Muon Decay in Nuclear Emulsion at 25 000 Gauss*

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Positive pions from the 90-Mev pion beam of the Nevis cyclotron were stopped in nuclear emulsion which was in a magnetic field of 25 000 gauss. The asymmetry parameter for the angular distribution of the positrons which came from the decay muons was measured. The result that $P\xi=-0.87\pm0.04$ implies that either the asymmetry parameter ξ is different from the value of -1 predicted by the V-A theory or that there is about 13% depolarization of positive muons in nuclear emulsion at 25 000 gauss.

I. INTRODUCTION

WHEN experimenters measure an angular distribution of positrons from the decay of positive muons which have a polarization of magnitude P at the time of decay, they measure an angular distribution which usually is represented by the form $1+a\cos\theta$ where θ is the angle between the initial directions of the muons and positron and $a=(\xi/3)|P|$. ξ is the asymmetry parameter intrinsic to muon decay. The V-A universal theory of Fermi interactions,[1–3] which has been very successful in describing weak interactions, predicts that $\xi=-1$. Clearly, only if the value of P is known can the value of ξ be determined from measurements of the positron angular distribution. Many experiments, using various techniques, have been performed to measure a. In most of these experiments the value of P was unknown or only poorly known either because the initial polarization of the muons was unknown, as was the case when muon beams from cyclotrons were used, or because the extent of depolarization of the muons before they decayed was not known.

When muons stop in nuclear emulsion in the absence of any external magnetic field, they are depolarized by more than 50% before they decay. However, it has been shown by Orear et al.,[4] and also by Sens et al.,[5] that the presence of a large magnetic field applied in the direction of the muon spin direction can prevent most of this depolarization. These experiments gave good reason for the belief that in fields of 9000 gauss or more the muon depolarization in nuclear emulsion is negligible.

If one could apply a large enough magnetic field so that the depolarization were rendered negligible, then the measurement of the asymmetry parameter for muons coming from pions which decay at rest in this large magnetic field would provide a direct measurement of ξ. One such experiment had been carried out by Barkas et al.[6] who used a 14 200 gauss magnetic field and obtained an "a value" of -0.23 ± 0.05. This is two standard deviations away from the value of one-third expected theoretically if there were no depolarization. However, since the experiments of Orear and Sens indicated that the muon depolarization at this field is less than the 30% indicated by the Barkas experiment, it was of interest to measure the asymmetry parameter in nuclear emulsion under a larger magnetic field and with greater precision.

This paper is a report on a measurement of the muon decay asymmetry parameter based on observations of more than 95 000 muon decays in nuclear emulsion. These muons came from pions which decayed at rest in a magnetic field of 25 000 gauss. A preliminary report of this experiment has already been published.[7]

II. THE EXPERIMENT

A. Description

The mesons which were used in this experiment were obtained from the 90-Mev π^+ beam of the Nevis cyclotron. The pions were slowed down and brought to rest in a stack of nuclear emulsion. This stack was composed of approximately 100 Ilford G-5 pellicles, each 600 microns in thickness and measuring one inch by two-thirds inch. These pellicles were placed in such a way that the pions entered the two-thirds inch edge as shown in Fig. 1. Those muons which were present in the beam had a range too large for them to stop in the stack. The stack was placed inside a magnetic field

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[4] J. Orear, G. Harris, and E. Bierman, Phys. Rev. **107**, 322 (1957).

[5] J. C. Sens, R. A. Swanson, V. L. Telegdi, and D. D. Youvanovitch, Phys. Rev. **107**, 1465 (1957).

[6] W. H. Barkas, P. C. Giles, H. H. Heckman, F. W. Inman, and F. M. Smith, Phys. Rev. **107**, 911 (1957).

[7] G. Lynch, J. Orear, and S. Rosendorff, Phys. Rev. Letters **1**, 471 (1958).

of 25 000 gauss with the magnetic field direction being in the plane of the pellicles. The pellicles were exposed to an integrated flux of about 5×10^4 pions/cm^2, the vast majority of which stopped in the central one-half of the pellicles.

If all of the decay muons in these plates were traveling in the direction of the magnetic field, the angular distribution of the positrons from the muon decay would have the form $1+a\cos\phi$ where ϕ is the angle between the direction of motion of the positron and the magnetic field direction. But the muons from pion decays have an isotropic distribution, as they must if the pion has zero spin. A muon which makes an angle β with the magnetic field will precess about the magnetic field direction so that when it decays nothing is known about the component of the spin perpendicular to the magnetic field direction. Only the component of the spin in the magnetic field direction is known. For this reason the only angles worth measuring are the angles that the muon and positron make with the magnetic field. In the magnetic field direction the muon has a polarization $\cos\beta$. Thus the angular distribution has the form $1+a\cos\phi\cos\beta$. In this experiment no attempt was made to verify this distribution. The only object was to measure the asymmetry parameter a on the assumption that this distribution is valid.

