is 79. Since the 81.6-kev transition in Ta^{175} decays with an $M1$–$E2$ mixture which is typical of intraband transitions, it is believed that this state is the $\frac{7}{2}$ member of the $K=\frac{5}{2}$ rotational band. The $I(I+1)$ energy dependence predicts that the energy of the 9/2 member should be 187 kev, which is very close to the observed level at 186.0. This level decays mostly by an $M1$–$E2$ mixed transition of 104.3 kev to the 81.6-kev level and by a weaker 186.0-kev crossover. This pattern is entirely consistent with an assignment of the 186-kev level as the 9/2 member of the ground rotational band associated with the $\frac{5}{2}-$ (512) neutron orbital.

The state at 348.8 kev decays by $M1$ transitions to the 9/2- and $\frac{7}{2}$-members of the ground-state band and must therefore be a $\frac{7}{2}$- or 9/2-state belonging to a different intrinsic configuration, presumably the $\frac{7}{2}-$ (514). The state at 207.8 kev must have spin $\frac{5}{2}$ or $\frac{7}{2}$ with negative parity. None of the other observed transitions can be unambiguously assigned to the decay scheme.

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Scattering of High-Energy Nucleons by a Nonlocal Potential*

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The scattering of high-energy nucleons by a simple nonlocal potential is examined in the Born approximation. It is shown that an energy dependent local potential is not fully equivalent to a nonlocal potential. The latter potential introduces an additional angular dependence in the differential cross section which seems to be particularly significant in the backward directions.

THE generalized optical potential has been shown by Feshbach[1] to be both nonlocal and energy-dependent. In the interpretation of the scattering data by an optical model, the empirical potential was, however, commonly taken as local and energy-dependent. In an infinite medium, it is of course true that one can replace the nonlocality of the potential by an explicit energy-dependence.[2] But for the scattering from a finite nucleus, the situation is more complicated, and this simple equivalence should not be expected to hold. The purpose of this note is to show that at least in the Born approximation, the nonlocality of the potential does introduce some modification in the differential cross section also and hence is not reflected entirely by an additional energy dependence.

The nonlocal Schrödinger equation,

$$\hbar^2/2M\nabla^2\psi+E\psi=\int K(\mathbf{r},\mathbf{r}')\psi(\mathbf{r}')d\mathbf{r}', \tag{1}$$

with the phenomenological kernel

$$K(\mathbf{r},\mathbf{r}')=V[(\mathbf{r}+\mathbf{r}')/2]\delta_a(\mathbf{r}-\mathbf{r}') \tag{2}$$

has been used by several authors[3] to describe nuclear bound state problems and scattering phenomena at low energies. Using the effective mass approximation for finite nuclei,[4] it was found that Eq. (1), with the particular choice of kernel given by (2), can indeed yield a fairly satisfactory description in the low-energy region; thus, in this study, we shall retain this special form of the phenomenological kernel. Since our attention will be restricted toward high-energy nucleon-nuclear scattering, the effective mass approximation is no longer valid; hence our results will depend somewhat on the choice of the form factor $\delta_a(\mathbf{r}-\mathbf{r}')$ to be used in Eq. (2).

The nonlocal scattering amplitude f_{NL} may be obtained by examining the asymptotic behavior of ψ in the usual way. The result is

$$f_{\mathrm{NL}}=-(\tfrac{1}{4}\pi)\int \exp(-i\mathbf{k}\cdot\mathbf{r})V[(\mathbf{r}+\mathbf{r}')/2] \times\delta_a(\mathbf{r}-\mathbf{r}')\psi(\mathbf{r}')d\mathbf{r}d\mathbf{r}', \tag{3}$$

where $\mathbf{k}$ is the wave number of the scattered wave, and V contains the factor $2M/\hbar^2$. By introducing relative and "center-of-mass" coordinates and the Fourier

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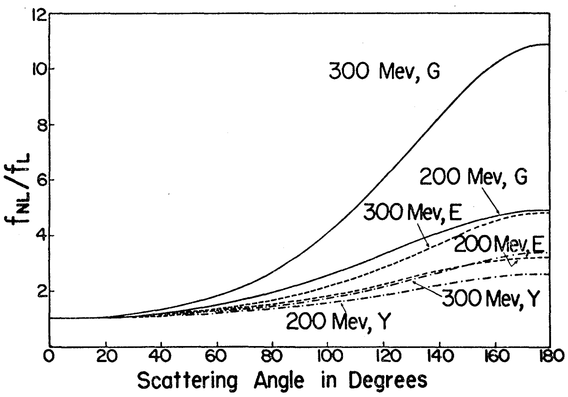

