

the improvement is obtained by taking cognizance of the different Hamiltonians for particles of opposite spins. They do not, however, list the individual differences of the separate orbitals, but only note the over-all change at the origin. The self-consistent field calculation carried out shows that the increase over the HF value is not due to a change in the $2s$ orbital, which remains virtually the same, but is in fact due to the net magnetic moment set up by the difference in the inner orbital charge densities as may be seen by Table II.

The results clearly indicate that the UHF approximation is considerably better than the HF approximation for the calculation of hyperfine splittings. The improvement is sufficient to outweigh the drawback of inequivalent orbitals and the resulting unaesthetic

lack of symmetry of the total UHF wave function. In this connection, it should be noted that the inner product of the UHF inner orbitals is still virtually unity. It is of particular interest to note that if one attempts to restore the over-all symmetry by projecting out the doublet, Ψ_2 , a much poorer value for the hyperfine splitting is obtained.

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Evaluation of the Interaction Effect in $n-p$ Capture*

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Recent information about the nuclear force has been used to recalculate the thermal $n-p$ capture cross section, and a value of 0.303 ± 0.012 b is obtained. The comparison of this number with the experimental cross section of 0.3315 ± 0.0017 b indicates an "interaction" magnetic dipole moment contribution of 0.028 ± 0.012 b.

I. INTRODUCTION

AN earlier paper under the above title¹ discussed the accuracy with which the thermal $n-p$ capture cross section could be computed from the usual phenomenological theory, the theory which ignores all "interaction effects." Interaction effects are those modifications of the two-body magnetic moment operator which are caused by the mesic nature of the nuclear interaction; in their absence the magnetic moment operator in $n-p$ capture is the sum of the spin magnetic moments of two free nucleons. Meson theory predicts a small value for the interaction effects, but it gives very little more information than that. However, further information is available from the $n-p$ capture experiment, the difference between the observed cross section and the predicted phenomenological cross section being a measure of the interaction effect. The present paper continues the task of assessing the accuracy which can be achieved by the theory.

In I it was concluded that an interaction effect indeed could be detected, and that it gave rise to an $8 \pm 5\%$ increase in the cross section. This determination was of

marginal accuracy. It seemed sufficiently in disagreement with the one-percent effect Sugawara found from meson theory,^{2,3} that he was led to criticize the values and errors chosen for the effective ranges in I. His choices for these quantities considerably increased the estimated error of the prediction and slightly decreased the discrepancy with experiment. Further discussion of the effective ranges will be given in the present paper in an attempt to settle the disagreement. For the sake of background it is interesting to note at this point that other experiments with the two-body system also point to large interaction effects; these effects also conflict with meson theory predictions, so a similar conflict in the present case need not seem surprising. The experiments in question concern deuteron photodisintegration and the deuteron magnetic moment. The photodisintegration seems at first sight to be in excellent agreement with the meson theory. Thus a negligible interaction modification of the medium-energy photodisintegration was found by Pearlstein and Klein,⁴ and careful phenomenological calculations⁵ show that this

² M. Sugawara (private communication).

³ L. Hulthén and M. Sugawara, *Encyclopedia of Physics* (Springer-Verlag, Berlin, 1957), Vol. 39. This article effectively summarizes Sugawara's work on the two-nucleon system.

⁴ L. D. Pearlstein and A. Klein, *Bull. Am. Phys. Soc.* **4**, 268 (1959); also *Phys. Rev.* (to be published).

⁵ J. J. de Swart and R. E. Marshak, *Phys. Rev.* **111**, 272 (1958).

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¹ N. Austern, *Phys. Rev.* **92**, 670 (1953). Henceforth this paper will be denoted I.

is in accord with experiment.⁶ Nevertheless, the excellent agreement with the photodisintegration experiment is possible only if the phenomenological theory uses 7% deuteron D state. Such a value implies a large interaction modification of the static magnetic moment of the deuteron, because without such a modification only 4% D state is obtained. At this point we see a disagreement with a prediction from meson theory, for Sugawara found interaction effects in the static magnetic moment to be small, and predicted $3 \pm 1\%$ D state.⁷ Evidently the statement that interaction-effect modifications of the static magnetic moment are negligible conflicts with the statement that such modifications of the photodisintegration are negligible. Further information from experiment is desirable. The present paper continues the program of seeking such information. It meets the criticisms made earlier,^{2,3} and also introduces into the phenomenological calculation a number of recent improvements in the input data. Substantially the same result as in I will be reached here.

