draw quantitative conclusions about the third resonance, corresponding to the pion scattering peak observed at 1-Bev lab pion energy.

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Nuclear Shell Effects in μ^- Capture*

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The total capture rate for μ^- mesons in complex nuclei can give some information on the spin-dependence of the weak interaction, by utilizing the variation from one nucleus to another of the spin-dependence of the nuclear transition. The calculation was carried out for N^{14}, O^{16}, and F^{19}, using shell-model wave functions which included configurational mixing in the unfilled shell. The result is not sufficiently spin sensitive to determine the Fermi and Gamow-Teller couplings separately at this stage, but it is in accord with the universal V-A hypothesis, if a conserved vector current pion-lepton interaction is included.

I. INTRODUCTION

WHILE the idea of a universal Fermi interaction, with the same form of coupling between many pairs of fermions, is not new,[1,2] the progress made in the past few years in the elucidation of the β-decay interaction and the unifying ideas of Gell-Mann and Feyman and others[3,4] have led to a fairly well-defined form, which can be tested for other processes. It has been remarked[3] that the present information on μ decay fits this form with considerable precision, though it does not, of course, determine it uniquely.

The μ-capture process is the one most closely analogous to β decay and it is therefore of interest to find what we can about the interaction Hamiltonian. Because of the Z^4 dependence of the capture rate,[5] the experiments on hydrogen, which would give the clearest answers, are not yet possible, so we must learn what we can from the results available. These are principally just the total capture rates which have been measured for a large number of elements with $Z>4$.[6–8] Recently some measurements have also been made on C^{12} of the capture rate to a particular final state.[9]

The total capture rate reflects principally the average coupling constant and an accurate value for this is one objective of such experiments. However, because the spin-dependence of the selection rules for the nuclear transition varies from nucleus to nucleus, mainly due to Pauli exclusion effects, we may learn something about the form of the interaction. This possibility was explored in a calculation by Tolhoek and Luyten,[10] who found, on the basis of a simple shell-model picture of the nucleus, that these shell selection rules produced variations of up to 50% in the nuclear transition probabilities. Their results are, as they say, of semi-quantitative significance only. It is our object to see what modifications can be made to improve on their approximations, and what limits can be placed on the coupling constants.

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[10] H. A Tolhoek and J. Luyten, Nuclear Phys. **3**, 679 (1957).

II. FORMULATION

It is superfluous for our purposes to consider an interaction Hamiltonian of a very general form. The experiment does not test conservation of parity or other such symmetries, and therefore a simple two-component neutrino theory can be assumed. Furthermore, the momentum transfer in this process, ~ 100 Mev/c, though large compared to that typical in β decay, is only of the same order as the momenta of the nucleons in the nucleus. With a type of experiment relatively insensitive to details, and our poor knowledge of the relativistic behavior of the nuclear wave functions, we cannot hope to probe the relativistic character of the interaction. We shall therefore take a nonrelativistic form for the nuclear Hamiltonian, except that we shall include a pseudoscalar interaction which, though vanishing in the nonrelativistic limit, probably has so large a coupling constant[11,12] ($g_P \approx 8g_A$) that it is important. The modification of the "bare" Hamiltonian by virtual pion effects has been extensively discussed elsewhere,[12,9] so we shall just quote results. Our Hamiltonian density then has the form

$$H=\sqrt{2}\sum_{i=1}^{A}\{g_F(\bar{\psi}_n\tau_i^-\psi_n)(\bar{\psi}_\nu(1-i\gamma_5/2)\gamma^0\psi_\mu) -ig_A(\bar{\psi}_n\tau_i^-\boldsymbol{\sigma}_i\psi_n)(\bar{\psi}_\nu(1-i\gamma_5/2)\gamma^5\boldsymbol{\gamma}\psi_\mu) -ig_P(\bar{\psi}_n\tau_i^-(\boldsymbol{\sigma}_i\cdot\mathbf{k}/2M_p)\psi_n)(\bar{\psi}_\nu(1-i\gamma_5)\gamma^5\psi_\mu)\},$$

which gives a total capture rate

$$\lambda=\frac{2\pi}{\hbar^4 c}|\psi_\mu(0)|^2\tfrac{1}{2}\{g_F^2R_F+g_T^2R_T\}\frac{4\pi\langle k^2\rangle_{\mathrm{Av}}}{(2\pi)^3},$$

