distributions for $r=1$ and $r=-1$ are:

$$1,\ \mathbf{p}\cdot\mathbf{q},\ (\mathbf{k}\cdot\mathbf{q})^2,\ (\mathbf{k}\cdot\mathbf{p})^2,\ (\mathbf{p}\cdot\mathbf{q})^2,\ \mathbf{q}\cdot\mathbf{k}\mathbf{k}\cdot\mathbf{p},\ (\mathbf{k}\cdot\mathbf{p}\times\mathbf{q})^2,\\ (\mathbf{p}\cdot\mathbf{q})(\mathbf{k}\cdot\mathbf{q})^2,\ (\mathbf{p}\cdot\mathbf{q})(\mathbf{k}\cdot\mathbf{q})(\mathbf{k}\cdot\mathbf{p}),\\ (\mathbf{p}\cdot\mathbf{k})^2(\mathbf{k}\cdot\mathbf{q})^2,\ (\mathbf{k}\cdot\mathbf{q})^4,\ (\mathbf{p}\cdot\mathbf{k})(\mathbf{k}\cdot\mathbf{q})^3. \quad (5)$$

The three terms present for $r=1$ and absent for $r=-1$ are of the form

$$(\mathbf{p}\cdot\mathbf{q})^2(\mathbf{k}\cdot\mathbf{q})^2,\ (\mathbf{k}\cdot\mathbf{p}\times\mathbf{q})^2(\mathbf{k}\cdot\mathbf{q})^2,\ (\mathbf{p}\cdot\mathbf{q})(\mathbf{p}\cdot\mathbf{k})(\mathbf{k}\cdot\mathbf{q})^3. \quad (6)$$

These terms would all arise from the $L=1$, $l=2$, $\lambda=2$ contingency for $r=1$. For $r=-1$, $L=1$ implies $l=1$ so that terms in which $\mathbf{p}$ occurs twice and $\mathbf{q}$ four times cannot appear in the angular distribution.

It is possible to discern the presence of these terms in the observed angular correlation via the following analysis. On averaging the angular distribution weighted with $[3(\mathbf{p}\cdot\mathbf{k})^2-1]$ over the directions of $\mathbf{p}$, the resulting angular correlation between $\mathbf{q}$ and $\mathbf{k}$ has the form

$$\alpha+\beta(\mathbf{q}\cdot\mathbf{k})^2+\gamma(\mathbf{q}\cdot\mathbf{k})^4. \quad (7)$$

Absence of the terms (6) implies $\gamma=0$. Deviation from a straight line plot in $(\mathbf{q}\cdot\mathbf{k})^2$ would serve to indicate a nonvanishing γ and further imply $r=1$. We wish to thank members of the Physics Department for their interest and helpful discussions, and in particular, Dr. M. Goldhaber, Dr. G. C. Wick, and Dr. C. N. Yang for their valued criticism.

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Electromagnetic Properties of π and K Mesons*

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A formalism is proposed which can give a smaller mass for the charged than for the neutral K meson, but a larger mass for the charged than for the neutral π meson. The theoretical prediction agrees with the experimental mass difference $M(K^0)-M(K^+)\approx 9.4$ electron masses if the rms radius of the charge distribution of the K meson is equal to 0.48×10^{-13} cm.

I. INTRODUCTION

ACCORDING to the principle of charge independence, the charged meson (meaning π meson) and the neutral meson should have the same mass before the electromagnetic interaction is switched on. The present explanation of the mass difference $M(\pi^{\pm})-M(\pi^0)$ is therefore based on the electromagnetic self-mass of the $\pi^{\pm}$, the electromagnetic self-mass of the π^0 being zero.

In the lowest-order perturbation theory, two Feynman diagrams give rise to the electromagnetic self-mass of the meson.[1] The first diagram is the familiar one corresponding to the virtual emission and reabsorption of a photon. In the second bubble diagram, the virtual photon is emitted and absorbed by the meson at the same point. The second diagram owes its existence to the requirements of gauge invariance of the interaction of mesons with the electromagnetic field.

Since both of these contributions are divergent, the integrals have been evaluated with an invariant cutoff function. Then the charged meson is found to be heavier than the neutral meson in agreement with experiments for the π mesons.[1] It has been found recently, however, that the neutral K meson is heavier than the charged K meson.[2] This poses a serious challenge to an explanation of the meson mass differences on electromagnetic grounds.

