Effect of a Pion-Pion Scattering Resonance on Nucleon Structure. II*

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(Received October 9, 1959)

It is shown that a resonance of suitable position and width in the $J=1$, $I=1$ state of the pion-pion system can bring the dispersion-theoretic calculation of the isotopic-vector part of the nucleon electromagnetic structure into agreement with experiment. The calculation of the isotopic-vector part of the nucleon form factors involves in first approximation the pion form factor and the matrix element for the production by two pions of a nucleon-antinucleon pair. For the pion form factor we used a semiphenomenological solution based on the work of Chew and Mandelstam and involving two parameters related to the position and width of the resonance. For the $\pi+\pi\rightarrow N+\bar{N}$ amplitude we used the results of the preceding paper.

I. INTRODUCTION

THE basic physical ideas used herein are discussed in an earlier Letter,[1] to which the reader is referred in lieu of an extensive introduction. The calculations outlined therein are carried out in this paper; namely, the dispersion-relation calculations of the electromagnetic properties of the nucleon carried out by Chew *et al.*[2] and by Federbush *et al.*[3] are modified to include the effect of pion-pion scattering.[4] Again we refer the reader to these papers for basic definitions and discussions of the dispersion-relation approach to the nucleon electromagnetic structure. As was done in these papers, we confine our remarks to the isotopic-vector part of the nucleon structure, rather than face the complexity of a three-pion intermediate state. Thus we are unable to say anything about the charge structure of the neutron.

For completeness we reproduce here, in essentially the same notation as in C, the dispersion relations for the form factors:

$$G_1{}^V(t)=\frac{e}{2}+\frac{t}{\pi}\int_{4\mu^2}^{\infty}\frac{g_1{}^V(t')dt'}{t'(t'-t)},\tag{1.1a}$$

$$G_2{}^V(t)=\frac{1}{\pi}\int_{4\mu^2}^{\infty}\frac{g_2{}^V(t')dt'}{t'-t},\tag{1.1b}$$

where $t=-(p'-p)^2=(p_0'-p_0)^2-(\mathbf{p}'-\mathbf{p})^2$, the square of the energy-momentum-transfer four vector.

In Sec. II our method is stated in detail. In Sec. III the results of our calculation are presented, and in Sec. IV these results are discussed.

II. RELATION TO THE PROCESS $\pi+\pi\rightarrow N+\bar{N}$ AND TO THE PION FORM FACTOR

In the preceding paper[5] partial-wave dispersion relations were derived from the Mandelstam representation[6] for the process $\pi+\pi\rightarrow N+\bar{N}$. The amplitudes $f_\pm{}^J(t)$ defined by Eqs. (3.15) and (3.16) of P and related to S-matrix elements of given nucleon and antinucleon helicities were found to possess simple analytic properties in the complex t plane. In order to relate the two-pion intermediate-state contribution to the nucleon form factors to these helicity amplitudes in the $J=1$ state, let us state their relation to the customary S- and D-wave amplitudes. Using Eq. (B5) of Jacob and Wick[7] and Eqs. (3.8), (3.13), and (3.14) of P, one finds[8]

$$f_+=E(3/2pq^3)^{\frac{1}{2}}[(\beta_S-\sqrt{2}\beta_D)/3],\tag{2.1a}$$

$$f_-=(3/2pq^3)^{\frac{1}{2}}[(\sqrt{2}\beta_S+\beta_D)/3],\tag{2.1b}$$

where $f_\pm=f_\pm{}^1$. The relation between β_S and β_D and the spectral functions $g_i{}^V$ is given by Eqs. (3.18) and (3.19) of F. Substituting Eqs. (2.1) into these formulas, one finds

$$g_i{}^V(t)=-(eF_\pi{}^*q^3/2E)\Gamma_i(t),\tag{2.2}$$

where

$$\Gamma_1(t)=(m/p_-{}^2)[(E^2/\sqrt{2}m)f_-(t)-f_+(t)],\tag{2.3a}$$

$$\Gamma_2(t)=(1/2p_-{}^2)[f_+(t)-(m/\sqrt{2})f_-(t)].\tag{2.3b}$$

In Sec. VI of P a method of approximate solution for $f_\pm(t)$ was given. For the problem at hand it is more convenient to apply this method to the functions $\Gamma_i(t)$ defined above, which have the same singularities in the complex t plane—namely, one branch cut from $-\infty$ to a, where $a=4\mu^2(1-\mu^2/4m^2)$ and another branch cut from $4\mu^2$ to ∞. The division by $p_-{}^2$ does not introduce a pole, since the factors in brackets in Eqs. (2.3) vanish to order p^2 at $p=0$. This fact can be seen form Eqs. (3.15) and (3.16) of P, which imply (remembering