Recent information about the two-body nuclear interaction is the most notable of the improvements since I. Most of I was devoted to a study of the influence of potential shape upon the computation of $\sigma_c(\text{theor})$. This influence was found to be small but not negligible, and there was as a consequence some uncertainty of $\sigma_c(\text{theor})$ because of the uncertainty about potential shape. However, the modern potentials of Signell-Marshak⁸ and Gammel-Thaler,⁹ certainly have substantially correct shapes. It is extremely unlikely that their shapes could be sufficiently wrong, in view of I, to cause any appreciable uncertainty in the cross section. Thus the uncertainty of $\sigma_c(\text{theor})$ due to potential shape has been eliminated.

The value of $\sigma_c(\text{exp})$ also has been determined to better accuracy over the past few years, several additional measurements having been performed. We adopt the value advocated by Baker and Wilkinson,¹⁰

$$\sigma_c(\text{exp}) = 0.3315 \pm 0.0017 \text{ barn.}$$

A still more recent experiment¹¹ also gives a result in agreement with the above value.

Other improvements since I will become apparent later on, and consist mainly in more conclusive attitudes regarding the effective ranges.

⁶ This agreement refutes earlier suggestions that an appreciable interaction effect was needed. See R. R. Wilson, Phys. Rev. **104**, 218 (1956); N. Austern, Phys. Rev. **108**, 973 (1957).

⁷ See Eq. (27.11) of reference 3.

⁸ P. Signell and R. Marshak, Phys. Rev. **109**, 1229 (1958); Signell, Zinn, and Marshak, Phys. Rev. Letters **1**, 416 (1958).

⁹ Gammel, Christian, and Thaler, Phys. Rev. **105**, 311 (1957); J. Gammel and R. Thaler, Phys. Rev. **107**, 291, 1337 (1957).

¹⁰ A. R. Baker and D. H. Wilkinson, Phil. Mag. **3**, 647 (1958); A. R. Baker, Proc. Roy. Soc. (London) **A248**, 539 (1958). These authors review the earlier experimental work.

¹¹ R. W. Stooksberry and M. F. Crouch, Phys. Rev. **114**, 1561 (1959). These authors obtain $\sigma_c(\text{exp}) = 0.330 \pm 0.008 \text{ b.}$

II. CALCULATIONS

In I the capture cross section was put into the form

$$\sigma_c(\text{theor}) = (\pi/2k)(e^2/Mc^2)(\omega/c)^3 \times (\mu_N - \mu_P)^2 N_g^2 a_s^2 |\mathfrak{M}|^2, \quad (1)$$

where

$$\mathfrak{M} = \int_0^\infty U_g U_s dr. \quad (2)$$

Here \mathfrak{M} is the overlap integral between the 3S part of the ground-state wave function, U_g , and the 1S zero energy wave function, U_s . These functions are normalized so that asymptotically

$$U_g \rightarrow e^{-r/r}, \\ U_s \rightarrow 1 - r/a_s.$$

The symbol k in (1) is the wave number for the incident particles in center-of-mass coordinates, at the standard neutron velocity, 2200 m/sec; ω is the frequency of the emitted light, a_s is the singlet scattering length, and N_g^2 is the ground-state normalization factor.

$$N_g^2 = 2\gamma/[1 - \gamma\rho_t(-\epsilon, -\epsilon)]. \quad (3)$$

Here γ is the bound-state damping parameter, defined so that $(\hbar^2\gamma^2/M) = \epsilon$, the deuteron binding energy, and $\rho_t(-\epsilon, -\epsilon)$ is the effective range for the bound state.