There is, however, an important distinction between the two contributions. In the first case, the nucleon-antinucleon pairs surrounding the meson can provide a natural cutoff; thus, as in the case of the electromagnetic self-mass of the nucleons, the use of the invariant cutoff functions that depend on a four-vector has a questionable meson-theoretical basis.[3] In the second case, since we disregard terms above the second order in the electromagnetic coupling constant e, the meson-nucleon interaction does not modify the interaction that occurs at one point and hence does not provide a natural cutoff. Hence we employ as usual an invariant cutoff function as a formal device to make a divergent integral finite in the second case.

In a previous article,[3] the Wick-Sorensen[4] (WS) method was used to obtain the electromagnetic self-mass of a physical nucleon. This paper presents an

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[1] R. P. Feynman and G. Speisman, Phys. Rev. **94**, 500 (1954); A. Petermann, Helv. Phys. Acta **27**, 441 (1954). Hereafter these papers will be referred to as FSP.

[2] Rosenfeld, Solmitz, and Tripp, Phys. Rev. Letters **2**, 110 (1959); Crawford, Cresti, Good, Stevenson, and Ticho, Phys. Rev. Letters **2**, 112 (1959).

[3] S. Sunakawa and K. Tanaka, Phys. Rev. **115**, 754 (1959) (hereafter referred to as ST).

[4] G. C. Wick, *Proceedings of the Seventh Annual Rochester Conference on High-Energy Nuclear Physics, 1957* (Interscience Publishers, New York, 1957); R. A. Sorensen (to be published). Hereafter these will be referred to as WS.

extension of the WS method to the contribution corresponding to the first diagram of the self-mass of the meson, in which the cloud of nucleon-antinucleon pairs associated with the physical meson gives rise to the meson form factor that provides a natural cutoff.[5]

An expression for the electromagnetic self-mass of a physical meson is obtained by a procedure analogous to that for the case of the physical nucleon, and a complete set of intermediate states is introduced. States with one, two and three mesons are considered. Then one obtains three terms for the electromagnetic self-mass of a charged meson. The first term (which is positive) corresponds to the contribution from the one meson state; the second term (which is negative) corresponds to the contribution from part of the state of two and three mesons; and the third term is the positive contribution from the aforementioned bubble diagram arising from the $AA\phi\phi$ term of the Hamiltonian of the interaction between the mesons and the electromagnetic field. Fortunately, the contribution of this third term with an invariant cutoff function is small compared to the second term with a meson form factor.

The interesting point is that the sign of the electromagnetic self-mass of the charged meson will be determined by the relative magnitude of the sum of the first and third terms relative to that of the second term. Thus there is a possibility of obtaining, with no additional assumption, a neutral meson that may be either heavier or lighter than the charged meson depending on the charge radius which is assumed.

In Sec. II, the formalism is briefly outlined, and the electromagnetic self-mass of a charged meson is obtained and finally in Sec. III, the result is discussed.

II. FORMALISM

The total Hamiltonian of the system of interacting nucleons, mesons and photons is given as

$$H=H_{nm}+H_\gamma+H_e, \qquad (1)$$

where H_{nm} represents the Hamiltonian of the interacting nucleon and meson field, including the counter-terms for mass renormalization and four-meson divergence; H_γ is the free Hamiltonian for the photon field; and H_e is the interaction Hamiltonian of the meson and nucleon field with the electromagnetic field, including the counter-term for the electromagnetic self-mass of the charged meson, the counter-term for the neutral meson being zero. The proton-neutron mass difference is neglected, and the mass difference between the charged and neutral meson may be neglected in H_{nm} that appears in Eq. (1).

[5] It should be noted that the π-meson structure plays an important role in nucleon structure. See Federbush, Goldberger, and Treiman, Phys. Rev. **112**, 642 (1958). For the case of K mesons, a similar analysis as that for the π mesons can be carried out throughout the manuscript, in which the "meson" and "nucleon" would be replaced by "K meson" and "baryon," respectively.

In a manner similar to ST,[3] the expression for the self-mass $\delta\mu$ of the charged meson (to the second order in the electromagnetic coupling constant and with $\hbar=c=1$) is found to be

$$[\tfrac{1}{2}\delta\mu^2-4e^2\langle(A_\mu(x)A_\mu(x))_+\rangle_0]/2E_k$$
$$=-\tfrac{1}{2}i\int dy\,\langle(A_\mu(x)A_\nu(y))_+\rangle_0[\langle k|(j_\mu(x)j_\nu(y))_+|k\rangle$$
$$-\langle 0|(j_\mu(x)j_\nu(y))_+|0\rangle], \qquad (2)$$

where $|k>$ is the physical meson state with energy E_k, in which the isotopic spin variable is suppressed, and $|0>$ is the physical vacuum state. The subscript $+$ means one should take the chronological P-product of the parenthesis.