where

$$R_T=\sum_{n'}\int\frac{d\Omega_{\mathbf{k}}}{4\pi\langle k^2\rangle_{\mathrm{Av}}}|\langle n'|\sum_{i=1}^{A}\tau_i^-\boldsymbol{\sigma}_i e^{i\mathbf{k}\cdot\mathbf{r}_2}|0\rangle|^2 \times(m_\mu-E_{n'}+E_0)^2,$$

and R_F is given by the same expression with $\boldsymbol{\sigma}_i$ replaced by 1. $\mathbf{k}$ is the neutrino momentum, and $\langle n'|$ is the nuclear wave function of the n'th state. We have written

$$g_F^2=g_V^2,\quad g_T^2=g_A^2+\tfrac{1}{3}[g_P^2(k^2/4M_p^2)-2g_Ag_P(k/2M_p)].$$

The factor[8]

$$|\psi_\mu(0)|^2=\langle\rho\rangle_0=(Z^3/\pi)(m_\mu e^2/\hbar)^3\alpha,$$

where[5] $z^4\alpha=Z_{\mathrm{eff}}^4$, gives the density of the μ-meson wave function at the nucleus; α is a measure of the change in the point charge Dirac wave function caused by the finite size of the nucleus. It has recently been recalculated for many elements by Sens *et al.*[8] The variation in the μ-meson wave function over the nucleus is neglected. Thus

$$\lambda=2.33\times10^{99}\hat{k}^2Z^3\alpha[g_F^2R_F+g_T^2R_T]\ \mathrm{sec}^{-1} =\eta_Z[g_F^2R_F+g_T^2R_T]$$

where

$$\hat{k}^2=\langle k^2\rangle_{\mathrm{Av}}/100\ \mathrm{Mev}/c.$$

III. NUCLEAR MATRIX ELEMENTS

As in β decay, the principal difficulty in the calculation of μ-capture processes is always in the evaluation of the nuclear matrix elements. Tolhoek and Luyten[10] calculated total transition rates for nuclei from $A=40$ to $A=48$. They used a simple shell-model picture to evaluate the matrix elements for particular transitions and summed over the first few of these.

In this region of the periodic table accurate measurements are available. However, theoretical predictions are more reliable for light elements for which spin-dependent effects, which come from unfilled shells, should also be larger. The region around O^{16} seems to us most promising in that measurements with 10% accuracy are available, while the shell-model is at its most highly developed. Coulomb effects are small and the configurations present have been ascertained in some detail. We therefore chose to calculate the capture rate for N^{14}, O^{16}, and F^{19}.

We may avoid the explicit sum over final states by using closure on the nuclear matrix elements. This has the additional advantage that only the ground-state wave function is then required, which is less uncertain than those for the excited states. This approximation involves neglecting the variations of k in the matrix element and in the phase space factor.

The neutrino space wave function appears as a form factor in the nuclear matrix element and can be included exactly. A multipole expansion, with $kR\sim1$, is not necessary. The harmonic shape of the shell-model potential well makes calculation simple; this is an incidental advantage of light nuclei. The pseudoscalar contribution is an exception to this in that its d-wave nature is neglected, which is equivalent to taking the first multipole term exactly, but ignoring interference effects in the higher terms.

The capture rate is proportional to the mean square neutrino momentum. Unfortunately, very little information is available from which to determine this. Some measurements of nuclear excitation energies in μ capture have been made by observing the neutron multiplicity, but these are not very extensive. Calculations are unreliable, as the results are sensitive to the high momentum components of the nuclear wave function, which are rather uncertain. A reasonable estimate gives an excitation energy ~15 Mev, that is $k\sim85$ Mev/c.

The wave functions for N^{14} and F^{19} are those of Elliott[13]

[11] L. Wolfenstein, Nuovo cimento **8**, 882 (1958).
[12] M. L. Goldberger and S. B. Treiman, Phys. Rev. **111**, 355 (1958).
[13] J. P. Elliott, Proc. Roy. Soc. (London) **A218**, 345 (1954). We are grateful to Dr. Elliott for the communication of some unpublished results.

and Elliott and Flowers,[14] respectively. They include mixed configurations in the unfilled shell. The configurations that are important for N^{14} are $^{13}D_1$ (95.9%), $^{13}S_1$ (0.6%), and $^{11}P_1$ (3.5%), while for F^{19} they are d^3 (12%), d^2s (59%), and s^3 (29%). The value of the shell-model well radius is not very well determined, as the parameter to which the energy is sensitive is the ratio of it to the range of the two-body force.[15] There is reason to believe[16] from Coulomb energies that it may jump by $\sim 15\%$ on crossing the closed O^{16} shell. We shall take the radius $b=1.86\times 10^{-13}$ cm for N^{14} and O^{16} and a value 10% larger for F^{19}.