The effective range $\rho_t(-\epsilon, -\epsilon)$ is not directly measured in the scattering experiments, but rather it is $\rho_t(0, -\epsilon)$ which is measured. The value of the latter quantity is³

$$\rho_t(0, -\epsilon) = 1.704 \pm 0.027 \times 10^{-13} \text{ cm.}$$

To find $\rho_t(-\epsilon, -\epsilon)$ it is necessary to make the shape dependent correction

$$\rho_t(-\epsilon, -\epsilon) = \rho_t(0, -\epsilon) + 2\gamma^2 \rho_t^3 P_t. \quad (4)$$

The triplet shape dependent parameter, P_t , will be discussed in detail below. It does not matter to which energy the ρ_t in the second term of (4) is referred since that term is very small.

It is convenient to rearrange (1). Thus we define the dimensionless quantity

$$Q = (\pi/2k)(e^2/Mc^2)(\omega/c)^3(\mu_N - \mu_P)^2(2\gamma)(a_s^2) = 1145 \pm 11. \quad (5)$$

Most of the uncertainty in Q comes from the uncertainty in a_s . Using Q and Eq. (4) we obtain

$$\sigma_c(\text{theor}) = Q[1 - \gamma\rho_t(0, -\epsilon) - 2(\gamma\rho_t)^3 P_t]^{-1} |\mathfrak{M}|^2. \quad (6)$$

Next \mathfrak{M} may be rearranged in the manner suggested by Bethe and Longmire.¹²

$$\mathfrak{M} = \mathfrak{M}_0 - \frac{1}{4}[\rho_s + \rho_t(-\epsilon, -\epsilon)] - \frac{1}{2}N_g^{-2}P_D + C,$$

¹² H. A. Bethe and C. Longmire, Phys. Rev. **77**, 647 (1950). Also see I.

so that

$$\mathfrak{M} = \mathfrak{M}_0 - \frac{1}{4}[\rho_s + \rho_t(0 - \epsilon)] - \frac{1}{2}\gamma^2 \rho_t^3 P_t - \frac{1}{2}N_g^{-2} P_D + C. \quad (7)$$

Here ρ_s is the singlet effective range and P_D is the D -state probability. The 3D state enters in (7) in a very simple manner, essentially only reducing the amount of 3S state which can participate in the reaction. The quantity \mathfrak{M}_0 in (7) is the zero range result, and is computed in terms of the asymptotic wave functions,

$$\begin{aligned} u_g &= e^{-\gamma r}, \\ u_s &= 1 - r/a_s; \\ \mathfrak{M}_0 &= \int_0^\infty u_g u_s dr = (5.098 \pm 0.025) \times 10^{-13} \text{ cm}. \end{aligned} \quad (8)$$

Most of the correction for finite range is contributed by the term in (7) involving the effective ranges. The quantity

$$C = \frac{1}{2} \int_0^\infty [(u_g - u_s)^2 - (U_g - U_s)^2] dr, \quad (9)$$

also contributes a further correction for finite range and incorporates all of the uncertainty about potential shape. Evidently C tends to be rather small, so that it is not a difficult quantity to evaluate to sufficient accuracy. It is apparent that most of the uncertainty in $\sigma_s(\text{theor})$ comes from the uncertainty in ρ_s , P_t , and P_D .

(a) Evaluation of C

The function U_g is most readily available in the literature in the form published by Moravcsik,¹³ who presented an analytic fit to the Gartenhaus ground-state wave function,¹⁴ advocated by Signell and Marshak.⁸ Moravcsik's form for U_g is¹⁵

$$U_g = (1 - e^{-2.5r})(1 - e^{-1.59r})(e^{-0.232r} - e^{-1.90r}). \quad (10)$$

No such convenient expression was available for U_s , so we performed a numerical integration of the Schrödinger equation, using the 1S_0 potential of Gammel and Thaler.⁹ The result of this integration is very accurately of Hulthén form, and is fitted by the expression

$$\begin{aligned} U_s &= 0, & r < 0.4f \\ U_s &= 1 + 0.02907r - 1.06e^{-1.71(r-0.4)}, & r > 0.4f. \end{aligned} \quad (11)$$

This fit is accurate to better than one percent except very near the core radius, where U_s is small anyhow. Having these functions it is straightforward to compute C . The result is

$$C = +0.048 \times 10^{-13} \text{ cm}. \quad (12)$$

Thus C is only a one-percent correction to \mathfrak{M}_0 .