The P-product of Eq. (2) is restricted to the region $(x_0-y_0)>0$ by inserting a factor 2, and the contraction is replaced by

$$\langle(A_\mu(x)A_\nu(y))_+\rangle_0=\tfrac{1}{2}\delta_{\mu\nu}D_F(x-y)$$
$$=-\delta_{\mu\nu}\frac{i}{(2\pi)^4}\int\frac{d^4k}{k^2}e^{ik\cdot(x-y)}$$
$$=i\delta_{\mu\nu}D^{(+)}(x-y)\quad (x_0-y_0)>0, \qquad (3)$$

and the variable of integration is changed from y to $x-y=z$. A sum over a complete set of states $|n>$ of the Hamiltonian H_{nm} is introduced on the right-hand side of Eq. (2). Then

$$[\tfrac{1}{2}\delta\mu^2-2e^2D_F(0)]/2E_k$$
$$=-\tfrac{1}{2}i\int_0^\infty dz\,D_F(z)\sum_n[\langle k|j_\mu(z)|n\rangle\langle n|j_\mu(0)|k\rangle$$
$$-\langle 0|j_\mu(z)|n\rangle\langle n|j_\mu(0)|0\rangle], \qquad (4)$$

where the sum over states is to be regarded as half the sum over "in" and "out" states. Since baryon number is conserved, the intermediate states of Eq. (4) consist of states with various numbers of mesons and nucleon-antinucleon pairs.

In the lowest-order perturbation theory without a cutoff, there are two contributions for the process in which a virtual photon is emitted and absorbed by the meson. They are the contributions for the process in which the intermediate meson after emission is a particle or its charge-conjugate particle. For instance, if the initial meson is positively charged there is the contribution for the process in which the intermediate meson is also positively charged, i.e., it is a particle. There is the other contribution for the process in which the positively charged meson is annihilated by the negatively charged meson that is created in a meson pair, while the positively charged meson of the pair becomes the final meson. The intermediate meson here is a charge-conjugate particle.

There is a clear correspondence between the term in which the intermediate meson is a particle and the contribution of the one-meson state of the first term of Eq. (4). Our object now is to extract from the infinite sum the contribution that corresponds to the term in which the intermediate meson is a charge-conjugate particle. This latter contribution is obtained from the three-meson state of the first term and the two-meson state of the second term of Eq. (4).

We are omitting an infinite number of terms as well as parts of terms. This procedure can be directly compared with the standard perturbation method (with a cutoff) of evaluating the self-mass to order e^2, on which the present electromagnetic explanation of π meson mass difference is based.[1]

Let us examine the intermediate states of the first term of the right-hand side of Eq. (4). The vacuum state gives no contribution because of charge conservation and invariance under charge conjugation. In a similar way to that of ST, the state with three mesons may be rewritten as[6]

$$\begin{aligned}\sum_{\mathbf{q},l,S}\langle k|j_\mu(z)|qlS\rangle\langle qlS|j_\mu(0)|k\rangle &=\sum_{l,S}\langle k|j_\mu(z)|klS\rangle\langle klS|j_\mu(0)|k\rangle \\ &\quad+\sum_{\mathbf{q}\neq k,l,S}\langle k|j_\mu(z)|qlS\rangle\langle qlS|j_\mu(0)|k\rangle \\ &=\sum_{l,S\neq k}\langle 0|j_\mu(z)|lS\rangle\langle lS|j_\mu(0)|0\rangle \\ &\quad+2\sum_l\langle 0|j_\mu(z)|lk\rangle\langle lk|j_\mu(0)|0\rangle+\cdots \\ &=\sum_{l,S}\langle 0|j_\mu(z)|lS\rangle_c\langle lS|j_\mu(0)|0\rangle_c \\ &\quad+\sum_l\langle 0|j_\mu(z)|kl\rangle_c\langle kl|j_\mu(0)|0\rangle_c+\cdots. \end{aligned}\tag{5}$$