The scanning procedure which was adopted was designed to minimize the scanning time necessary to obtain a result of a given precision. Thus, the attempt was not to obtain the most information per event, but rather to obtain the most information per scanner hour. This was practical because there were more events in the plates than were needed. When the time required to find events is small, a method which requires the scanner to measure only projected angles can be more efficient with regard to scanner time than a method which requires the scanner to measure the space angles ϕ and β. Furthermore since very little information is contained in those π-μ-e events in which the muon or the positron directions have small components in the field direction, there is an advantage to be gained by not recording these events.

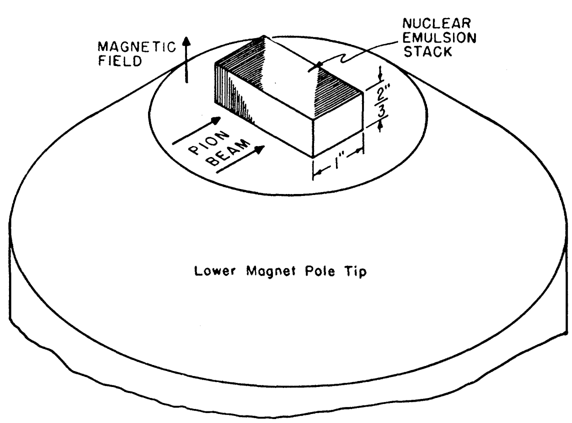

FIG. 1. Exposure geometry. Upper pole tip is not shown.

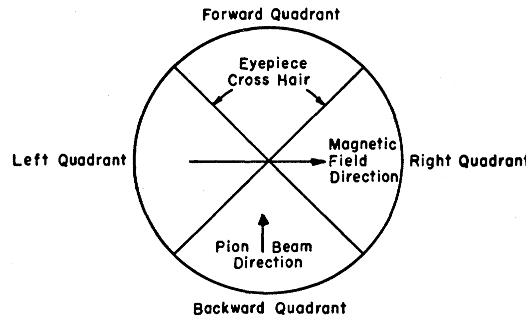

FIG. 2. Orientation of microscope eyepiece cross hair.

The following scanning procedure was adopted at Cornell. A diagonal cross hair was placed in an eyepiece of the microscope and the plate placed on the microscope stage in such a way that the magnetic field direction bisected the left and right quadrants formed by the cross hair. This is shown in Fig. 2. The plates were scanned for muon endings, using a magnification of about 500. Upon finding a muon ending, the scanner recorded it only if the muon ending was more than 30 microns from the surface of the processed emulsion (which corresponds to about 75 microns in the unprocessed emulsion) and only if both the initial projected directions of both the positron and muon lay in the quadrants bisected by the magnetic field. In other words, those events in which either the muon or the positron started out in either the forward or backward quadrant were not recorded. Clearly only one quarter of the events had muon and positron directions both in the side quadrants. Therefore, only one quarter of the events which satisfied the depth criterion were recorded. In the scanning no muons were followed from one plate to another. When the muon came from a pion which had stopped outside the pellicle in which the muon had stopped, which was the case for about one-half of the events, the initial direction of the muon was approximated by its direction upon entering the pellicle.

The scanners recorded four things for each recordable event: 1. The position of the muon ending, 2. Whether or not the muon came from a pion which decayed in the same plate, 3. Whether the muon went left or right, and 4. Whether the positron went left or right. From these data the uncorrected asymmetry parameter was estimated by the expression: $a=2[(S-O)/(S+O)]$, where S is the number of recorded events in which the muon and positron directions are in the same quadrants and O is the number of events in which they are in opposite quadrants. This expression for a can be derived[8] by integrating over the distribution $1+a\cos\phi\cos\beta$ to find the expected number of events which occur in the S

[8] G. R. Lynch, Ph.D. thesis, Cornell University, 1959 (unpublished).

and O categories. It is only fortuitous that this integration yields the same numerical constant as is used when estimating a from simple forward backward counts.

In addition to the scanning procedure outlined above, the scanners were instructed to be on the alert for the decay of a muon into three charged secondaries. Most of the scanning was done at Cornell University. However, some of the stack was scanned at Columbia University where the scanning procedure was basically the same as the one at Cornell.

B. Corrections

There are a number of systematic errors for which the experimentally measured asymmetry parameter must be corrected. For each effect a correction δ will be determined. This quantity δ is the fractional amount by which the measured asymmetry parameter must be changed in order to correct for the effect in question.