The result (12) for C is not unexpected. The modern U_g and U_s functions are of the general sort explored in

¹³ M. J. Moravcsik, Nuclear Phys. **7**, 113 (1958).

¹⁴ S. Gartenhaus, Phys. Rev. **100**, 900 (1955).

¹⁵ Where not otherwise stated all numerical quantities in this paper have the dimensions 10^{-13} cm , or 10^{+13} cm^{-1} .

I_1 and values for C there were of the same order as in (12). It is satisfying that nothing unusual has happened.

One could ask how C changes if the Signell-Marshak U_s is used, or if the Gammel-Thaler U_g is used. These computations have not been done explicitly. Nevertheless, the two sets of nuclear forces are in very good agreement for just the states in question, so since the wave functions are not responsive to fine details of the forces no additional calculations appear necessary.

One precaution must be taken in the computation of C . This quantity is the only one in $\sigma_s(\text{theor})$ which is obtained from the fundamental nuclear forces. For all other quantities in (6) and (7) best experimental values will be used. Consistency demands, however, that best experimental values of the parameters in the asymptotic wave functions u_g and u_s not be used in (9). The calculated U_g and U_s have scattering lengths and effective ranges which differ from experiment by several percent, an error which was not worth correcting for this calculation. This error is compensated without further calculation if asymptotic functions appropriate to the *computed* U_g and U_s are used in (9), leaving only a small error in a small quantity.

(b) Value of P_D

The quantity $\frac{1}{2}N_g^{-2}P_D$ in (7) is a very small correction in the matrix element. In agreement with modern ideas⁸ we adopt

$$P_D = 0.07 \pm 0.02,$$

giving

$$\frac{1}{2}N_g^{-2}P_D = 0.046 \pm 0.013 \times 10^{-13} \text{ cm}. \quad (13)$$

A reduction of P_D to 0.03, the older value, would make no appreciable change in the result of this paper.

(c) Value of ρ_s

Determinations of the n - p singlet effective range from neutron scattering experiments are difficult to achieve to the required accuracy. The results obtained thus far seem to cluster⁸ near $2.4 \times 10^{-13} \text{ cm}$, but are of uncertain accuracy. In this situation it appears best to rely upon charge independence for the value of ρ_s , since very accurate cross section measurements can be achieved in the p - p system.

Charge independence was first postulated on the basis of a comparison of n - p and p - p singlet state scattering; this comparison *assuming* equality of the force ranges of the two systems and then comparing the depths. It might appear more important to check charge independence by a direct measurement of the n - p range than to build further broad assumptions upon it. But charge independence is not based on this experiment alone. Other evidence gives adequate support to it, and it has become customary to go on from that evidence to accept in full the idea that charge independence is precisely correct for the meson-nucleon coupling. Then the present information about the

nucleon-nucleon interaction merely is *not in conflict* with the invariance postulated for the meson coupling. One may seek to derive properties of the two-nucleon system from charge independence, in this point of view, rather than use them to establish it.

Following the point of view that it is the meson theory which is precisely charge independent, we see that the $n-p$ and $p-p$ singlet state interactions are not expected to be exactly equal, despite various meticulous comparisons between them which have implied such an expectation. Mesons are charged, so electromagnetic corrections to the nuclear force must occur. Evidences of this are seen in the four percent $\pi^0-\pi^\pm$ mass difference, and in the difference between the singlet scattering lengths, which requires the $n-p$ singlet force to be two percent stronger than the $p-p$ force^{3,9} *if the ranges are taken equal*. The departures from charge independence are expected to be slight, of course, but they can be of the order of several percent, and they are extremely difficult to obtain from meson theory.¹⁶ Even the sign of the effect is unknown. It would appear that a correct attitude for finding ρ_s from the $p-p$ information must make some use of the observation that the $n-p$ force is a little the stronger, but cannot treat the other effects in any other way except to make some allowance for the error they cause.