* This work done under the auspices of the U. S. Atomic Energy Commission.
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[1] W. R. Frazer and J. R. Fulco, Phys. Rev. Letters **2**, 365 (1959). Hereafter called L.
[2] G. F. Chew, R. Karplus, S. Gasiorowicz, and F. Zachariasen, Phys. Rev. **110**, 265 (1958). Hereafter called C.
[3] P. Federbush, M. L. Goldberger, and S. B. Treiman, Phys. Rev. **112**, 642 (1958). Hereafter called F.
[4] It has been called to our attention that similar physical ideas are contained in W. G. Holladay, Phys. Rev. **101**, 1198 (1956).
[5] W. R. Frazer and J. R. Fulco, preceding paper [Phys. Rev. **117**, 1603 (1960)]. Hereafter called P.
[6] S. Mandelstam, Phys. Rev. **112**, 1344 (1958) and Phys. Rev. **115**, 1741 and 1752 (1959).
[7] M. Jacob and G. C. Wick, Ann. Phys. **7**, 404 (1959).
[8] We use the same notation as in P. The mass of the pion is set equal to unity.

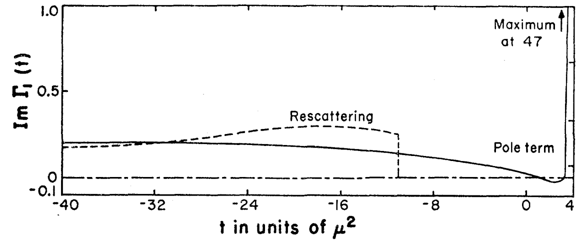

FIG. 1. The spectral function $\mathrm{Im}\Gamma_1(t)$, occurring in Eq. (2.5), for the nucleon charge structure. The contributions of both the pole term and the (3,3) resonance (rescattering correction) are shown.

that at $p=0$, A_J and B_J vanish as p^J)

$$f_+^J(4m^2)/mJ=f_-^J(t)/[J(J+1)]^{\frac{1}{2}}+O(p^2). \qquad (2.4)$$

In addition to having the same singularities as the functions $f_\pm(t)$, the $\Gamma_i(t)$ have the same phase. In constructing the solution for $f_\pm^J$ given in P, we conjectured that in the region $4\mu^2\leq t\leq 16\mu^2$ the phase of these amplitudes is equal to the pion-pion scattering phase shift in the corresponding angular-momentum and isotopic-spin state. For the amplitudes $f_\pm$ that enter into the nucleon electromagnetic structure problem this phase condition is as well founded as the dispersion-relation approach itself. This statement follows from the reality of the g_i's, which is in turn implied by the reality of the G_i's.[9] Now, in the region $4\mu^2\leq t\leq 16\mu^2$ the only contribution to the weight functions g_i comes from the two-pion intermediate state, so that Eq. (2.2) is exact. Therefore if g_i^V is real, then $\Gamma_i(t)$ must have the same phase as $F_\pi(t)$, which can be proved to be the pion-pion scattering phase shift in the $J=1$, $I=1$ state (hereafter designated δ).[10] Then the method described in L and P allows us to write, in the approximation of neglecting all but the two-pion intermediate state,

$$\Gamma_i(t)=\frac{F_\pi(t)}{\pi}\int_{-\infty}^{a}\frac{dt'\,\mathrm{Im}\Gamma_i(t')}{(t'-t)F_\pi(t')}. \qquad (2.5)$$