The subscript c means that only "connected graphs" are to be taken. When Eq. (5) is combined with the state with one meson, the first term on the right-hand side of Eq. (4) yields

$$\begin{aligned}\sum_S\langle k|j_\mu(z)|S\rangle_c\langle S|j_\mu(0)|k\rangle_c &+\sum_{l,S}\langle 0|j_\mu(z)|lS\rangle_c\langle lS|j_\mu(0)|0\rangle_c \\ &+\sum_l\langle 0|j_\mu(z)|kl\rangle_c\langle kl|j_\mu(0)|0\rangle_c.\end{aligned}\tag{6}$$

The two-meson state of the second term on the right-hand side of Eq. (4) yields

$$\sum_{l,S}\langle 0|j_\mu(z)|lS\rangle_c\langle lS|j_\mu(0)|0\rangle_c.\tag{7}$$

[6] The factor 2 that appears in the right-hand member of Eq. (5) is a statistical factor [G. C. Wick, Revs. Modern Phys. **27**, 339 (1955)]. The author is indebted to Dr. S. Sunakawa for discussions on this point.

The intermediate states with no meson and one meson drop out because of the properties of vacuum, charge conservation and charge conjugation invariance.

On substituting Eqs. (6) and (7) into Eq. (4), we obtain the basic relation

$$\begin{aligned}[\tfrac{1}{2}\delta\mu^2-2e^2D_F(0)]/2E_k &=-\tfrac{1}{2}i\int_0^\infty dz\,D_F(z)[\sum_S\langle k|j_\mu(z)|S\rangle_c\langle S|j_\mu(0)|k\rangle_c \\ &\quad+\sum_l\langle 0|j_\mu(z)|kl\rangle_c\langle kl|j_\mu(0)|0\rangle_c].\end{aligned}\tag{8}$$

The physical meson states are characterized by their momenta and are on the mass shell. From Lorentz and gauge invariance and isotopic spin requirements, the matrix elements that appear on the right-hand side of Eq. (8) may be written (when the isotopic spin variables are reinserted) as

$$(4E_kE_s)^{\frac{1}{2}}\langle ki|j_\mu(z)|Sj\rangle_c=ie(k+S)_\mu F[(k-S)^2]e^{-i(k-S)\cdot z}\epsilon_{3ij}/\sqrt{2}\tag{9}$$

$$(4E_kE_l)^{\frac{1}{2}}\langle 0|j_\mu(z)|kilj\rangle_c=ie(k-l)_\mu F[(k+l)^2]e^{i(k+l)\cdot z}\epsilon_{3ij}/\sqrt{2}\tag{10}$$

where F is the meson form factor.[5] The form factor that appears in Eq. (10) depends on a time-like four-momentum transfer which is in the nonexperimental region. We assume that the form factor used in the experimental region [Eq. (9)] will have the same form in the nonexperimental region [Eq. (10)]. It is noted here that the same isotopic spin treatment can be given for the charged K mesons as for the charged π mesons.

Substituting Eqs. (3), (9), and (10) into Eq. (8) and integrating over the variables z,s and z,l, one obtains for $\mathbf{k}=0$, without loss of generality,

$$\begin{aligned}\delta\mu^2=-\frac{8e^2i}{(2\pi)^4}\int_{-\infty}^{\infty}\frac{d^4p}{p^2}F^2(p^2)+\frac{e^2}{2(2\pi)^3}\int_0^\infty\frac{d^3p}{\omega E_p} &\times\left(\frac{2\mu(E_p+\mu)F^2[2\mu(E_p-\mu)]}{\omega+E_p-\mu}\right. \\ &\left.-\frac{2\mu(E_p+\mu)F^2[-2\mu(E_p+\mu)]}{\omega+E_p+\mu}\right),\end{aligned}\tag{11}$$

where $\omega=|\mathbf{p}|$ and the cutoff function that has the same form as the meson form factor has been inserted in the first term on the right-hand side.