FIG. 1. The correction factor as a function of the scattering angle. G, E, and Y stand for Gaussian, exponential, and Yukawa functional dependence of the nonlocal potential, respectively.

transforms of δ_a and ψ, Eq. (3) may be further reduced and yields

$$f_{NL}=-(\tfrac{1}{4}\pi)\int \exp(-i\mathbf{k}\cdot\mathbf{R})V(\mathbf{R})\tilde{\delta}_a[(\mathbf{k}+\mathbf{k}')/2] \times\exp(i\mathbf{k}'\cdot\mathbf{R})\tilde{\psi}(\mathbf{k}')d\mathbf{k}'d\mathbf{R}. \quad (4)$$

Since the transform $\tilde{\delta}_a$ is a slowing varying function of its argument when the nonlocality is small (i.e., δ_a in Eq. (2) is sharply peaked), the integration over $\mathbf{k}'$ in Eq. (4) may be approximately carried out if we replace $\mathbf{k}'$ in $\tilde{\delta}_a$ by the incident wave number $\mathbf{k}_0$; then,

$$f_{NL}\cong\{\tilde{\delta}_a[(\mathbf{k}+\mathbf{k}_0)/2]/\tilde{\delta}_a(\mathbf{k}_0)\} \times\left\{-\tfrac{1}{4}\pi\int \exp(-i\mathbf{k}\cdot\mathbf{R})V(\mathbf{R})(2\pi)^{\frac{3}{2}} \times\tilde{\delta}_a(\mathbf{k}_0)\psi(\mathbf{R})d\mathbf{R}\right\}. \quad (5)$$

We note that Eq. (5) is exact in the first Born approximation. If, for example, we take $\tilde{\delta}_a(\mathbf{r}-\mathbf{r}')$ as a normalized Gaussian, i.e.,

$$\delta_a(\mathbf{r}-\mathbf{r}')=(\pi a^2)^{-\frac{3}{2}}\exp[-(\mathbf{r}-\mathbf{r}')^2/a^2], \quad (6)$$

then Eq. (5) becomes

$$f_{NL}\cong\exp[(k_0^2a^2/4)\sin^2(\theta/2)] \times\left[-\tfrac{1}{4}\pi\int \exp(-i\mathbf{k}\cdot\mathbf{R})V_L(\mathbf{R})\psi(\mathbf{R})d\mathbf{R}\right], \quad (7)$$

where $V_L(\mathbf{R})=V(\mathbf{R})\exp(-k_0^2a^2/4)$. Taking the depth of $V(\mathbf{R})$ as -80 Mev and $a^2\cong0.67$ obtained by Green and Wyatt[5] in their extensive study of the nuclear bound state problem and nucleon-nuclear scattering phenomena at low energies with a nonlocal potential of the same type as considered here, the depth parameter of V_L has approximately the same value as that of the empirical local optical potential. Thus, we write

$$f_{NL}\cong\exp[(k_0^2a^2/4)\sin^2(\theta/2)]f_L. \quad (8)$$

The factor $f_{NL}/f_L=\exp[(k_0^2a^2/4)\sin^2(\theta/2)]$ expresses the approximate effect of the nonlocal interaction, which is seen to be a function of both the incident energy and the scattering angle. Qualitatively similar expressions for the correction factor result if we choose a normalized exponential or Yukawa functional dependence for δ_a. Figure 1 shows the correction factors as a function of scattering angle for incident energies of 200 Mev and 300 Mev for those choices of δ_a which all have the same root mean square spread. The general trend of the three sets of curves is seen to be rather similar. The correction is negligibly small in the forward directions but becomes appreciable in the backward directions. We point out, however, that the correction factor is mainly calculated in the Born approximation, therefore the nature of this factor at the backward angles has qualitative significance only.

In this note, we have assumed that the optical potential is primarily nonlocal. In the actual case, as is noted by Feshbach, it is also intrinsically energy-dependent. This then indicates that our calculations may have overestimated the importance of the nonlocal nature of the potential in the backward directions. A more detailed account of the scattering formalism for nonlocal potentials will be given in a forthcoming paper by W. E. Frahn and H. Fiedelday.

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