The quantity $\mathrm{Im}\Gamma_i(t)$ is given in terms of pion-nucleon scattering by Eqs. (2.3) and by Eqs. (5.7) and (5.8) of P. In the region $0\leq t\leq a$, the only contribution comes from the single-nucleon poles in pion-nucleon scattering:

$$[\mathrm{Im}\Gamma_1(t)]_N=(m/2p_-^3q_-)\{2\pi f^2m^3[3z_0^2-1 \\ -(p_-^2/m^2)(z_0^2-1)]\}, \qquad (2.6a)$$

$$[\mathrm{Im}\Gamma_2(t)]_N=-(1/4p_-^3q_-)2\pi f^2m^3(3z_0^2-1), \qquad (2.6b)$$

where the subscript N indicates that these terms come from the single-nucleon intermediate state in pion-nucleon scattering. These terms are plotted in Figs. 1 and 2. Notice that the anomalous-moment weight function has a large peak close to $t=4\mu^2$, which dominates the integral in Eq. (2.5). Therefore the anomalous moment receives a dominant contribution from a region in which we can calculate the weight function $\mathrm{Im}\Gamma_2$ quite accurately. Since $F_\pi=1$ at $t=0$, the large peak in $\mathrm{Im}\Gamma_2$ permits the approximation, accurate to about 10%, of setting $F_\pi=1$ under the integral sign in Eq. (2.5) for Γ_2. In this approximation we have simply

$$g_2^V(t)\approx|F_\pi(t)|^2[g_2^V(t)]_0,$$

where $[g_2^V(t)]_0$ means the weight function calculated with pion-pion scattering neglected, as in C and F.

The situation is not so favorable for the charge structure. The narrow peak in $\mathrm{Im}\Gamma_1$ does not dominate; the more distant contributions are comparable. Therefore our calculations are not so reliable for the charge structure as for the anomalous magnetic moment and its structure.

In the region $-9.36\leq t\leq 0$, there is an additional contribution to $\mathrm{Im}\Gamma_i$ from elastic pion-nucleon scattering (often called the rescattering correction):

$$[\mathrm{Im}\Gamma_1(t)]_{\pi N}=\frac{m}{8\pi p_-^3q_-}\int_{(m+\mu)^2}^{L(t)}ds' \\ \times\{(p_-/q_-)za_1^{(-)}+b_1^{(-)}(m/2)[(3z^2-1) \\ -(p_-^2/m^2)(z^2-1)]\}, \qquad (2.7a)$$

$$[\mathrm{Im}\Gamma_2(t)]_{\pi N}=-\frac{1}{16\pi p_-^3q_-}\int_{(m+\mu)^2}^{L(t)}ds'\,\{(p_-/q_-)za_1^{(-)} \\ +b_1^{(-)}(m/2)(3z^2-1)\}, \qquad (2.7b)$$

where a_1 and b_1 are given by Eq. (5.10) of P. As pointed out in P, the polynomial expansion given in Eq. (5.10) converges only for $t\lesssim -26$. We shall evaluate this term in the same approximation as is used in C and F; i.e., setting all partial cross sections equal to zero except for the (3,3) state, and approximating the latter by a delta function [see Eq. (16.3) of C]. In this approximation the corresponding $\mathrm{Im}\Gamma_i$, which are nonzero from $-\infty\leq t\leq -11$, are shown in Fig. 1 and Fig. 2.

The threshold for inelastic processes occurs at

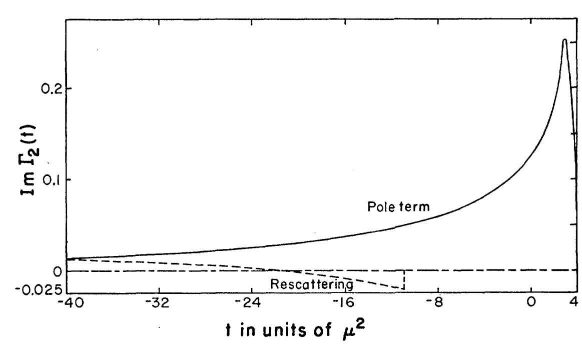

FIG. 2. The spectral function $\mathrm{Im}\Gamma_2(t)$, occurring in Eq. (2.5), for the nucleon anomalous magnetic moment. The contributions of both the pole term and the rescattering corrections are shown.

[9] D. R. Yennie, M. M. Lévy, and D. G. Ravenhall, Revs. Modern Phys. **29**, 144 (1957), Appendix.

[10] See, for example, Appendix II of S. Fubini, Y. Nambu, and V. Wataghin, Phys. Rev. **111**, 329 (1958).