As was mentioned earlier, the scanning procedure used was such that the pion decay was observed for only about one-half of the recorded events. For the rest of the events the initial muon directions were approximated by the directions which the muons had when they entered the pellicle in which they decayed. This introduced an error in the measured asymmetry parameter because the muons scattered through an unknown angle η before entering the pellicle. This scattering effect does not change the number of muons recorded because just as many muons scatter into criterion as scatter out of criterion. The average polarization of the muons which were recorded is decreased because of this scattering Since the measured asymmetry parameter is proportional to the average muon polarization, the change in average polarization must be taken into account.

In order to determine what correction to the asymmetry parameter was needed to account for this multiple scattering effect, detailed multiple scattering calculations[8] were carried out using the exact multiple scattering distribution of Goudsmit and Saunderson. The correction depends upon the thickness ϵ of the region near the surface of the pellicle in which muon endings were not recorded. The larger the value of ϵ, the smaller is the correction. On the average the value of ϵ used was 75 microns Corresponding to this value of ϵ the multiple scattering correction is $\delta_{ms}=1.5\%$.

In an attempt to obtain larger grain densities the pellicles were processed before being mounted on glass. Because of this there was lateral shrinkage, and this shrinkage was on the average four percent greater in the magnetic field direction than it was in the direction perpendicular to the magnetic field. This necessitated a lateral shrinkage correction $\delta_{ls}=-1.2\%$.

To compensate for the uncertainty in the direction of the magnetic field a correction $\delta_m=0.2\%$ was estimated.

A small fraction of the muon stopping in the pellicles came from pions which decayed in flight. Because of the fact that only for one-half of the events is the pion decay seen, some of these unwanted muons were recorded. Since these muons had no polarization in the magnetic field direction, they increased the number of muons recorded without affecting the $S-O$ difference on the average, and a correction equal to the fraction of muons coming from pion decays in flight is necessary. To determine this number 553 muons which were in criterion were followed into the next pellicle to see if the pion decayed there, and ten were found to have come from pion decays in flight, necessitating a decay in flight correction of $\delta_{df}=0.9\pm0.4\%$.

The data must also be corrected for scanning efficiency. The efficiency for finding positrons was very good and no correction was necessary for this effect. Of the approximately 24 000 events for which the pion decay was observed and the muon was within the criteria, there were only 9 cases where the positron was not observed. In addition no positron was found on 0.2% of the endings of tracks which looked like muons but were not observed to come from pions. A few of these were probably protons and a few more were endings of cosmic-ray negative muons. Also, there were 133 endings which were labeled as probable proton endings. A few of these may have been muons.

The efficiency for finding events varied within a range of from 80 to 100% from one scanner to another. The average efficiency for finding recordable events was measured by means of a random rescanning to be $(95\pm1)\%$. Missing events constitute a bias only if the missed events are preferentially of one type. For example, the missed events might be preferentially steep events. The average value of $\cos\beta$, where β is the angle the muon makes with the magnetic field direction, is a measure of the average polarization of those muons in this direction. This in turn is proportional to the average "a value." For recordable events this average polarization should be $\frac{1}{2}$. For those events which had been missed and were found in the random rescanning, the average value of $\cos\beta$ was measured, and was found to be 0.63 ± 0.05. From this one obtains the correction needed to correct for missed events of $\delta_{me}=-0.6\pm0.4\%$.

As a check against possible recording mistakes, approximately one-half of the recorded events were re-examined. From the results obtained it was concluded that the few recording mistakes which existed caused a negligible error in the measured asymmetry parameter. A number of checks were made of the data to determine if they were self-consistent. The data of the different scanners were compared with one another, and a plate by plate analysis of the data was made. In all cases the data demonstrated self-consistency.

Due to radiative effects, there are small corrections which must be made to the theory. In both pion and muon decay internal bremsstrahlung photons are

emitted in addition to the other decay products. In pion decay these radiative effects can cause the muon not to be completely polarized. This effect is believed to be less than 0.1%.[9] In muon decay, radiative effects reduce the effective value of ξ. Kinoshita and Sirlin[10] calculated this correction to be $\delta_r = 0.3\%$.

The corrections can now be accumulated to determine the final correction. Those which have been determined are listed below.

$$\begin{aligned} \delta_{ms} &= 1.5\pm0.2\% \\ \delta_{ls} &= -1.2\pm0.1 \\ \delta_{m} &= 0.2\pm0.1 \\ \delta_{df} &= 0.9\pm0.4 \\ \delta_{me} &= -0.6\pm0.4 \\ \delta_{r} &= 0.3 \\ \hline & 0.11\pm0.6\% \end{aligned}$$

This final correction as well as the uncertainty to the correction is quite small compared with the statistical error of four percent associated with the asymmetry parameter measurement.

C. Results

A total of 18 555 events were recorded at Cornell and from them the corrected asymmetry parameter estimate is $a = -0.292\pm0.015$. In addition, 5372 events were recorded at Columbia and from these the estimate is $a = -0.286\pm0.027$ The combined asymmetry parameter based on 23 927 events is $a = -0.290\pm0.013$. This corresponds to a value of $P\xi = -0.87\pm0.04$.

