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Matter and Point Set Theory

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A brief theoretical exposition of the concept of matter as a nowhere dense perfect set of positive measure is developed starting from the observation that matter is largely vacuous in all scales from subnuclear to extragalactic.

THE atomistic view that matter is composed of fundamental particles was constructed as an antithesis to the view that matter is a continuum. Quantum theory changes the way to work out the consequences of either of these two views or of any other underlying classical theory. However, the quantum principle does not eliminate the sharp distinction between a fundamentally particulate and a fundamentally continuum description of the localization of mass energy before quantization. Therefore it is of interest to note that it is *not necessary* to assume the existence of a smallest concentration of mass in order to avoid the assumption that matter is continuous. One can assume that matter is distributed as a *nowhere dense perfect set.* This concept can be explained and made plausible as follows.

On an extremely large scale galaxies appear as mass "points." As the scale of resolution is decreased these galaxies appear to be large aggregates of mass points which may be approximated as continua. With a further decrease in scale the continuum approximation becomes poor and the stellar components appear to be mass points. Further decrease in scale permits them to be approximated as continua while still further decrease yields atomic systems and nuclear systems. A *lack of connectivity* appears to be an essential feature of all systems and subsystems of matter.

Now let it be postulated that there exist matter-free intervals of space within any interval of space however small the latter may be chosen. In other words, the distribution of matter is regarded as similar to that of the points in a *nowhere dense set.* On the other hand, it is natural to think of the continuum of space as being available to nonlocalized energy. Thus, one is led to the view that the set of material points has the *power of the continuum.*[1] This condition would be fulfilled by a nowhere dense *perfect set* (i.e., a set identical with the set of its limit points). The classical example is Cantor's ternary set[2] which is the set of all points remaining from a closed interval when the open interior thirds of all remaining intervals are removed. Although Cantor's set has measure zero, a slight modification in the method permits the construction of a nowhere dense perfect set of positive measure.

Naturally nothing is known today about the consequences of such a description of nature after quantization. Judging from the pre-quantization version just described, one would expect the scale of resolving power of the instrument of observation to set a corresponding scale of "interval removal size."

The requirement that the set of material points constructed in this way have positive measure prevents the same proportion being removed from the intervals at successive stages of resolution. Thus, the characteristic sizes of the mass distribution (i.e., galactic, stellar, atomic, nuclear, etc.) cannot be in the same proportion to the volume outside these objects if the limiting set is to have positive measure. This conclusion is consistent with observation.

[1] F. Hausdorff, *Mengenlehre* (Dritte Auflage, New York, 1944).
[2] J. C. Burkill, The Lebesgue Integral (Cambridge University Press, Cambridge, 1951), pp. 9, 25.