$t=-9.36$. The exact expression for $\mathrm{Im}\Gamma_i$ consists of the two terms given above, plus all the possible inelastic processes, i.e.,

$$\mathrm{Im}\Gamma_i(t)=\Sigma_n\theta(t_n-t)[\mathrm{Im}\Gamma_i(t)]_n, \qquad (2.8)$$

where the index n runs over all possible intermediate states in pion-nucleon scattering, and where the various thresholds t_n are given by Eq. (5.3) of P. All these terms except the single-nucleon pole and elastic terms are ignored in our treatment, according to the "effective-range" approximation of neglecting distant singularities. One might expect *a priori* that the dominant contribution for small t to the functions $\Gamma_i(t)$ comes from the pole term, since its branch cut lies so close to the region of interest, $t\gtrsim 4\mu^2$. This is indeed the case for the anomalous magnetic moment and its structure.

Looking back at Eq. (2.5), we see that if we knew the pion form factor $F_\pi(t)$ we would have an explicit solution for $\Gamma_i(t)$, and therefore for the isotopic-vector form factors $G_i^V(t)$ of the nucleon. If the pion-pion scattering phase shift δ in the $J=1$, $I=1$ state were known, one could calculate an approximate expression for $F_\pi(t)$, valid for small t. The procedure of Sec. VI of P gives for this quantity

$$F_\pi(t)=e^{u_1'(t)}, \qquad (2.9)$$

where $u_1'(t)$ is defined in terms of δ by Eq. (6.2) of P

A theoretical treatment of pion-pion scattering by Chew and Mandelstam,[11] using the method of partial-wave dispersion relations, is now in progress. Consider their Eqs. (V.23) to (V.25) of reference 11 for the P-wave amplitude $A_1(\nu)/\nu$, where $\nu=\frac{1}{4}t-\mu^2$. Their method of solution involves expressing the amplitude as a ratio of numerator and denominator functions, the latter being a real analytic function except for the physical branch cut for $\nu>0$, the former having singularities only on the negative real axis. Now, since the pion form factor has the same right-hand branch cut as A_1, and has the same phase on this cut (neglecting four-pion and higher intermediate states), we can identify the form factor with the reciprocal of the denominator function[12]; i.e.,

$$F_\pi[t(\nu)]=D_1(-1)/D_1(\nu). \qquad (2.10)$$

This identification satisfies the physical criterion that $F_\pi=1$ for $\delta=0$.

In the treatment of Chew and Mandelstam the numerator function is expressed as an expansion in terms of all angular-momentum states of pion-pion scattering. The resulting set of coupled nonlinear integral equations has not yet been solved. We shall therefore adopt the more approximate and phenomenological approach outlined in L; namely, we approximate the effect of the left-hand branch cut by a pole of appropriate position and residue,

$$N_1(\nu)=\lambda/(\nu+\nu_0), \qquad (2.11)$$

where λ and ν_0 will be determined by comparison with experiment. Equations (2.10) and (2.11), together with Eq. (V.25) of reference 11, give

$$F_\pi[t(\nu)]=\left(\frac{\nu_0+\nu}{\nu_0-1}\right)\times\frac{\nu_r+1-(2/\pi)\Gamma}{\nu_r-\nu[1-\Gamma\alpha(\nu)]-i\theta(\nu)\Gamma[\nu^3/(\nu+1)]^{\frac{1}{2}}}, \qquad (2.12)$$

where, for $\nu>0$ and $\nu<-1$,

$$\alpha(\nu)=\frac{2}{\pi}\left(\frac{\nu}{\nu+1}\right)^{\frac{1}{2}}\ln(|\nu|^{\frac{1}{2}}+|\nu+1|^{\frac{1}{2}}), \qquad (2.13\text{a})$$

and, for $-1<\nu<0$,

$$\alpha(\nu)=\left(\frac{-\nu}{1+\nu}\right)^{\frac{1}{2}}\left[1-\frac{2}{\pi}\,\mathrm{ctn}^{-1}\left(\frac{1+\nu}{-\nu}\right)^{\frac{1}{2}}\right]. \qquad (2.13\text{b})$$

The constants ν_r and Γ are related to the position and residue of the pole by the equations

$$\Gamma=\lambda/[\lambda\alpha(-\nu_0)-1], \quad \nu_r/\Gamma=\nu_0/\lambda. \qquad (2.14)$$

If ν_r and Γ are positive and not too large, the real part of the denominator in Eq. (2.12) can vanish, corresponding to a P-wave resonance in pion-pion scattering. In this case, ν_r is approximately the position of the resonance and Γ is related to its width. We show in the next section that such resonant solutions do indeed result in good agreement with the nucleon-structure experiments.