In the course of the experiment more than 95 000 muon endings were observed and more than half of these were observed with care. No case of the muon decaying into three charged secondaries was observed.

III. REVIEW OF MUON ASYMMETRY PARAMETER MEASUREMENTS

There have been many experiments which have measured the asymmetry parameter a in μ-e decay. These measured values of a are equal to $\frac{1}{3}P_1P_2\xi$, where ξ is the asymmetry parameter intrinsic to muon decay, P_1 is the average polarization of the muons which are used for the measurement, and P_2 is the extent to which the muons are not depolarized in the material in which they are brought to rest. These experiments can be grouped into a number of types.

First, there are the counter experiments which used cyclotron muon beams. It was found in these experiments that the measured "a-value" depended greatly upon which stopping substance was used. But for a large class of substances, which included graphite, there was a maximum "a-value" within the statistical accuracy of about 10% with which most of the data were taken. This has generally been interpreted to indicate that P_2 is nearly unity for these substances. An alternate interpretation is that there exists a depolarization mechanism common to many of these substances and that all of them depolarize muons to about the same extent.

Experiments in which the muons came from pions which decayed at rest in the presence of very small magnetic fields which include both nuclear emulsion experiments and bubble chamber experiments have the advantage that P_1 is determined only by the pion decay and is equal theoretically to unity. Their disadvantage is that the stopping media are limited to substances which turn out to have values of P_2 which are considerably less than unity. Many of the nuclear emulsion experiments suffer from poor statistical accuracy. For this reason they are subject to the suspicion that the world average "a-value" of -0.111 ± 0.011 obtained from the twelve published emulsion experiments[11–22] which were performed at small magnetic fields may not represent the world average of performed experiments because experimenters are less apt to publish a result which is not statistically significant. Thus, there is a suspicion that the world average of emulsion "a value" measurements is too high. There have been a few emulsion experiments in which the emulsion has been placed in a large magnetic field in an attempt to reduce the depolarization of the muons. Orear et al.[4] used the Nevis cyclotron muon beam. The muons were stopped in nuclear emulsion on which was imposed a magnetic field of 9000 gauss in the direction of the muon spin. In this experiment both P_1 and P_2 were unknown. But P_2 was much nearer unity than in previous emulsion experiments. The experiment of Barkas et al.[6] used muons from pion decays at rest in a magnetic field of 14 200 gauss and obtained an "a-value" of -0.23 ± 0.05.

In addition to the present experiment there are three recent experiments which have considerably better

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[10] T. Kinoshita and A. Sirlin, Phys. Rev. **113**, 1652 (1959).

[11] J. I. Friedman and V. L. Telegdi, Phys. Rev. **105**, 1681 (1957).
[12] C. Castagnoli, C. Franzinetti, and A. Manfredini, Nuovo cimento **5**, 684 (1957).
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[14] P. H. Fowler, P. S. Freier, C. M. G. Lattes, E. P. Ney, and S. J. St. Lorant, Nuovo cimento **6**, 63 (1957).
[15] C. M. G. Lattes, P. H. Fowler, P. S. Freier, E. P. Ney, and S. J. St. Lorant, Bull. Am. Phys. Soc. **2**, 206 (1957).
[16] B. Bhowmik, D. Evans, and D. J. Prowse, Nuovo cimento **5**, 1663 (1957).
[17] G. B. Chadwick, S. A. Durrani, L. M. Eisberg, P. B. Jones, J. W. G. Wignal, and D. H. Wilkinson, Phil. Mag. **2**, 684 (1957).
[18] D. F. Davis, A. Engler, C. J. Goebel, T. F. Hoang, M. F. Kaplon, and J. Klarmann, Nuovo cimento **6**, 311 (1957).
[19] J. K. Bøggild, K. H. Hansen, and M. Scharff, Nuovo cimento **8**, 767 (1958).
[20] A. O. Vaisenberg and V. A. Smirnitskii, J. Exptl. Theoret. Physics (U.S.S.R.) **33**, 621 (1957) [translation: Soviet Phys.-JETP **6**, 477 (1958)].
[21] I. I. Gurevich, V. M. Kutukova, A. P. Mishakova, B. A. Nikol'skii, and L. V. Surkova, J. Exptl. Theoret. Physics (U.S.S.R.) **34**, 280 (1958) [translation: Soviet Phys.-JETP **34**(7), 195 (1958)].
[22] Kh. P. Babaian, N. A. Marutian, K. A. Matevosian, and M. G. Sarinian, J. Exptl. Theoret. Physics (U.S.S.R.) **35**, 561 (1958) [translation: Soviet Phys.-JETP **35**(8), 387 (1959)].