Shape-dependent corrections to effective range theory cause appreciable uncertainties in the $p-p$ effective range, $r_{0s}(p-p)$, because of the need to extrapolate to zero energy from the several Mev energies of the scattering experiments. It seems not to have been noticed before that the procedure which then generates the *corresponding* $n-p$ effective range, $r_{0s}(n-p)$, gives a result which is much less shape dependent. This happens because the interplay between Coulomb and nuclear forces in $r_{0s}(p-p)$ also depends on shape, so that removing the Coulomb force to get $r_{0s}(n-p)$ introduces a further shape correction¹⁷ which accidentally cancels the first. In detail, $r_{0s}(n-p)$ is found from the formulae of Blatt and Jackson¹⁸ for $n-p$ scattering, using in these formulae the intrinsic range b and well-depth parameter s obtained from the $p-p$ scattering. In this way $r_{0s}(n-p)$ is not found from $r_{0s}(p-p)$, but both rather are found from the same nuclear force. Table I shows a portion of Table 13 of reference 3, with an extra column added for $r_{0s}(n-p)$. The value of $r_{0s}(n-p)$ seems very well determined.

The actual $n-p$ singlet effective range, ρ_s , may now be estimated in terms of $r_{0s}(n-p)$ by making a suitable adjustment to get the correct $n-p$ scattering length. A two-percent increase in s , the well-depth parameter, easily accomplishes the necessary large increase of the singlet scattering length to the $n-p$ value, because the

TABLE I. Effective ranges from $p-p$ scattering.

Well shape	b (10^{-13} cm)	s	$r_{0s}(p-p)$ (10^{-13} cm)	$r_{0s}(n-p)$ (10^{-13} cm)
Square	2.583	0.8893	2.559 ± 0.017	2.74
Exponential	2.513	0.9073	2.673 ± 0.017	2.74
Yukawa	2.480	0.9216	2.774 ± 0.017	2.76

system is near resonance. The associated change in r_{0s} is approximately a two-percent *reduction*, a slight effect. An alternative way to change the scattering length would be to increase the intrinsic range b , keeping s constant. This method makes no use of the resonance, however, so both b and r_{0s} are *increased* in the same proportion as the scattering length. Adjustment of s is much more reasonable.

The result of reducing $r_{0s}(n-p)$ by two percent from the results of Table I appears to be our most reasonable value for ρ_s . We append to this a generous allowance for the errors caused by the other effects mentioned earlier. Thus we adopt

$$\rho_s = (2.68 \pm 0.30) \times 10^{-13} \text{ cm.} \quad (14)$$

(d) Value of P_t

The value of the triplet shape dependent parameter P_t , to be used in this paper should for reasons of consistency be computed for the Gartenhaus potential.¹⁴ Although this calculation has not been done, the general properties of the Gartenhaus potential are sufficiently clear so that P_t can be estimated to reasonable accuracy from earlier calculations. Such a procedure also has the advantage of not appearing too involved with the specific form of the particular potential used.

In I the central value of P_t was chosen to be zero, with a suitable spread of error about this value because of the uncertainty in potential shape. Hulthén and Sugawara criticized this choice, and for their discussion of $n-p$ capture quote¹⁹ a value $P_t = 0.048 \pm 0.089$. Their value of P_t appears to have both too large a central value and too large an error. It was obtained merely as an average between the P_t values for square and Yukawa well shapes, ignoring tensor force effects and ignoring repulsive core effects. Other information in reference 3 bears on this question in a detailed way, showing that P_t for the elementary Yukawa shape is very much out of line with values that can be regarded as reasonable. Figure 16 of reference 3 shows that a repulsive core of any appreciable size not only reduces P_t , whatever the well shape outside the core, but also causes the various values for different well shapes to cluster together more closely. Likewise, Fig. 19 of reference 3 shows that consideration of the tensor force also drops the value of P_t . These facts suffice to fix P_t in a suitable range when one considers the general characteristics of the modern forces^{8,9} and thus obviates

¹⁶ A. Sugie, Progr. Theoret. Phys. (Kyoto) **11**, 333 (1954).

¹⁷ For example, see the discussion given by R. G. Sachs, *Nuclear Theory* (Addison-Wesley Publishing Company, Inc., Reading, 1953), p. 131.