III. RESULTS

The integration of Eqs. (2.5) and (1.1) has been performed with the aid of an IBM-650 computer. The results are given in this section. It can be seen from Eq. (2.3) and the limitations imposed by unitarity on the functions $f_\pm(t)$ that these integrals should converge if the pion form factor given by Eq. (2.12) is used. However, the polynomial expansion we have made in treating the rescattering correction, and our approximate solution, Eq. (2.5), both violate the unitarity restrictions. This is, of course, because our treatment is supposed to be valid only at low energies. Therefore we cut off all integrations and regard as reliable only that portion of our results which turns out to be insensitive to the position of the cutoff.

A. The Nucleon Anomalous Magnetic Moment

Let us first consider the dominant contribution to the anomalous magnetic moment, which comes from the pole term, Eq. (2.6), as a function of the two parameters ν_r and Γ. Over the range of these parameters that we

[11] G. F. Chew and S. Mandelstam, Lawrence Radiation Laboratory Report UCRL-8728, April, 1959 (unpublished).

[12] G. F. Chew, Lawrence Radiation Laboratory (private communication, 1959).

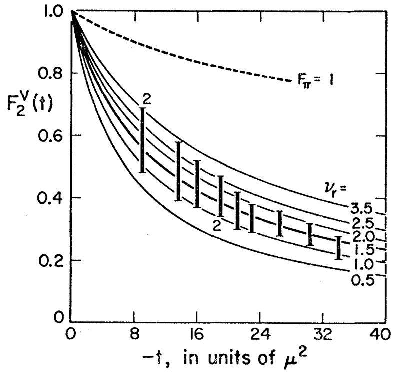

FIG. 3. The isotopic-vector anomalous magnetic moment structure as a function of the position of the resonance. The width is held fixed at a value $\Gamma=0.4$, which produces agreement with the observed moment. Experimental points marked "2" are those at which two measurements have been made, in order to determine F_1 and F_2 separately. The errors shown include the uncertainty in the ratio F_1/F_2. The curve labeled $F_\pi=1$ is the prediction of the dispersion relations in the case of no pion-pion scattering.

considered, it turns out that the anomalous moment μ_V depends only on Γ, whereas the radius $\langle r_2^2\rangle_V$ of the anomalous-moment distribution is controlled by ν_r. A value of $\Gamma=0.4$ gives $\mu_V=1.83e/2m$, in agreement with experiment. Raising Γ to 0.5 decreases μ_V to 1.34. For $\Gamma=0.4$ the dependence of the form factor $F_2^V(t)$ on ν_r is shown in Fig. 3. For comparison, the prediction of the dispersion relations in the case of no pion-pion scattering is also shown. The heavy curve, which fits the experimental points best, has $\nu_r=1.5$ and corresponds to $\langle r_2^2\rangle_V=0.55$ (rms radius 1.05×10^{-13} cm), somewhat higher than that obtained with the models of Hofstadter *et al.*[13] The resonance positions $\nu_r=2.5$, 2.0, and 1.0, which give reasonable fits to the data, give rms radii of 0.91, 0.97, and 1.13×10^{-13} cm, respectively.

In all the above results, the cutoff on the integration in Eq. (2.5) over the left-hand branch cut was set near $t=-26$, the point beyond which the polynomial expansion in Eq. (2.7) fails to converge. The value of the rms radius and of $F_2^V(t)$ are practically independent of the cutoff position, which we allowed to vary over the wide range $-4m^2\leqslant t\leqslant -8$. Moreover, this extreme

TABLE I. Pole and rescattering contributions to μ_V as functions of cutoff (expressed in pion mass units).