statistical accuracy as well as larger measured "*a* values" than previous experiments. The present experiment, the experiment of Ali-Zade et al.,[23] and the experiment of Plano et al.[24] used muons from pions which decayed at rest and used a large magnetic field in an attempt to prevent depolarization. In this experiment and in the experiment of Ali-Zade et al. nuclear emulsion in magnetic fields of 25 000 and 27 000 gauss, respectively, were used. Ali-Zade's group obtained $P_2\xi=-0.97\pm0.06$. In Plano's bubble chamber experiment the stopping substance was liquid hydrogen in a magnetic field of 7000 gauss and the result was $P_2\xi=-0.93\pm0.065$. The experiment of Bardon et al.[25] used muons from decays in flight of pions of known energy. By using those muons which make the maximum possible angle with the pion beam, muons of known energy and polarization were obtained. In this case muons were stopped in bromoform, a substance in which previous experiments had indicated, but with poor accuracy, that there is little depolarization of the muons. Their measured value was $P_2\xi=-0.97\pm0.05$.

The values for $P_2\xi$ of -0.97 ± 0.06, -0.93 ± 0.065, and -0.97 ± 0.05 obtained in these three experiments are in good agreement with the theoretical prediction that $\xi=-1$. They are also consistent with the value of -0.87 ± 0.04 obtained in this experiment.

IV. DEPOLARIZATION OF MUONS

A. Mechanisms

The mechanisms by means of which positive muons are depolarized in the material in which they are brought to rest are not very well understood. The subject of the depolarization of Fermions as they pass through matter was first given a comprehensive treatment by Wolfenstein[26] who showed that protons suffer negligible depolarization as they are slowed down and brought to rest. More recently Ford and Mullin[27] demonstrated that the depolarization of 4-Mev muons as they are brought to rest is less than 0.1%.

After the muon comes to rest depolarization may occur. Before one can understand the muon depolarization, one must know what position the muon assumes within the stopping material. This question is particularly hard to answer for an amorphous substance such as nuclear emulsion. By weight and by stopping power nuclear emulsion is predominantly silver bromide. However, in terms of numbers of atoms, nuclear emulsion is about 25% silver bromide and 75% gelatin. When a muon is nearly stopped, only the valence electrons are effective in bringing the muon to rest. Therefore, one can expect that most of the muons stop in gelatin.

The muons probably do not exist as simple muonium atoms to any appreciable extent. The experiments which have measured the precession rate of muons in nuclear emulsion did not find a precession rate corresponding to the muonium magnetic moment. This means that if the muons exist as simple muonium atoms, they are depolarized very quickly ($\sim10^{-7}$ second) by local fields, and those muons which remain polarized at zero external magnetic field are not in muonium atoms. Nevertheless, it is instructive to consider the depolarization of the muon when muonium is formed because the muonium system can serve as a model for other more complicated systems. Since no electron can be appreciably closer to the muon than the electron is in the muonium atom, the depolarization obtained for the muonium atom should give a good idea of the upper limit of the depolarization obtainable by means of this type of mechanism for any configuration of the muon.

The muonium depolarization calculation shows that for an S state muonium atom there is 50% depolarization within a time on the order of $h/\Delta E=3\times10^{-11}$ second, where ΔE is the hyperfine splitting. However, this depolarization can be reduced by the presence of a large external magnetic field. The depolarization as a function of magnetic field H (in gauss) is given[4] by

$$1-P=\frac{1}{2}\left[\frac{1}{1+(H/1600)^2}\right].$$

At 25 000 gauss the depolarization is about 0.2%. The magnetic field serves to decouple the muon and electron spins and inhibits the mixing of spin states. In a magnetic field which is very large compared with 1600 gauss the magnetic energy of the electron is large compared with the hyperfine interaction energy. However, it would take fields large compared to 10^5 gauss to make the magnetic energy of the muon larger than the hyperfine energy. Thus, the effect of a magnetic field of around 10^4 gauss is to fix the electron spin direction and thereby indirectly fix the muon spin direction by means of angular momentum conservation, rather than directly to fix the muon spin direction.

If a proton entered a solid, it certainly would not exist in the form of nascent hydrogen. Nor does the muon form an isolated muonium atom. A more reasonable expectation is that a proton would attach itself to a molecule. A muon in a hydrogen-like molecule would not depolarize at all because in the ground state of the hydrogen molecule the electron spins are paired and there is no hyperfine splitting. In order to depolarize a muon which is in a magnetic field of a few thousand gauss there must be an unpaired electron.

If this unpaired electron is in an S state, it will not

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[23] S. A. Ali-Zade, I. I. Gurevich, U. P. Dobretzov, B. A. Nikol'skii, and L. V. Surkova, J. Exptl. Theoret. Physics (U.S.S.R.) **36**, 1327 (1959) [translation: Soviet Phys.-JETP **36**(9), 940 (1959)].