The FSP expression on which the present explanation of the meson mass difference is based, written here in

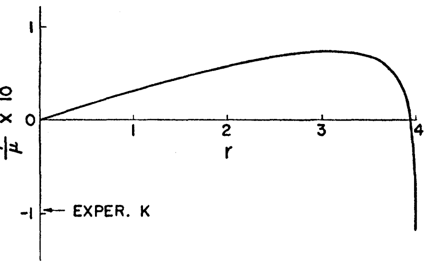

FIG. 1. Graph of $\delta\mu/\mu$, the mass difference between charged and neutral mesons divided by the meson mass, plotted against $r=\Lambda^2/\mu^2$ on the basis of Eq. (14).

order to remark on its relation to our expression (11), is

$$\delta\mu^2=-\frac{2e^2i}{(2\pi)^4}\left(4\int_{-\infty}^{\infty}\frac{d^4k}{k^2}F^2(k^2)-\int_{-\infty}^{\infty}\frac{d^4k}{k^2}\frac{(k+2p)^2F^2(k^2)}{[(k+p)^2+\mu^2]}\right). \quad (12)$$

If the second term on the right-hand side of Eq. (12) is integrated with respect to k_0, the resulting expression is related to the second and third terms on the right-hand side of Eq. (11) in a manner analogous to that of the proton-neutron mass difference.[3] This relation will not be elaborated further.

The meson form factor that appears in the second term on the right-hand side of Eq. (11) is due to the virtual nucleon-antinucleon pairs associated with the physical meson. For the sake of illustration, let us carry out the integration of Eq. (11) for the Yukawa model given by

$$F(p^2)=\Lambda^2/(\Lambda^2+p^2)=[1+(p^2a^2/6)]^{-1}=[1+(p^2/r\mu^2)]^{-1}, \quad (13)$$

where $a^2=6/\Lambda^2$ is the mean-square radius of the electromagnetic charge distribution of the meson and $r=\Lambda^2/\mu^2$.

Inserting Eq. (13) into Eq. (11), integrating the first term, and solving in terms of r and $\alpha=e^2/4\pi$ (for $\mu=1$), we obtain the expression

$$\frac{\delta\mu}{\mu}=\frac{\alpha}{\pi}\left(\frac{r}{2}+\int_0^{\infty}\frac{pdp}{E_p}\frac{(E_p+1)}{(E_p+p-1)}[1+(2/r)(E_p-1)]^{-2}-\int_0^{\infty}\frac{pdp}{E_p}\frac{(E_p-1)}{(E_p+p+1)}[1-(2/r)(E_p+1)]^{-2}, \quad (14)$$

where $E_p=(1+p^2)^{\frac{1}{2}}$. Equation (14) is valid for $r\leqslant 4$, since one encounters a singularity[3] in the form factor of the last term when $r>4$. This result is applicable to both the π and K mesons.[5]

The numerical evaluation is carried out in steps of 0.2 from $p=0$ to 2 and in steps of 4 from $p=2$ to 18 by use of Simpson's rule. The result for $\delta\mu/\mu$ vs r is plotted in Fig. 1. The experimental value of $\delta\mu/\mu$ is given by $\delta\mu_K/\mu_K=[M(K^+)-M(K^0)]/M(K)=-0.97\times10^{-2}$ for the K meson and $\delta\mu_\pi/\mu_\pi=[M(\pi^+)-M(\pi^0)]/M(\pi)=3.2\times10^{-2}$ for the π meson.

III. RESULTS

The WS method of evaluating the proton-neutron mass difference has been extended to the calculation of part of the contribution to the self-mass of mesons. The effect of nucleon-antinucleon pairs around the meson has been taken into account by introducing a form factor for the charge distribution of the meson. This form factor which enters in a natural way is characterized by a rms radius.

When one assumes a Yukawa model for the meson form factor, one can explain the K meson mass difference if the charge distribution has a rms radius of $a_K=0.48\times10^{-13}$ cm, a reasonable value. With the same Yukawa model Eq. (13), one can obtain the correct sign but not the correct magnitude of the mass difference between charged and neutral π mesons.[7]

It is emphasized that we have omitted an infinite number of terms including the nucleon-antinucleon pair state from the sum given in Eq. (4). We cannot estimate whether or not these terms give a negligible contribution. Therefore, the present attempt should be regarded as tentative.

The numerical result is not so important as the fact that both possible signs of difference between meson masses can be obtained with the present formalism.

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[7] For the π mesons, the maximum value of $\delta\mu_\pi/\mu_\pi$ we can obtain from Fig. 1 is about $\delta\mu_\pi/\mu_\pi\simeq0.73\times10^{-2}$ at $r\simeq3$. This is much smaller than the experimental value 3.2×10^{-2}. A different meson form factor can improve the value of $\delta\mu_\pi/\mu_\pi$, but the rms radius will again be too large because of the requirement $r\leq4$.