Left-hand cutoff	Pole contribution to μ_V (in units of $e/2m$)	Rescattering correction to μ_V (in units of $e/2m$)
-8	1.34	...
-32	1.83	-0.05
-100	2.20	0.39
$-4m^2$	1.84	0.60

[13] We are indebted to Dr. S. D. Drell and Dr. F. Bumiller for providing us with a graph from which the experimental points in Figs. 3 and 4 are taken. For a review of the experiments see R. Hofstadter, F. Bumiller, and M. R. Yearian, Revs. Modern Phys. **30**, 482 (1958).

variation in the cutoff produced a deviation in μ_V of less than 25% from the value $\mu_V=1.83$ (see Table I). None of the quantities calculated shows any appreciable sensitivity to a variation of the cutoff of the integration in Eq. (1.1) over the range $m^2\leqslant t\leqslant 16m^2$. Furthermore, over 80% of the calculated μ_V comes from below the threshold at $t=16$ of the lowest neglected intermediate state.

The contribution of the (3,3) resonance in pion-nucleon scattering to μ_V, shown in Table I, is negligible for a cutoff around the point $t=-26$. This contribution becomes appreciable only when the cutoff is moved extremely far out into the region of divergence of the Legendre polynomial expansion. The large rescattering corrections obtained in F can now be seen (as was already surmised in F) to arise from this divergent expansion. Whereas more sophisticated methods of analytic continuation of the pion-nucleon scattering amplitude than we have used will be necessary to evaluate the rescattering corrections precisely, our method indicates that they are not large.

TABLE II. Variation of calculated nucleonic charge and rms charge radius with cutoff (expressed in pion mass units).

	$\langle r_1^2\rangle_V$		Calculated nucleonic charge, $G_1^V(0)/e$ (should equal $\frac{1}{2}$)	
Left-hand cutoff	Normalized to calculated charge (μ^{-2})	Normalized to observed charge (μ^{-2})	From pole term	Rescattering correction
-8	0.58	0.17	0.15	...
-32	0.54	0.40	0.37	0.35
-100	0.51	0.91	0.89	0.65
$-4m^2$	0.50	1.34	1.34	0.66

B. The Nucleon Charge Structure

The interpretation of our results for the charge structure is less straightforward. If one calculates $G_1^V(t)$ from the subtracted form of the dispersion relation, Eq. (1.1a), the result varies wildly with change in the position of the cutoff on the integral in Eq. (2.5), and is therefore unreliable. However, Chew has raised the possibility of using the unsubtracted form of the dispersion relation,[14]

$$G_1^V(t)=\frac{1}{\pi}\int_{4\mu^2}^{\infty}\frac{g_1^V(t')dt'}{t'-t}. \tag{3.1}$$

Whereas unitarity has guaranteed the convergence of all the integrals we have calculated up to this point, it gives us no such assurance for Eq. (3.1). If nevertheless one assumes that this integral does have meaning, one can calculate the nucleonic charge in terms of the spectral functions. If we then calculate $G_1^V(t)$ from Eq. (3.1) and normalize to the *calculated* rather than

[14] Geoffrey F. Chew, University of California Radiation Laboratory Report UCRL-8194, February, 1958 (unpublished).

to the observed charge, we find that the nucleon charge structure is practically independent of the cutoff, since the shape of $g_1^V(t)$ is determined almost entirely by $F_\pi(t)$.

Whereas the integral in Eq. (2.5) over the left-hand branch cut receives important contributions from distant regions and is unreliably calculated, the integral in Eq. (3.1) is dominated by the low-energy region because of the factor $|F_\pi|^2$ in g_1^V. Therefore the unreliability of the left-hand integration is reflected primarily in the normalization of g_1^V and not in the calculated charge radius.

We find that to within 5%, $F_1^V(t)=F_2^V(t)$, a conclusion which agrees with experiment within the large uncertainties involved. The variation of the rms charge radius with the left-hand cutoff is shown in Table II for the calculations with subtracted and unsubtracted dispersion relations. Also shown are the calculated

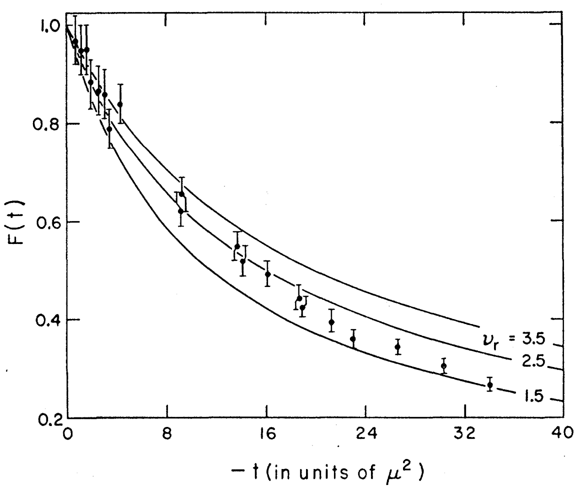

FIG. 4. A comparison of the calculated isotopic vector form factors $[F_1^V(t)\approx F_2^V(t)]$ for $\Gamma=0.4$ with the experimental proton form factors, under the assumption (see text) that $F_1^p(t)=F_2^p(t)$. This comparison is probably meaningful only for small values of t.