[24] R. J. Plano and A. Lecourtois, Bull. Am. Phys. Soc. **4**, 82 (1959); and invited lecture by R. J. Plano at New York Physical Society Meeting, 1959 (unpublished).

[25] M. Bardon, D. Berley, and L. M. Lederman, Phys. Rev. Letters **2**, 56 (1959).

[26] Lincoln Wolfenstein, Phys. Rev. **75**, 1664 (1949).

[27] G. W. Ford and C. J. Mullin, Phys. Rev. **108**, 477 (1957).

be able to depolarize the muon to any greater extent than the muon is depolarized in the ground state of the muonium atom. Suppose, however, the electron is not in an S state. Can the addition of a spin-orbit effect enhance depolarization? Some indication of what occurs in such a situation may be obtained by an investigation of a P-state muonium atom. Though other systems may differ from a P state muonium atom in the magnitude of the fine structure and hyperfine structure splittings, the effect is qualitatively the same. However, at large fields the depolarization of the muon is much less in the $2P$ state than it is in the $1S$ state.[8] Thus, it appears that even though muonium is not formed, the muonium depolarization calculation serves as an upper limit for the depolarization which would occur by this type of mechanism no matter which molecular structure the muon joins.

When the muon is almost stopped, it can be expected to pick up an electron for a short time and then lose it and repeat this process a few times before coming to rest. This successive muonium formation has been suggested[5] as a possible explanation of the fact that the measured depolarization is considerably greater than the depolarization calculated by means of the muonium model. However, in view of the fact that the precession time for the muon in muonium is larger than the slowing down time, there is not enough time to depolarize the muon during slowing down by even as much as it would be depolarized by a single muonium formation.

After it comes to rest the muon may suffer repeated collisions with surrounding molecules, and in some of these collisions the muon and the molecule with which it collides may exchange electrons. Each such exchange is effectively the formation of a new muonium atom. Even if each electron exchange causes only a 0.2% depolarization, 50 such exchanges could cause 10% depolarization. If this mechanism were an important one, measurements of the asymmetry parameter would show a decrease as a function of the time during which the muon is at rest before it decays. The measurements of Swanson[28,29] showed that no large time dependence of the "a value" exists for nuclear emulsion. If all of the muons have the same relaxation time his experiment rules out the possibility that a 20% depolarization occurs by means of this mechanism. It does not rule out the possibility of a 10% depolarization. It is possible, however, that only some of the muons come to rest in such a position that this mechanism is effective, and for these few it could be so effective that at zero magnetic field these muons are fully depolarized in less than 10^{-7} second. If the relaxation time were this small the time dependence would not have been observed. This type of spin relaxation effect would be temperature dependent and the depolarization might be reduced by lowering the temperature of the nuclear emulsion.

[28] Robert A. Swanson, Phys. Rev. **112**, 580 (1958).
[29] Robert A. Swanson (private communication).

B. Experiments

In discussing the experiments which have measured the depolarization of muons in various materials the viewpoint will be taken here that the V-A theory of Fermi interactions is correct. Then all measurements of the muon decay asymmetry parameter can be taken as measures of the depolarization P_2 of the muons at the time of decay. The experiments which use muons from pions which decay at rest can be used directly to determine P_2. The cyclotron experiments can be used only after some measure of the cyclotron muon beam polarization is obtained. Both the Chicago and the Columbia cyclotron groups[28,30] measured the asymmetry parameter using propane and nuclear emulsion as stopping substances. These "a values" can be used to obtain the depolarization in any other stopping substance used in the cyclotron experiments by making use of the "a values" measured in nuclear emulsion and propane directly. For example, P_2 for graphite, the material for which the most accurate measurements were made, is given by

$$P_2(\text{graphite}) = P_2(\text{emulsion})\frac{a\ (\text{graphite})}{a\ (\text{emulsion})}.$$

In this manner four values of P_2(graphite) were obtained as follows: $P_2=0.72\pm0.12$ obtained from the Columbia[31] data and the emulsion world average, $P_2=0.88\pm0.13$ obtained from the Chicago data and the emulsion world average, $P_2=0.77\pm0.12$ obtained from the Chicago data and the propane world average, $P_2=0.79\pm0.11$ obtained from the Columbia[31] data and the propane world average. The average of these values is $P_2=0.79\pm0.07$. This result is at variance with the heretofore generally accepted belief that there is negligible depolarization of muons in graphite. This belief was supported by the fact that Sens et al.[5] did not succeed in increasing the asymmetry parameter with a magnetic field when graphite was used as a stopping material, whereas a marked asymmetry parameter increase was obtained for nuclear emulsion and for vitreosil (fused SiO_2). However, the accuracy of this experiment was such that a 20% effect might not have been observed. It seems unlikely that this result that there is $21\pm7\%$ depolarization in graphite is due to systematic errors in the experiments because there is such good agreement among the four ways of calculating P_2. The one systematic error which is suspected is the one mentioned in the last section that the world average of emulsion experiments may be too high. If this were so, the estimate of the depolarization in graphite would need to be even larger than this estimate of 21%.