IV. RESULTS

The results of the calculation are shown in Table I, where

$$\zeta=\lambda_{\text{expt}}/\eta z R_F=g_F^2+\xi g_T^2 \quad\text{and}\quad y=g_T^2/g_F^2.$$

It is clear that the combined experimental and theoretical results are not sufficiently accurate to put any restriction on g_F and g_T as independent variables. We shall therefore make the assumption that $g_F=g_V^\beta$, the β-decay vector coupling constant and determine y. Dispersion relation analysis[12] suggests that the momentum dependence of the vector interaction will be small, so that this is a reasonable way to test an universal hypothesis.

The universal Fermi theory, with the usual estimate of the pion-induced pseudoscalar interaction ($g_p\approx 8g_A$), gives from β decay $y=1.30$. The additional pion-lepton interaction required for a conserved vector current leads to the following modifications of the coupling constants[9]

$$g_V\to g_V \quad g_A\to g_A-g_V(1+\mu_p-\mu_n)k/2M_p$$
$$g_P\to g_P-g_A-g_V(1+\mu_p-\mu_n),$$

where μ_p and μ_n are the anomalous magnetic moments of the proton and neutron, respectively. There are further relativistic terms, $\sim k/2M_p$, but not $\propto k$, which are not included here. With this additional interaction we expect $y=1.55$. Thus these results lend some support to the idea of a conserved vector current. However, if there is no conserved current the equality of the coupling constants of β decay and μ decay is not understood, so that our assumptions are more questionable.

Among the experiments, the result for F^{19} seems to possess some uncertainty, as the measurement is made on a compound (KHF_2) in which the potassium is a strong absorber of muons. The O^{16} measurement is done on water.

V. CRITIQUE

The estimation of corrections in this calculation is difficult, as often they are sensitive to factors which are not really considered in the first approximation. A case in point is that of the relativistic corrections. It is relatively easy to account for those proportional to $\mathbf{p}_n-\mathbf{p}_p=\mathbf{k}$, as this is known, but those proportional to $\mathbf{p}_n+\mathbf{p}_p$ are hard to estimate as they depend on the high-momentum components of the nuclear wave function. The exclusion principle favors such terms, so that they may well be the more important. Similarly, the accuracy of the closure approximation depends on high nuclear excitations being improbable, which in turn is governed by the high nuclear momenta. The only firm justification for both approximations is the small number of fast neutrons found experimentally.

The validity of shell-model wave functions for this process must also be examined. Their main shortcoming is the lack of any two-body correlations, apart from those coming from the exclusion principle. However, the operators for this process are not sensitive to short-range correlations ($kb\sim 1$), so that this should not be serious. Similarly the absence of higher configurations for the closed shells, such as are needed to account for the O^{17} quadrupole moment, may be justified as these states lie at higher energies. In F^{19} there are additional uncertainties that arise from crossing the closed O^{16} shell. The question of the well radius has already been mentioned. There is also the possibility that $\hat{k}^2$ is larger because there are more low-lying excited states.

Each of these sources seems capable of introducing errors of 5–10%, so that the choice of these nuclei as a balance point between theoretical and experimental uncertainty still seems reasonable. However, it seems that this balance is not enough to give any real limitation on the form of the interaction. As one should expect, wave functions which include more configurations smooth out the sharper selection effects. It seems to us unlikely that this method can be improved to the point where it would usefully limit g_T/g_F, essentially because $\xi=3$. Other elements (C^{12}, B^{10}) might be tried, but the uncertainties seem general. As a test for universality it may be useful, and it may also be used in reverse to throw some light on the usefulness of the shell model for processes with large momentum transfer. Other

TABLE I. Comparison of results. The symbols are defined in the text.