nucleonic charge and the rescattering correction thereto. As in the case of the anomalous-moment structure, the rescattering corrections do not have a large effect on the charge structure if one normalizes to the calculated charge.

If one accepts the result that $F_1^V(t)\approx F_2^V(t)$ and uses the fact that the neutron charge radius has been shown experimentally to be extremely small, one can conclude that $F_1^p(t)\approx F_2^p(t)$ for small t. The resulting comparison with experiment is shown in Fig. 4. Higher values of ν_r are favored by this comparison than by Fig. 3, but the theoretical situation is not as clear.

On the other hand, the result $F_1^V(t)\approx F_2^V(t)$ can be combined with experimental information to set a limitation on the charge form factor of the neutron. We assume $G_2^S(t)\approx 0$, or $G_2^n(t)\approx -G_2^p(t)$, since $G_2^S(0) = -0.06\ e/2m$, and define $F_1^p(t)/F_2^p(t)=b(t)$. Then it

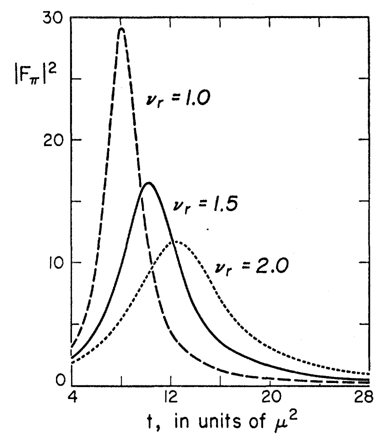

FIG. 5. The square of the magnitude of the pion form factor for $t\geqslant 4$, for $\Gamma=0.4$ and for three positions of the resonance.

follows that

$$F_1^n=F_2^p(b-1). \tag{3.2}$$

It is known experimentally that at $t=-9$ and $t=-19$, $b\approx 1.2\pm 0.2$.[15]

C. The Pion Form Factor and Pion-Pion Scattering

The parameters $\nu_r=1.5$ and $\Gamma=0.4$, which give the best fit to the isotopic-vector anomalous magnetic moment and its structure, imply a pion electromagnetic form factor as shown in Figs. 5 and 6, calculated from Eq. (2.12). From Eq. (2.12) one can calculate the expression for the pion radius, valid in the approximation $\nu_0\gg 1$,

$$\langle r_\pi^2\rangle \approx \frac{3}{2}\frac{1-(8/3\pi)\Gamma}{\nu_r+1-(2/\pi)\Gamma}. \tag{3.3}$$

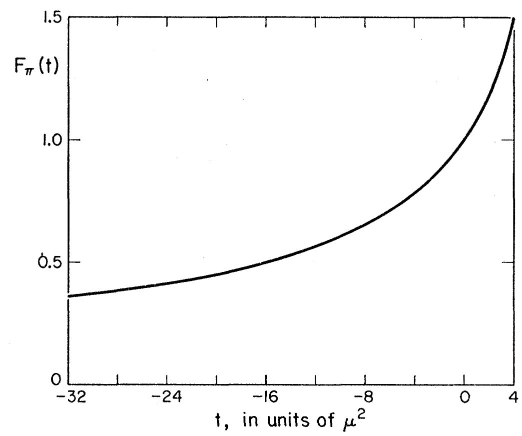

FIG. 6. The pion form factor in the region in which it is measurable in principle by electron-pion scattering experiments, for the parameters $\nu_r=1.5$ and $\Gamma=0.4$ that give the best fit to the anomalous-moment structure.

[15] R. Hofstadter, Stanford University (private communication, 1959).