In Table I is listed the values of P_2 for all of the

[30] Marcel Weinrich, Ph.D. thesis, Columbia University, 1958 (unpublished).
[31] The Columbia data used were the relative values tabulated in Fig. 7 of Weinrich's thesis.[30]

TABLE I. Depolarization measurements.

Stopping substance	Data used	P_2
Nuclear emulsion (zero field)	12 emulsion experiments[a]	0.333±0.033
Propane (zero field)	4 bubble chamber experiments[b–e]	0.57 ±0.05
Nuclear emulsion (14 200 gauss)	Barkas et al.[f]	0.69 ±0.15
Liquid hydrogen (zero field)	Abashian et al.[g]	0.75 ±0.14
Graphite	Cyclotron experiments plus emulsion and propane data	0.79 ±0.07
Nuclear emulsion (25 000 gauss)	Present experiment	0.87 ±0.04
Liquid hydrogen (7000 gauss)	Plano et al.[h]	0.93 ±0.065
Bromoform (zero field)	Bardon et al.[i]	0.97 ±0.05
Nuclear emulsion (27 000 gauss)	Ali-Zade et al.[j]	0.97 ±0.06

[a] See references 11–22.
[b] I. A. Pless, A. E. Brenner, R. W. Williams, R. Bizzarri, R. H. Hildebrand, R. H. Milburn, A. M. Shapiro, K. Strauch, J. C. Street, and L. A. Young, Phys. Rev. **108**, 159 (1957).
[c] M. H. Alston, W. H. Evans, T. D. N. Morgan, R. W. Newport, and P. R. Williams, Phil. Mag. **2**, 1143 (1957).
[d] V. V. Barmin, V. P. Kanavets, B. V. Morozov, and I. I. Pershing, J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 830 (1958) [translation: Soviet Phys.-JETP **34(7)**, 573 (1958)].
[e] A. I. Alkhanian, V. G. Kirillov-Ugriumov, L. P. Kotenko, E. P. Kuznetsov, and I. S. Popov, J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 253 (1958) [translation: Soviet Phys.-JETP **34(7)**, 176 (1958)].
[f] See reference 6.
[g] A. Abashian, R. K. Adair, R. Cool, A. Erwin, J. Kopp, L. Leipuner, T. W. Morris, D. C. Rahm, R. R. Rau, A. M. Thorndike, W. L. Whittemore, and W. J. Willis, Phys. Rev. **105**, 1927 (1957).
[h] See reference 24.
[i] See reference 26.
[j] See reference 25.

substances in which it has been measured directly, as well as the value of P_2 for carbon which has just been calculated. Three substances, bromoform, nuclear emulsion at 25 000 gauss, and liquid hydrogen at 7000 gauss appear to cause less depolarization than graphite. As a matter of fact, the original measurements on bromoform by the Columbia group did give a considerably larger "*a* value" than the graphite "*a* value." However, this was regarded as a statistical fluctuation rather than a real effect, and the bromoform measurements were never carried out with the precision that was used for many of the other substances. It would be instructive if someone were to do more accurate "*a*-value" measurements for bromoform to see if the asymmetry parameter for bromoform is greater than that for graphite by the factor of 1.23±0.12 which is indicated by present experiments.

In Fig. 3 is plotted the measured value of P_2 in nuclear emulsion as a function of magnetic field. Orear et al.[4] measured an "*a* value" at 9000 gauss which was nearly identical to the "*a* value" which had been measured for graphite with the same Nevis cyclotron muon beam. Therefore, a depolarization equal to that which was calculated for carbon has been plotted at this point. The data of Sens et al. were not reported as "*a*-values," but rather as the ratio of positrons counted in a fixed counter in the forward direction to muons stopping. These reported values were grouped into a few intervals and the group averages were converted to "*a* values." These "*a* values" were multiplied by three and divided by the Chicago cyclotron muon beam polarization to obtain P_2. The muon beam polarization was obtained by averaging the ratio of the propane "*a* value" measured with the Chicago cyclotron muons to the world average propane "*a* value" with the analogous ratio for nuclear emulsion data. This average is $P_1=0.83\pm0.09$. No attempt is made to fit the points with a curve. However, the depolarization expected from single muonium formation is plotted as a comparison. The data are certainly not consistent with this muonium curve.