	N^{14}	O^{16}	F^{19}
R_F	1.83	2.07	2.29
$(1/3)R_T$	2.24	2.07	2.31
$\xi=R_T/R_F$	3.66	3	3.06
$\eta z R_F$	0.84×10^{102}	1.37×10^{102}	1.79×10^{102}
λ_{expt} (sec^{-1})	0.86 ± 0.11[a] (0.93 ± 0.11)[b]	1.59 ± 0.14[a] (1.38 ± 0.12)[b]	2.54 ± 0.22[a] 2.72 ± 0.20[c]
ζ	10.2×10^{-98}[a] (11.1×10^{-98})[b]	11.6×10^{-98}[a] (10.8×10^{-98})[b]	12.2×10^{-98}[a] 13.1×10^{-98}[c]
y $(g_F=g_V^\beta)$	1.2	1.6	1.7

[a] See reference 8. [b] See reference 6. [c] See reference 7.

[14] J. P. Elliott and B. Flowers, Proc. Roy. Soc. (London) **A229**, 536 (1955).
[15] J. Swiatecki, Proc. Roy. Soc. (London) **A205**, 238 (1952).
[16] J. P. Elliott and A. M. Lane, Phys. Rev. **96**, 1160 (1954).

methods now available are more sensitive to the form of the interaction.

It is of some interest to compare this work with other calculations of the total capture rate in the closure approximation[17] even though these have a somewhat different purpose, namely to find the gross variation of the rate over the periodic table as well as an estimate of the total effective coupling constant ζ. They ignore the local variations in which we are interested by using an average nuclear model. The rather small variations that we find in a model that, if anything, has too sharp features suggests that for $Z>8$, this is a reasonable approximation. It is still not easy to make a good calculation and in neither case does the predicted Z variation seem soundly based.

Primakoff includes all the angular momentum states in the same inexact fashion used here, that is, neglecting spin-orbit interference effects that arise from the effective pseudoscalar term. This approximation is probably as good as taking one extra term in the multipole expansion. However, he makes a more serious approximation in treating the exclusion principle as a correction term to the capture on Z free protons, just linear in the relative neutron excess. This correction, as one expects, cancels 75–90% of the main term so that a 5% error or variation with Z of the mean nucleon correlation distance d (d^3 is the measure of the exclusion principle that enters) changes the result by 100%. Such an effect can come, for instance, from the change in importance of the nuclear surface as Z increases. Tolhoek has improved on this by assuming that the Pauli cancellation is complete in the zeroth order of the multipole expansion; he then calculates the next order. However, for large Z, electromagnetic effects, which will prevent the cancellation being exact, and the slow convergence of the multipole expansion reduce the reliability of the result. The experimental Z dependence is insensitive and fits both results adequately. This suggests that detailed calculations on a few selected light nuclei may be more reliable for fixing the total effective coupling, the uncertainties due to local fluctuations being smaller than the difficulties inherent in theories which cover the whole range of Z.

[17] H. Primakoff, Revs. Modern Phys. **31**, 802 (1959); and H. A. Tolhoek, Nuclear Phys. **10**, 606 (1959). We are grateful to the authors for private communication of their results.

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Charge-Dependent Corrections to Pion-Nucleon Scattering*†

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If account is taken of the mass difference between neutral and charged pions and of the possibility that the three coupling constants (π^0-n, π^0-p, $\pi^\pm$ nucleon) may differ, then the pion-nucleon system no longer conserves isotopic spin. This effect has been investigated using Chew-Low theory with a p-state interaction. For each J value there are ten scattering amplitudes, replacing the two of the charge-independent case. Only eight of these amplitudes are independent due to time reversal invariance, and the mass difference effect can be related to a change in the energy scale. The amplitudes are determined as solutions to a set of linear integral equations which may be solved approximately in the one-meson approximation. Corrections to the differential cross sections are then calculated. These corrections go through a maximum at about 125 Mev and can affect the magnitude of the π^- cross sections by as much as 35% in this region, as well as the slope of the π^- cross section in the region 125–175 Mev. The effect on the π^+ cross section is small. Attempts are made to correlate the calculation with available data.

I. INTRODUCTION

CHARGE independence in pion scattering is only an approximation. It is known, for example, that the electromagnetic interaction destroys charge independence, and it is the purpose of this calculation to determine the nature of the contributions to be expected from charge-dependent contributions, without explicitly introducing the e-m field. It is assumed that at low energies these effects will manifest themselves as changes in the pion masses and coupling constants.

The fact that the mass of the neutral pion is about 3% less than that of the charged pions is a clear indication of a breakdown of charge independence. This can

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