¹⁸ J. M. Blatt and J. D. Jackson, Phys. Rev. **76**, 18 (1949).

¹⁹ See reference 3, Sec. 37.

the need to make more specific calculation of P_t . We take

$$P_t = 0.00 \pm 0.05. \quad (15)$$

(e) Calculation of σ_c (theor)

Upon assembling the values from the preceding subsections, the result is found to be

$$\sigma_c(\text{theor}) = (0.303 \pm 0.006)(1 - 0.326dP_D - 0.112d\rho_s + 0.140dP_t) \text{ b}, \quad (16)$$

the more interesting errors being displayed explicitly in (16). The error in the leading coefficient comes from the factors not explicitly displayed and is due to the uncertainties in a_s , $\rho_t(0, -\epsilon)$, and γ , each contributing roughly the same amount.

If the various numerical estimates of error are inserted in (16) and combined on a random basis, the final result is

$$\sigma_c(\text{theor}) = (0.303 \pm 0.012) \text{ b}, \quad (17)$$

this to be compared with the measured value

$$\sigma_c(\text{exp}) = (0.3315 \pm 0.0017) \text{ b}.$$

III. CONCLUSIONS

The interaction effect increase of the capture cross section appears well established. It has the value

$$(0.028 \pm 0.012) \text{ b},$$

and is about ten percent of the uncorrected cross section. Such a large effect is entirely unexpected.

Nevertheless it does not appear possible to manipulate ρ_s , the least known of the quantities in $\sigma_c(\text{theor})$, to such an extent as to make the effect disappear. Thus ρ_s would have to decrease to 1.86 to make the effect vanish, or to 2.20 to make the magnitude of the effect equal that of the estimated uncertainty. Evidently a direct measurement of ρ_s would be very helpful.

A large interaction effect change in a magnetic multipole is not impossible and does not violate "Siegert's theorem." Nevertheless the effect found is surprising. It is worthwhile to note that this effect cannot be attributed to the $(\mathbf{L} \cdot \mathbf{S})$ interaction between nucleons, as the magnetic dipole operator which that force implies has no matrix elements between the states in question.

Note added in proof.—It has been suggested that the n - p capture must have a contribution from the electric quadrupole transition from the continuum 3S state to the 3D part of the ground state. This quadrupole process does not interfere with the magnetic dipole process, hence it is expected to give a small change in the cross section. Explicit calculation shows the ratio of $E2$ and $M1$ cross sections to be about 10^{-9} , so the $E2$ cross section is negligible. We are grateful to Dr. C. J. Goebel for reminding us of the $E2$ process, and for discussions of its magnitude.

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Decay of Os^{182} and Os^{183} . I. Gamma and Beta Spectroscopy*

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Four activities, produced by the bombardment of tungsten with alpha particles having energies up to 48 Mev, were identified. They have half-lives of 9.9 ± 0.3 , 13.67 ± 0.1 , 21.1 ± 0.3 , and >500 hours. From the excitation functions and other measurements these are attributed to the decay of Os^{183m} , Os^{183} , and Os^{182} , respectively. Measurements on the gamma-ray spectra and conversion line spectra are reported. The spectra are complex, and a total of 251 conversion lines was observed. In many cases the decay of individual gamma rays and conversion lines was studied.

1. INTRODUCTION

UNTIL a short time ago the only information reported on the decay properties of the K -capturing nuclei Os^{182} and Os^{183} was that the half-lives were 24 ± 1 hour and 12 ± 0.5 hours (Stover¹). More

recently Foster, Hilborn, and Yaffe² reported a half-life of 21.9 ± 0.1 hours for Os^{182} and two distinct half-lives of 15.4 ± 0.3 hours and 10 ± 1 hour for Os^{183} . They also report a number of transitions which they assign to these various decays.

The present work was undertaken because it was thought to be of interest to study nuclei in the transition

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¹ B. J. Stover, Phys. Rev. **80**, 99 (1950).

² J. S. Foster, J. W. Hilborn, and L. Yaffe, Can. J. Phys. **36**, 555 (1958).