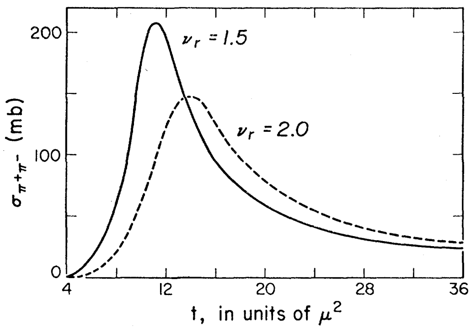

FIG. 7. The total cross section for $\pi^+-\pi^-$ scattering which would correspond to $\Gamma=0.4$ and two values of ν_r if there were no scattering in states other than $J=1$, $I=1$.

For all values of the parameters investigated, this approximation is an excellent one; i.e., for $\nu_r=1.5$ and $\Gamma=0.4$, $\nu_0=652$. The pion radius corresponding to these parameters is $\langle r_\pi^2\rangle=0.44$.

Since the pion form factor is closely related to the pion-pion scattering amplitude in the $J=1$, $I=1$ state [see Eqs. (10) and (11) of L], one can calculate the cross section which would be implied by the above results if there were no scattering in other states. The total cross section for $\pi^+-\pi^-$ scattering calculated under these hypotheses is shown in Fig. 7.

IV. CONCLUSIONS

We conclude that a resonance in the $J=1$, $I=1$ state of pion-pion scattering characterized by the position $\nu_r=1.5$ and the width $\Gamma=0.4$ would give complete agreement with experiment for the isotopic-vector anomalous magnetic moment and its structure, and, with some ambiguity, for the proton charge structure. The position and width of the resonance are not very precisely determined; furthermore, the contributions of higher-mass intermediate states we have neglected will certainly have some effect on the parameters. It is, however, difficult to imagine a mechanism other than the proposed resonance that would resolve the discrepancies which existed between dispersion theory and experiment.[16]

ACKNOWLEDGMENTS

We are indebted to Dr. Geoffrey F. Chew for his advice throughout this work and to Dr. Stanley Mandelstam, Dr. Sidney Drell, and Dr. Marshall Baker for helpful discussions. We also thank Mr. Michael Lourié for carrying out the numerical calculations on the IBM-650 computer.

[16] For an analysis of these discrepancies see S. D. Drell, *1958 Annual International Conference on High-Energy Physics at CERN*, edited by B. Ferretti (CERN, Geneva, 1958).

PHYSICAL REVIEW VOLUME 117, NUMBER 6 MARCH 15, 1960

Electron-Neutrino and Electron-Antineutrino Scattering

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(Received October 12, 1959)

Cross sections for electron-neutrino and electron-antineutrino scattering are given as a function of recoil electron energy, averaged over a reactor spectrum of antineutrinos.

If it ultimately becomes feasible to observe elastic electron-neutrino and electron-antineutrino scattering, the observation must depend on the ionization produced by the recoil electron. The following note estimates relevant cross sections and energy distributions.[1]

The conventional assumption of a universal Fermi interaction with lepton conservation and two-component neutrinos would yield a neutrino-electron interaction of the form

$$H=g\{\bar{\psi}_e\gamma_\mu(1-\gamma_5)\psi_\nu\}\{\bar{\psi}_\nu\gamma_\mu(1-\gamma_5)\psi_e\} \qquad (1a)$$

$$=g\{\bar{\psi}_e\gamma_\mu(1-\gamma_5)\psi_e\}\{\bar{\psi}_\nu\gamma_\mu(1-\gamma_5)\psi_\nu\} \qquad (1b)$$

$$=-g\{\bar{\psi}_e\gamma_\mu(1-\gamma_5)\psi_e\}\{\bar{\psi}_{\bar{\nu}}\gamma_\mu(1+\gamma_5)\bar{\psi}_{\bar{\nu}}\}. \qquad (1c)$$

Fierz transposition leads from Eq. (1a) to (1b); and Eq. (1c) is appropriate to antineutrino-electron scattering, as would be induced by the flux from a reactor. The cross section from Eq. (1c) for an electron at rest is

$$d\sigma=(8/\pi)(\mathrm{gm})^2N^{-3}(N+1-E)(E-1)dE, \qquad (2)$$

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[1] Only elastic scattering is considered; the inelastic process $\bar{\nu}+e \rightarrow \mu+\bar{\nu}$ has a threshold of order $\mu^2/2m\approx 10$ Bev, and it is difficult to imagine sources for such energetic neutrinos and antineutrinos with measurable intensity.