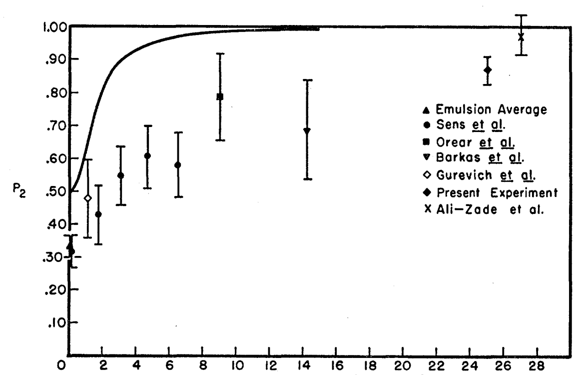

FIG. 3. Plot of P_2 for nuclear emulsion vs external magnetic field. The depolarization of muons stopping in nuclear emulsion is $(1-P_2)$.

The values of the polarization shown in Table I and in Fig. 3 are all based on the assumption that the intrinsic muon asymmetry parameter ξ is equal to minus one. However, if this is not true, then the quantity which is tabulated and plotted here is $-P_2\xi$ rather than P_2.

V. CONCLUSION

The result of this experiment that $-P\xi=0.87\pm0.04$ for muons stopping in nuclear emulsion at 25 000 gauss is more than three standard deviations away from the value of -1 which is predicted by the V-A theory for completely polarized muons. There is less than one chance in 300 in obtaining a discrepancy this large by chance alone. This means either than the theory is incorrect and that $|\xi|<1$, or that there is about 13%

depolarization of muons in nuclear emulsion at 25 000 gauss.

The experimental information available at present is still quite consistent with a value of $\xi=-0.9$. More accurate measurements are necessary to remove this ten percent uncertainty in the value of ξ. In view of the success which the V-A theory has had, the most likely explanation seems to be that the muons do depolarize even at this large field. Those depolarization mechanisms which have been treated adequately do not give rise to any appreciable depolarization at 25 000 gauss. However, the nature of depolarization mechanisms is not understood sufficiently to rule out the possibility that substantial depolarization occurs. In fact, what experimental evidence exists supports this conclusion.

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Low-Energy Pion Phenomena*

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The relation between low-energy pion-nucleon scattering and pion photoproduction is examined. Correct extrapolation to threshold of both the π^+ and π^- photoproduction data gives agreement with theory. A recent new method for analyzing the scattering data is applied giving $a_1=0.178$, $a_3=-0.087$, and reasonable agreement with the Panofsky ratio $P=1.5$ is obtained. An inner Coulomb correction to the scattering data helps to improve this agreement. The possibility of detecting a $\pi-\pi$ interaction by low-energy pion scattering is examined. A new dispersion relation connects the s- and p-wave phase shifts at low energies; this relation excludes some well-known sets of phase-shift curves.

I. INTRODUCTION AND SUMMARY

THE violation of the well-known connection between low-energy pion scattering and threshold pion photoproduction via the Panofsky ratio have given some stimulus to theoretical studies of these low-energy phenomena.

In 1958 the situation was clarified by Cini et al.,[1] who asserted that the data were in agreement with a Panofsky ratio $P=1.5$ and a threshold π^-/π^+ ratio $r=1.3$. This agreement was achieved by two steps:

(i) Following a suggestion of Bernardini,[2] the extrapolation of the π^+ photoproduction cross section to threshold was improved by allowing for the retardation term. This *increased* the threshold value.

(ii) It was suggested that the pion scattering crossing relations gave a new plot for the scattering phase shifts. This led to the very low value $a_1-a_3=0.24$ (in units $\hbar=c=\mu=1$) where a_1, a_3 are the $T=\frac{1}{2}$ and $T=\frac{3}{2}$ scattering lengths.

A brief survey of the data and of these arguments is given in Sec. II below. Comments on this scheme include the following:

(a) Beneventano et al.[3] asserted that the increased threshold value for π^+ photoproduction was now in disagreement with the threshold photoproduction measurements of Adamovič et al.[4] (using $\gamma+D$) if we wished to retain $r=1.3$. We show in Sec. II that on using the correct extrapolation for both the π^+ and π^- photoproduction data, and using the correct values of Adamovič's results, this difficulty disappears. For this extrapolation we use the dispersion relation of Chew, Goldberger, Low, and Nambu.[5]

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[1] M. Cini, R. Gatto, E. L. Goldwasser, and M. A. Ruderman, Nuovo cimento **10**, 242 (1958).

[2] G. Bernardini, Suppl. Nuovo cimento **1**, 104 (1955).

[3] M. Beneventano, G. Bernardini, G. Stoppini, and L. Tau, Nuovo cimento **10**, 1109 (1958).

[4] See *1958 Annual International Conference on High-Energy Physics at CERN*, edited by B. Ferretti (CERN Scientific Information Service, Geneva, 1958), p. 49.

[5] G. F. Chew, M. L. Goldberger, F. Low, and Y. Nambu, Phys. Rev. **104**, 1345 (1956).